ADAPTIVE FILTERS DESIGN AND ANALYSIS USING LEAST SQUARE AND LEAST PTH NORM

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ABSTRACT

Adaptive filters are considered nonlinear systems; therefore their behavior analysis is more complicated than for fixed filters. As adaptive filters are self-designing filters, their design can be considered less involved than in the case of digital filters with fixed coefficients. This paper presents simulation of Low Pass FIR Adaptive filter using least mean square (LMS) algorithm and least Pth norm algorithm. LMS algorithm is a type of adaptive filter known as stochastic gradient-based algorithms as it utilizes the gradient vector of the filter tap weights to converge on the optimal wiener solution whereas Least Pth does not need to adapt the weighting function involved and no constraints are imposed during the course of optimization. The performance of both approaches is compared.

KEYWORDS: *Adaptive filters, FIR, Least Pth norm, LMS, Matlab.*

I. INTRODUCTION

Adaptive filter is a filter that self-adjusts its transfer function according to an optimization algorithm driven by an error signal. Because of the complexity of the optimization algorithms, most adaptive filters are digital filters. An adaptive filter is required when either the fixed specifications are unknown or the specifications cannot be satisfied by time-invariant filters [1]. An adaptive filter is a nonlinear filter since its characteristics are dependent on the input signal. However, if we freeze the filter parameters at a given instant of time, than adaptive filters considered are linear in the sense that their output signals are linear functions of their input signals[2]. As the signal into the filter continues, the adaptive filter coefficients adjust themselves to achieve the desired result, such as identifying an unknown filter or canceling noise in the input signal. Adaptive filtering can be considered as a process in which the parameters used for the processing of signals changes according to some criterion. Adaptive filters are dynamic filters which iteratively alter their characteristics in order to achieve an optimal desired output. An adaptive filter algorithmically alters its parameters in order to minimize a function of the difference between the desired output and its actual output. To define the self-learning process, select the adaptive algorithm used to reduce the error between the output signal y(k) and the desired signal d(k) [3].There are various algorithms involved for the filtering depending upon the applications and the requirements. To construct an adaptive filter it has to be considered that which method is to be used to update the coefficients of selected filter and whether to use an IIR filter or FIR filter. For designing an adaptive filter algorithm plays a vital role. The algorithm has to be practically implemented, has to adapt the coefficients quickly and provide the desired performance.

The paper provides a logical organization; a top-down approach is used. Firstly a general idea regarding adaptive filters is provided; than various algorithms involved in designing of adaptive filters are discussed, from which least square and least Pth norm algorithms are described. Further a low pass FIR adaptive filter is proposed using Least Pth norm algorithm and the results are compared with least square algorithm which is then provided with conclusion and future directions.

II. ADAPTIVE FILTERING ALGORITHMS

Adaptive filtering can be classified into three categories: adaptive filter structures, adaptive algorithms, and applications. The choice of algorithm is highly dependent on the signals of interest, the operating environment, as well as the convergence time required and computation power available. An adaptive digital filter can be built up using an IIR (Infinite impulse response) or FIR (Finite impulse response) filter. Adaptive FIR filter structure is most commonly used adaptive FIR filter structure and is the transversal filter which implements an all-zero filter with a canonic direct form (without any feedback). FIR is inherently stable because its structure involves forward paths only, no feedback exists. The presence of feed back to the input may lead the filter to be unstable and oscillation may occur. For this adaptive FIR filter structure, the output is a linear combination of the adaptive filter coefficients. Alternative adaptive FIR filter structures improve performance in terms of convergence speed [4].For simple implementation and easy analysis; most adaptive IIR filter structures use the canonic direct form realization. Some other realizations are also presented to overcome some drawbacks of canonic direct form realization, like slow convergence rate and the need for stable monitoring.

An algorithm is a procedure used to adjust adaptive filter coefficients in order to minimize the cost function. The algorithm determines important features of adaptive procedure, such as computational complexity, convergence to suboptimal solutions, biased solutions, objective cost function and error signal. The algorithm used in equalization is LMS and is known for its simplification, low complexity and better performance in different running environments [5]. Further symmetric approach can be employed to reduce the complexity with partial serial MAC based approach to optimize speed and area [6]. Fractionally spaced equalizer (FSE) can be used to compensate for channel distortion before aliasing effects occur due to symbol rate sampling. FSE is used to reduce computational requirements and to improve convergence [7].

Further Fast Block Least Mean Square (FBLMS) is one of the fastest and computationally efficient adaptive algorithms. Distributed Arithmetic further enhances the throughput of FBLMS algorithm with reduced delay, minimum area requirement and reduced hardware multipliers. Distributed arithmetic (DA) is a bit level rearrangement of a multiply accumulate to hide the multiplications [8]. But the reduced hardware complexity of higher order filters was at the expense of increased memory and adder requirement. And the technique is suitable for higher order filters. It is a powerful technique for reducing the size of a parallel hardware multiply-accumulate that is well suited to FPGA designs. DA is one of the efficient techniques, in which, by means of a bit level rearrangement of a multiply accumulate terms; FFT can be implemented without multiplier.

The unconstrained optimization problem of Non-recursive filter to minimize the difference between actual and desired response of magnitude is solved using least squares design method for L2p norm [9]. Least square error design method for the optimal design of FIR filter showed that as the order of the filter is increased the ripple content in the stop band diminishes. Also the design using least Pth norm showed that the ripple content disappears and smoothen the response and give a constant response in stop band. The least Pth norm method doesn't need to update weighting functions, no constraints are imposed and design can start anywhere in parameter space [10]. Mixed-norm digital filter design methods provide the capability to design filters that have minimum deviation in the pass bands (using the $L\infty$ norm) and minimum broadband noise power in the stop-bands (using the L_2 norm). Filters that tradeoff between these two extremes (e.g., L_4 norm) are also possible [11].The method allows for the rapid design of mixed-norm FIR filters by using an unconstrained optimization method.

III. LEAST SQUARE AND LEAST PTH NORM

When designing systems, it is important to have a systematic approach so that the design can be done timely and efficiently, which ultimately leads to lower cost. Among different algorithms for updating coefficients of an adaptive filter, LMS algorithm is used more because of its low computational processing tasks and high robustness. This algorithm is a member of stochastic gradient algorithm. It uses Mean Square Error (MSE) as a criterion. LMS uses a step size parameter, input signal and the difference of desired signal and filter output signal to frequently calculate the update of the filter coefficients set. The convergence time in case of LMS depends upon the step size parameter. If step size is small it will take long convergence time and smaller MSE. On the other hand large step size results faster convergence but large MSE. But if it is too large it will never converge. Thus the choice of step size determines the performance characteristics of adaptive algorithm in terms of convergence rate and amount of steady-state mean square error (MSE). The performance of LMS is a tradeoff between step size and filter order. The performance is also a tradeoff between convergence rate and MSE. To eliminate the tradeoff between convergence rate and MSE, one would use a variable step-size [12]-[13]-[14].

The commonly used algorithm LMS provides low complexity and stability. Further the need of filter to minimize the difference between actual and desired response of magnitude is solved using least Pth design method. But for FIR filters to a target frequency response one can apply a rectangular window to the impulse response. However, the resulting ringing is usually not acceptable and is not an optimal choice. For matching non-noisy target frequency responses, Least Pth is considered. The Pth optimization as a design tool is not new. It was used quite successfully for the minimax design of IIR filters. The method does not need to update the weighting function, and it is an unconstrained convex minimization approach. The approach has advantages as filter quality, mathematical verification of the properties such as causality, stability, etc using the pole zero and magnitude plots. The Least Pth norm algorithm has a larger gradient driving it to converge faster when away from the optimum. However, the LMS will have more desirable characteristics in the neighborhood of the optimum. The Least Pth norm algorithm is defined by the following cost function:

$$Jn = E[en^p] \tag{1}$$

Where the error $en = dn + wn - cTnxn$ (2)

dn is the desired value, **c***n* is the filter coefficient of the adaptive filter (with **c***opt* is its optimal value), **x***n* is the input vector and *wn* is the additive noise.

IV. SIMULATION RESULTS

The optimal design of FIR filter using least Pth norm is implemented under MATLAB and is compared with least square algorithm. The filters vary in terms of desired filter characteristics and consequently in the number of coefficients depending upon the order of the filter. Simulation results are presented for the case of ten coefficient filter and a twenty coefficient filter. Comparisons are made with the Least square and least Pth norm algorithms. Figure 1, 2 and 3 shows the simulated results for 10 coefficients Low pass FIR filter Figure 4, 5 and 6 shows the simulated results for 20 coefficient Low pass FIR filter. Figure 1 and 4 shows magnitude response with the sample frequency of 48 KHz for 10 coefficient and 20 coefficient Low pass FIR filter, Figure 2 and 5 shows pole/zero plot specifying the stability aspect for 10 and 20 coefficient filters. The magnitude response shows that filter implemented using least Pth norm with (p=4) converges faster and the filter implemented using least mean square converges slow. As the value of p increases the ripples are smooth.

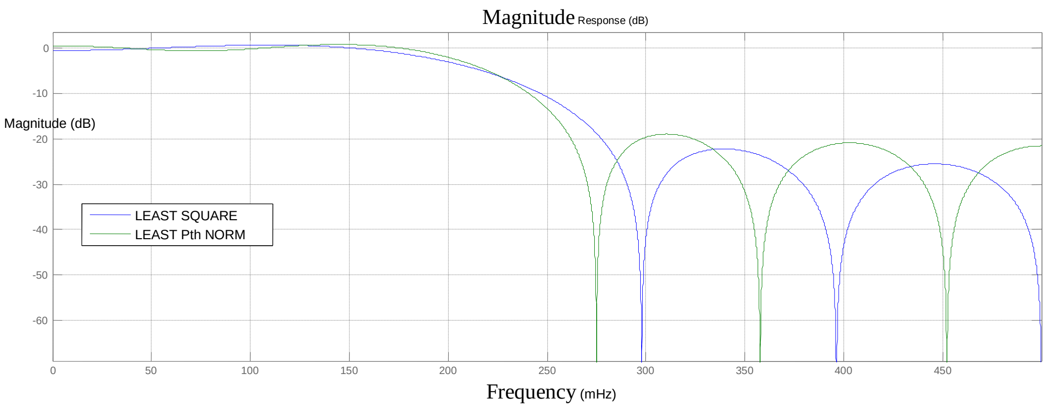

Figure 1 Magnitude Response (10 coefficient)

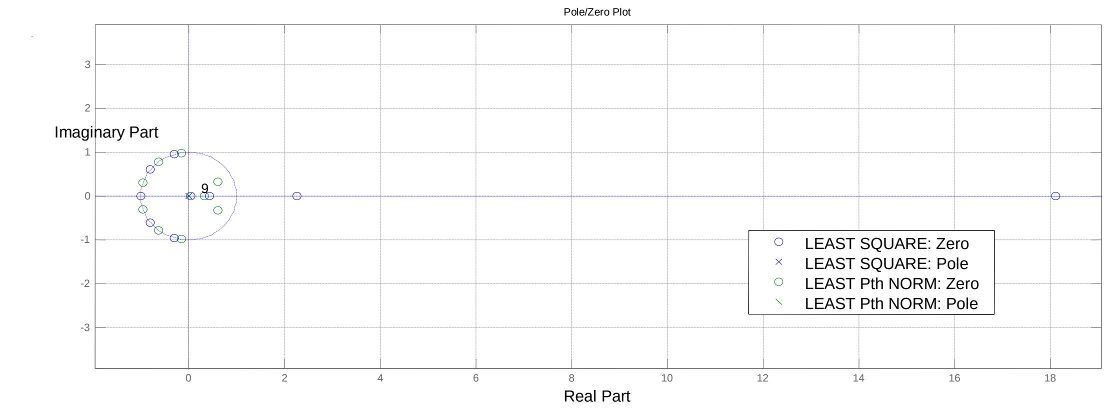

Figure 2 Pole and Zero Plot (10 coefficient)

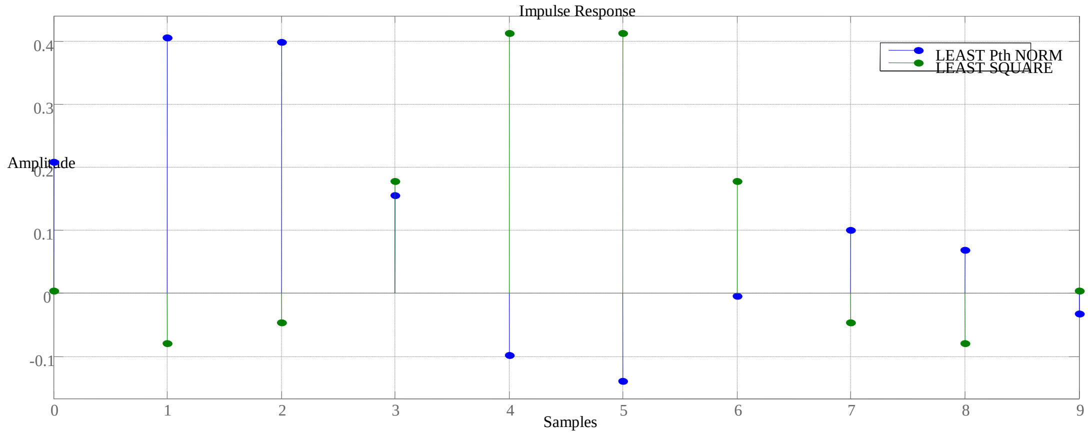

Figure 3 Impulse Response (10 coefficient)

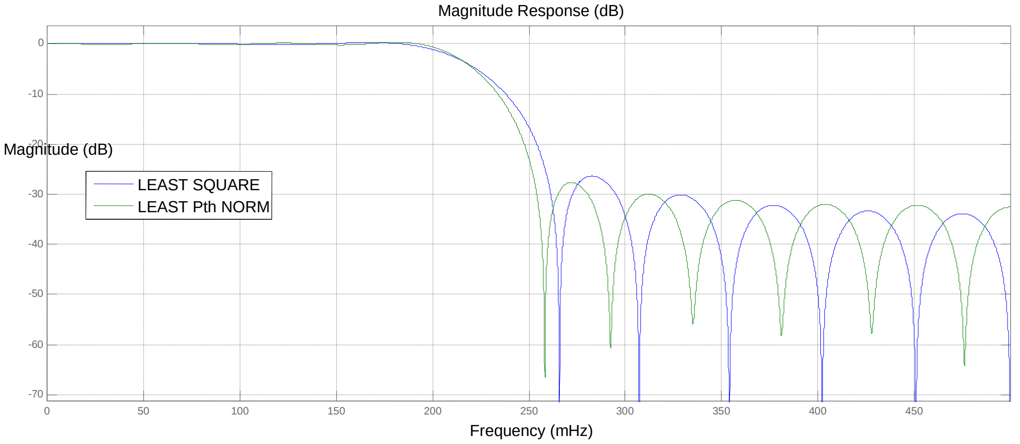

Figure 4 Magnitude Response (20 coefficient)

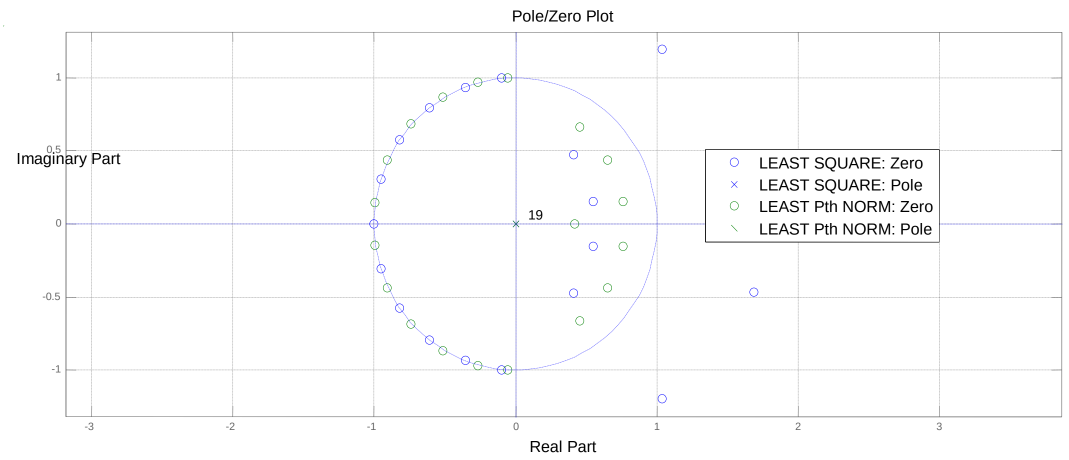

Figure5 Pole and zero plot (20 coefficient

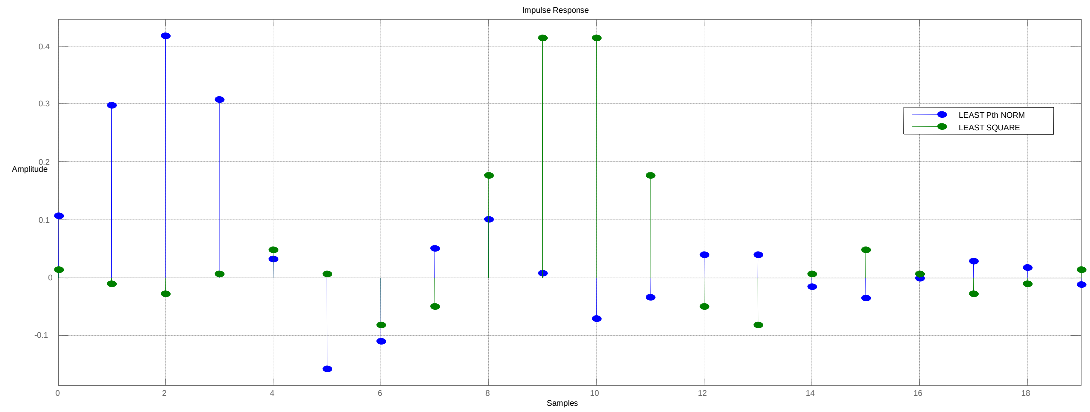

Figure 6 Impulse Response (20 coefficient)

Table: 1 Simulation results of Least Square and Least Pth

Description	Least-square		Least Pth Norm	
	10 coefficients	20 coefficients	10 coefficients	20 coefficients
Filter Gain	.014	.004	.208	.107
Magnitude (at 10KHz)	-3.9	-2.4	-3.05	-1.8
Phase (at 10KHz)	-5.9	-12.4	-2.3	-3.3

V. CONCLUSION

The aim of the paper was to compare the performance parameters of Least square and Least Pth Norm algorithm for adaptive filtering. Here two Low pass FIR filters using 10 coefficients and 20 coefficients were simulated using MATLAB. As the coefficient order increases both in Least square or Least Pth norm the multipliers, adders, multiplications per unit and additions per input sample increases. The Least Pth provided the better results. The least Pth provided better gain as compared to least square. The ripple content disappears in a similar way in both the cases but in case of least Pth norm ripples smoothen the response and give a constant response in stop band.

VI. FUTURE SCOPE

Least Pth norm can be used to design optimal FIR filters using optimization algorithms for both linear phase and non-linear phase.

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