Inflation and Earnings Uncertainty and Volatility Forecasts: A Structural Form Approach*

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Abstract

We propose a new structural form methodology for understanding the fluctuations and predictability of volatilities and covariances of asset returns. The methodology is applied to a model in which investors learn about the joint movements in inflation and real earnings through business cycles. The econometrician extracts investors' beliefs about fundamentals from the prices and volatilities of stocks and bonds. The duration of episodes of enhanced investor uncertainty are forecastable and lead to forecasts of volatilities that are more precise than those obtained from a reduced form specification that includes lagged volatility and several macroeconomic variables. The model's success stems largely from endogenous and time-varying weights that investors assign to news about real fundamental growth and discount rates.

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Introduction

One of the most compelling findings in empirical finance research over the past three decades is that volatilities of asset returns are highly persistent. The persistence property has been exploited by the GARCH family of models that uses the information in current volatilities to forecast future volatilities and has revolutionized the practice of portfolio management, risk management, and derivatives.[1] Despite the success of the volatility persistence paradigm, the empirical literature on the economic mechanisms that cause this persistence is still at a relatively early stage. In addition, the GARCH literature says little about the levels of asset prices themselves. In fact, an even earlier school of thought starting with the work of Hayek (1945) and in its modern form under the Efficient Markets Hypothesis of Fama (1970) argues that asset prices aggregate investors' information. In its *weak form*, the theory asserts that the information contains the history of past prices and volatilities. Therefore, the information in GARCH forecasts is a subset of all the information in prices relevant for volatility forecasts. Since investors are likely risk averse, their forecasts of volatility will affect current market clearing prices. Therefore, in principle, an econometrician should be able to back out the volatility forecasts of investors from asset prices. In this paper, we propose a structural form approach to volatility forecasting that explicitly extracts investors' volatility forecasts from asset prices and show that these forecasts substantially improve upon those obtained from a reduced form specification that includes lagged volatility and several macroeconomic variables.

The canonical GARCH method of volatility forecasting is

$$f(\sigma^2(t)) = \beta_0 + \beta_1 f(\sigma^2(t-1)) + \beta_2 X(t-1) + \epsilon(t), \tag{1}$$

where $\sigma^2(t)$ is the variance of the asset in period t, $f(\cdot)$ is a possibly nonlinear function often used to make variance positive in this equation, $X(t)$ is a vector of explanatory variables, and and $\epsilon(t)$ is the shock to variance at time t. We call (1) the reduced form approach to volatility forecasting. The vector $X(t)$ typically contains macroeconomic or price (returns) information. For example, in a reduced form setup, Glosten, Jagannathan, and Runkle (1993) show that short-term interest rates positively forecast stock market volatility. Another popular X variabiable is asset returns, especially in periods when these returns are negative, resulting in the so-called "leverage effect" [see, for example, Black (1976)]. Schwert (1989) shows that recession dummies can forecast stock volatility better than several other X variables, and Gennotte and Marsh (1993) find predictive power in lagged volatilities of fundamentals. More recently the literature has moved towards studying the

[1] The concept of ARCH (Autoregressive Conditional Heteroskedasticity) modeling was introduced in Engle (1982) and extended to GARCH in Bollerslev and Engle (1986). See Bollerslev, Chou, and Kroner (1992) for a relatively early survey of its applications and Anderson, Bollerslev, and Diebold (2004) for more recent advances.

role of macroeconomic factors in understanding the observed nonlinearities in volatility dynamics. For example, Boyd, Hu, and Jagannathan (2005) and Anderson, Bollerslev, Diebold, and Vega (2007) show that the response of volatility to macroeconomic news depends on the state of the business cycle, so that $X(t)$ should contain more complicated functions of real fundamentals that can capture the conditional state of the economy.

We raise three questions with respect to such forecasting models. First, what is the role of including lags of variances in the equation? Do the lags capture the effects of persistence in the explanatory variables $X(t)$, or are they a parsimonius way to capture the effect of all missing explanatory variables? Second, what is the explicit mechanism by which these X factors impact volatility? For example, despite there being fairly strong evidence for the effect of interest rates on volatility, the explanations offered for the significance are anecdotal and not precisely quantified. Finally, why do various interest rates measures that proxy for monetary policy and are perhaps the best predictors of the macroeconomy [see, for example Estrella and Mishkin (1998)] fail to be sufficient for forecasting volatility on economy-wide stock indexes, leaving room for other variables such as returns and the current state of the economy to provide incremental forecasting power?

To address these questions we propose a new and different structural form approach to volatility forecasting. In contrast to the GARCH literature, our approach starts from an economic specification of fundamentals and investors' pricing kernel – the main ingredients of an asset pricing model – and explicitly solves for asset price levels and their volatilities in closed-form as functions of the state variables of the model. The model is then fit not only to fundamental data, but also to asset price levels and their volatilities with the use of overidentifying moments. Thus, in our framework the impact of the X variables on volatility is explicit and completely specified. It is important to note that our approach is different from the traditional GARCH literature in not only specifying a functional form for the volatilities of assets returns, but also in the way we fit our model parameters to financial data. In the former literature, the volatility of historical asset returns is not observed by the econometrician, who formulates alternative theoretical dynamic relationships for the volatility process to maximize the likelihood of observing the asset returns process. We instead follow an alternative approach in the volatility literature of treating realized volatility as an observed process.[2] This permits us to use volatility as an overidentifying moment in our estimation procedure described above. Forecasts of volatility can be optimally constructed using Monte Carlo simulations using the assumed dynamics of the state variables of the economy and the estimated parameters of the structural form model. In addition, our estimation procedure allows for the econometrician's information set to be quite different from that of investors, by allowing for signals and state variables being observed only by the latter. Thus, in our model, prices aggregate the signals received by investors as discussed in the opening paragraph.

The economic mechanism in play in our analysis builds on the theoretical Bayesian learning models of David (1997), Veronesi (1999), and Veronesi (2000), but significantly generalizes these models by building in a joint macroeconomic relationship between nominal and real variables. Suppose that inflation and the growth rates of real fundamentals in the economy jump erratically between states and are not observable by investors. Instead, investors must learn about these shifts from observations of fundamental growth, prices, and other signals. The states of inflation and real growth are not independent, and information on the state of inflation helps investors formulate more precise forecasts of real growth (and vice versa). Due to the noise in the fundamental processes, investors take time to learn about these shifts, and during this learning period they give more weight to news and update their beliefs more rapidly. Asset prices, as the expected discounted value of future fundamentals, will thus also become more volatile. If an econometrician is able to estimate the frequency of shifts in average growth rates and the level of noise in fundamentals, then she can

[2] In this sense we follow the approach of Schwert (1989), who approximates volatility at the monthly and quarterly frequencies as the squared daily average returns in these periods, but does not have a structural specification of volatility. More recently, there has been increasing interest in using higher frequency intraday data to approximate volatility in any given period [see, for example, Anderson, Bollerslev, Diebold, and Labys (2003)]. We continue to use daily returns to construct our volatility measures since we are interested in studying the relationship between volatility and the macroeconomy for a long sample during which intradaily data are not available.

predict how long the episode of increased uncertainty (volatility) will persist.[3] Similarly, in periods of low uncertainty, she can estimate the time for the next shift and episode of high volatility. Since investors' beliefs of the state of the fundamentals are common factors in the valuation of stocks and bonds, covariances between their returns are also forecastable. In our empirical estimation, we find that shifts of fundamental average growth rates occur at the business cycle frequency, causing forecastability of volatilities and covariances up to a year.

The biggest success of our structural form model is in forecasting stock market volatility one year ahead. Our model-based forecast doubles (from 18% to 38%) the explained future variation of stock volatility relative to reduced form models that include lags and a host of macroeconomic control variables, such as interest rate measures, fundamental volatilities, the state of the economy, stock returns, and survey-based measures of investors' uncertainty. The forecast $\bar{R}^2$ improves to 47% if the fourth quarter of 1987 in excluded, since the extreme volatility following the crash of 1987 seems to be largely unrelated to fundamentals. The set of controls used represents almost all the variables used in the volatility forecasting literature. It is important to emphasize why our model's forecast is more successful than these other forecasts. First, our model utilizes the information aggregation property of asset prices to capture the forward-looking information of investors in the economy. Second, our structural model endogenously determines time-varying relative weights given to news about inflation and earnings growth through different stages in the business cycle, as it has been found by other authors. This relative weighting is important for our model to match volatility fluctuations, because as we show, pure measures of inflation and earnings uncertainty

[3]By "uncertainty" we mean a situation in which investors are less than perfectly sure of some aspect of the conditional data generating process (DGP) in any time period. In the model in this paper, investors only have uncertainty of the drifts of the fundamental processes in the economy, which jump between a finite set of states. Investors are Bayesian, and update their beliefs from observations of fundamentals and other signals. This definition of uncertainty is different from its use in the work of Bansal and Yaron (2004), who refer to uncertainty as the conditional volatility of consumption growth, which affects risk premiums in asset pricing models. In complete information settings similar to our paper, several authors have written regime-switching models of the term structure and asset returns [see, for example, Bansal and Zhou (2002), Ang and Bekaert (2002), Dai, Singleton, and Yang (2003), Guidolin and Timmermann (2008)]. Our work is different in two respects: (i) we do not model regimes of interest rates, but instead fundamental regimes (both real and inflationary), and (ii) the regimes are unobserved, so that volatility of interest rates is endogenously determined in contrast to the above papers. Our definition is similar to, but distinct from the ambiguity literature [see, for example, Gilboa and Schmeidler (1989) or Epstein and Wang (1994)] in which investors do not have perfect knowledge of the DGP as in our model, but are also unable to put conditional probabilities on the different possibilities. In such situations, investors are not Bayesian.

have low explanatory power for explaining stock volatility fluctuations. Finally, the model helps to tease out investors' expectations and uncertainty of fundamentals on a finer filtration than the simple boom/bust states of a macroeconomic business cycle. In particular, our model calibration shows that an important component of volatility, particularly in the 1960s and the late 1990s, has been the possibility of the U.S. economy entering a new growth phase, which has also led to lofty stock price valuations. The bursts of volatility in these episodes defies the common wisdom that volatility is negatively correlated with stock prices, the so-called leverage effect. Our model naturally generates this positive conditional correlation between stock prices and volatility, as investors' upward revision of their belief in a "new economy" regime raises both the expected earnings growth (which pushes up prices) and the uncertainty about such growth (which pushes up volatility).

The model is successful in forecasting bond volatility as well, although its performance is less spectacular than for stocks. In particular, model-implied bond volatility explains 73% and 66% of the variation of one-year and five-year bonds, respectively. This is an increase in R^2 of 10% and 6% over lagged volatility, respectively. However, unlike the case of stocks, a forecast regression with both model-implied volatility and lagged volatility results in both variables being significant, showing that our model does not capture all of the factors that affect bond return volatility. We also find though that when we regress realized volatility on our model-based volatility forecast and a set of macro variables, our variable keeps its strong statistical significance, while other macro variables often become insignificant. This shows that our model-based forecast captures most of the information in macroeconomic variables, although not all.

Similar findings apply to the conditional covariance between stocks and bonds. In short, our model-implied forecast of covariance alone explains 60% and 50% of the variation of realized covariance for one-year and five-year bonds, respectively. In addition, it is always highly significant in a regression with both lagged covariances and other macro variables, although some of these other variables are significant as well. The main difficulty of the model, we find, is to explain the strong negative covariance between stocks and bonds in the beginning of the current decade. Although our fitted model does generate negative covariance, the level is far smaller than that realized in the data. Outside of this period, our model forecasts covariances over the remaining 40 years of our sample quite accurately. In line with our results, Baele, Bekaert, and Inghelbrecht (2007) are also unable to explain the realized negative covariance during this period using a broader set of macroeconomic variables than our paper, but with a different empirical methodology that does not explicitly extract information from asset prices. Following the suggestion in their paper we pursue the alternative channels of "flight-to-safety" and "flight-to-liquidity" to explain the negative covariance in this period, but are unable to explain its extreme movements with popular measures of these variables. We provide a further interpretation of this finding in the conclusion.

The paper develops as follows. In Section 1 we provide our economic model of fundamentals, asset prices, and asset price volatilities. In Section 2 we describe our empirical methodology to estimate the model and discuss its implications for the time series of fundamentals and asset prices. Section 3 contains our main findings: it performs the empirical tests on the forecastability of volatility and the covariation of asset returns. In Section 4, we provide a discussion of the three-way empirical relationships between fundamental uncertainty, asset volatilities, and valuation ratios. Section 5 concludes. An appendix contains the details of our empirical methodology to estimate the parameters of our continuous time model with discrete data using the Simulated Method of Moments (SMM).

1. Structure of the Model

In this section we describe the structural model, which is fully described by Assumptions 1 through 6. The model has several of the features in David (2006), who introduces a model to price defaultable bonds.

Assumption 1. The price of the single homogeneous good in the economy, Q_t, evolves according to the process

$$\frac{dQ_t}{Q_t} = \beta_t \, dt + \sigma_Q \, dW_t, \tag{2}$$

where $W_t = (W_{1t}, W_{2t}, W_{3t}, W_{4t})'$ is a four-dimensional vector of independent Weiner processes, the 1×4 constant vector $\sigma_Q = (\sigma_{Q,1}, \sigma_{Q,2}, 0, 0)$ is assumed to be known by investors, and the process followed by β_t is described below.

Assumption 2. Real earnings, E_t, evolves according to the process

$$\frac{dE_t}{E_t} = \theta_t \, dt + \sigma_E \, dW_t, \tag{3}$$

where the 1×4 constant vector $\sigma_E = (0, \sigma_{E,2}, 0, 0)$ is assumed known by investors and the process for θ_t is described below.

Assumption 3. Investors observe unbiased signals, S_t, of the drift rate of earnings that follow the process

$$\frac{dS_t}{S_t} = \theta_t \, dt + \sigma_S \, dW_t, \tag{4}$$

where the 1×4 constant vector $\sigma_S = (\sigma_{S,1}, \sigma_{S,2}, \sigma_{S,3}, \sigma_{S,4})$ is assumed known by investors. The signals are not observed by the econometrician.

We use these fundamentals and the signal process to price all assets in our model. In order to do so, we need to specify a stochastic discount factor to be used to discount real payoffs. We make the following assumption:

Assumption 4. There exists a real pricing kernel M_t taking the form

$$\frac{dM_t}{M_t} = -k_t dt - \sigma_M \, dW_t, \tag{5}$$

where $\sigma_M = (\sigma_{M,1}, \sigma_{M,2}, \sigma_{M,3}, \sigma_{M,4})$ is a 1×4 constant vector of the market prices of risk, and $k_t = \alpha_0 + \alpha_\theta \, \theta_t + \alpha_\beta \, \beta_t$ is the real short rate of interest conditional on the hidden state variables θ_t and β_t. The kernel M is used to price real claims and determine the expected real returns of all securities. We restrict the real rate of interest to be a linear function of the two (hidden) state variables of the model.

The kernel is observed by investors but not by the econometrician. Using it, agents price a generic real payout flow of $\{D_t\}$ as

$$M_t P_t = E\left[\int_t^\infty M_s \, D_s \, ds | \mathcal{F}_t\right], \tag{6}$$

where $\mathcal{F}_t$ is investors' filtration to be described in Assumption 5. As in several recent papers [for example, Berk, Green, and Naik (1999) and Brennan and Xia (2002)], we specify an exogenous pricing kernel process to formulate equilibrium relationships among endogenous financial variables. The linear dependence of the real rate on the drifts of fundamentals has a theoretical basis in general equilibrium models. For example, in a Lucas (1978) economy where investors have power utility $U(C,t) = e^{-\phi t} \frac{C^{1-\gamma}}{1-\gamma}$, we would have $C_t = E_t$, $M_t = U'(E_t)$, and hence $k_i = \phi + \gamma \theta_i + \frac{1}{2}\gamma(1-\gamma)\sigma_E \sigma_E'$ and $\sigma_M = \gamma \sigma_E$. In this case, the real rate is not affected by the inflation drift, that is, $\alpha_\beta = 0$. Our specification generalizes the real rate process to economies where expected inflation affects the real cost of borrowing. This specification of the real rate can be supported in an equilibrium if we allow a storage technology that can be bought or sold, is in perfectly elastic supply, and offers a safe instantaneous return of k_t. We provide further motivation for its dependence on expected inflation next.

The literature on the effect of expected inflation on the real rate of return is extensive, and it is beyond the scope of this paper to build in all the effects without significantly complicating our analysis. David (2006) provides further description of these effects, some of which we describe below. At the micro level there is a fairly large literature on various tax and accounting channels through which expected inflation affects the real return on capital [Feldstein (1980a), Feldstein (1980b), Auerbach (1979), and Cohen and Hassett (1999)]. Among the several monetary channels that lead to the non-neutrality of money, of particular relevance are cash-in-advance models in which expected inflation is a tax on money balances and raises the real cost of transactions, thus affecting the real interest rate, capital accumulation, and business cycle variation [see Cooley and Hansen (1989) for an empirical analysis of the size of this effect]. Given the multiplicity of channels that affect the relationship between expected inflation and the real short rate, we treat the sign and size of this relationship as an empirical question that can be estimated from the joint time-series variation of fundamentals and asset prices. Our chosen functional form of the real rate can be seen as a reduced form for the above noted effects.

For notational convenience, we stack the fundamental processes (2) and (3). Let $Y_t = (Q_t, E_t)'$, so that

$$\frac{dY_t}{Y_t} \;=\; \varrho_t\, dt \;+\; \Sigma_2\, dW_t, \tag{7}$$

where $\frac{dY_t}{Y_t}$ is to be interpreted as "element-by-element" division, $\varrho_t = (\beta_t, \theta_t)'$, and $\Sigma_2 = (\sigma_E', \sigma_Q')'$ Similarly, we find it useful to add the signal and kernel processes to the stacked vector and write $X_t = (Q_t, E_t, M_t, S_t)'$, which has the drift vector $\nu_t = (\beta_t, \theta_t, -k_t, \theta_t)'$, and volatility matrix $\Sigma = (\sigma_Q', \sigma_E', -\sigma_M', \sigma_S')'$.

Assumption 5. The drift vector ν_t follows an N-state, continuous-time finite state Markov chain with generator matrix Λ, that is, over the infinitesimal time interval of length dt

$$\lambda_{ij} dt \;=\; \text{prob}\left(\nu_{t+dt} \;=\; \nu_j | \nu_t = \nu_i\right), \quad \text{for} \quad i \neq j, \qquad \lambda_{ii} = -\sum_{j\neq i} \lambda_{ij}.$$

The transition matrix over states in a finite interval of time, s, is $\exp(\Lambda s)$ [see, for example, Karlin and Taylor (1982)].

Assumption 6. Investors do not observe the realizations of ν_t but know all the parameters of the model.

Since investors do not observe ν_t, they need to infer it from the observations of past earnings, inflation, the signal, and the pricing kernel. This learning process will generate a distribution on the possible states $\nu_1, ..., \nu_N$ that in turn generates changes in "uncertainty" as they learn about the current state. At time t, investors' distribution over hidden states is summarized by the posterior probabilities

$$\pi_{it} \;=\; \text{prob}(\nu_t = \nu_i | \mathcal{F}_t),$$

where $\mathcal{F}_t$ is the filtration generated by observing the entire path $(X_s)_{0 \le s \le t}$. Let $\pi_t = (\pi_{1t}, ..., \pi_{Nt})$ be the vector of beliefs.

Lemma 1. *Given an initial condition $\pi_0 = \hat{\pi}$ with $\sum_{i=1}^N \hat{\pi}_i = 1$ and $0 \le \hat{\pi}_i \le 1$ for all i, the probabilities π_{it} satisfy the N-dimensional system of stochastic differential equations:*

$$d\pi_{it} = \mu_i(\pi_t)dt + \sigma_i(\pi_t)d\tilde{W}_t, \tag{8}$$

$$\textit{in which} \quad \mu_i(\pi_t) = [\pi_t \Lambda]_i, \quad \sigma_i(\pi_t) = \pi_{it}\,[\,\nu_i - \overline{\nu}(\pi_t)]'\,\Sigma'^{-1}, \tag{9}$$

$$\overline{\nu}(\pi_t) = \sum_{i=1}^N \pi_{it}\,\nu_i = E_t\left(\frac{dX_t}{X_t}|\mathcal{F}_t\right), \quad \textit{and}$$

$$d\tilde{W}_t = \Sigma^{-1}[\frac{dX_t}{X_t} - E\left(\frac{dX_t}{X_t}|\mathcal{F}_t\right)] = \Sigma^{-1}\,(\nu_t - \overline{\nu}(\pi_t))dt \;+\; dW_t. \tag{10}$$

Moreover, for every $t > 0$, $\sum_{i=1}^N \pi_t = 1$.

Proof. See Wonham (1964) or David (1993).

The filtering theorem for jumps in the underlying drift was first derived (to the best of our knowledge) in Wonham (1964). David (1993) provides a proof using the limit of Bayes' rule in discrete time. The first application of this theorem in financial economics, as well as several properties of the filtering process, are derived in David (1997). In a parallel development in mathematical finance, Elliott, Aggoun, and Moore (1995) provide results for this filtering process under an alternative equivalent measure that simplifies the filtering process for some purposes.

We make the following summary comments about the updating process (8): (i) The elements of the generator matrix, Λ, completely capture the transition probabilities between states. Absent any new information, beliefs tend to mean-revert to the unconditional stationary probabilities that are completely determined by Λ. For example, if there are only two states with $\lambda_{12} = 2$ and $\lambda_{21} = 1$, then the belief that the economy is in state 1 mean reverts to $1/3$. (ii) The diffusion term, $(\nu_i - \bar{\nu}(\pi_t))\,(\Sigma')^{-1}$, describes the magnitude of the change in beliefs due to uncertainty in each of the components: inflation, earnings, and real rates. For example, investors' probability of being in state i will react more to inflation shocks when the inflation rate in that state is further away from investors' conditional mean of inflation at that moment. In such periods investors revise their beliefs more rapidly. Their beliefs are also more volatile in economies with lower noise in fundamentals and higher precision of signals as captured in the $(\Sigma')^{-1}$ term. (iii) Agents update their beliefs about underlying states by observing not only the path of fundamentals and signals, but also their pricing kernel. This is analogous to the Lucas economy mentioned above in which the agent would learn about the drift rate of the marginal utility of consumption.

It is useful to note that investors' beliefs change with their inferred shocks, $d\tilde{W}$, in equation (10) as opposed to the true shocks, dW, that affect fundamentals. Similarly, they infer that the process $\{X_t\}$ follows: $dX_t/X_t = \bar{\nu}(\pi_t)dt + \Sigma d\tilde{W}_t$. At each point in time, substituting the definition of $d\tilde{W}_t$, we have $dX_t/X_t = \bar{\nu}(\pi_t)dt + \Sigma\left[\Sigma^{-1}(\nu_t - \overline{\nu}(\pi_t))dt \ + \ dW_t\right] = \nu_t dt + \Sigma dW$, which is the same process as in Assumptions 1 through 3. In the filtering literature, $d\tilde{W}_t$ is an "innovations" process under the investors' filtration and under the separation principle it can be used for dynamic optimization. See David (1997) for a discussion. As a special case we write

$$\frac{dY_t}{Y_t} \ = \ \bar{\varrho}(\pi_t)dt \ + \ \Sigma_2\, d\tilde{W}_t \ = \ \varrho(\pi_t)dt \ + \ \Sigma_2\, dW_t, \tag{11}$$

and the kernel under investors' filtration as $dM_t/M_t = -\bar{k}(\pi_t)dt - \sigma_M\, d\tilde{W}_t$, where the real rate in the economy, $\bar{k}(\pi_t)$, is its expected value conditional on investors' filtration.

1.1. Stock Prices and the Term Structure of Interest Rates

To evaluate nominal claims and nominal risk premiums we will also use the nominal pricing kernel, $N_t = M_t/Q_t$, which follows

$$\frac{dN_t}{N_t} \ = \ -r_t dt \ - \ \sigma_N\, dW_t, \tag{12}$$

where $r_t = k_t + \beta_t - \sigma_N\sigma_Q'$ and $\sigma_N = \sigma_M + \sigma_Q$. The nominal rate differs from the real rate by the sum of expected inflation and the inflation risk premium, which is the covariance between inflation and the nominal pricing kernel. With unobserved states, the projected (observable) nominal interest rate at time t is $\bar{r}(\pi) = \sum_{i=1}^{N} r_i\, \pi_{it}$, where $r_i = k_i + \beta_i - \sigma_N\sigma_Q'$ is the nominal rate that would obtain in the i^{th} state, were the states observable. The following proposition provides expressions for the price-earnings (henceforth P/E) ratio and the nominal bond price:

Proposition 1.

(a) The P/E ratio at time t is

$$\frac{P_t}{E_t}(\pi_t) \ = \ \sum_{j=1}^{N} C_j\, \pi_{jt} \ \equiv \ C \cdot \pi_t, \tag{13}$$

where the vector $C = (C_1, .., C_N)$ *satisfies* $C \ = \ A^{-1} \cdot \mathbf{1}_N$,

$$A = Diag(k_1 - \theta_1 + \sigma_M\, \sigma_E', k_2 - \theta_2 + \sigma_M\, \sigma_E', \cdots, k_N - \theta_N + \sigma_M\, \sigma_E') \ - \ \Lambda. \tag{14}$$

(b) The price of a nominal zero-coupon bond at time t with maturity τ *is*

$$B_t(\pi_t, \tau) = \sum_{i=1}^{N} \pi_{it}\, B_i(\tau), \tag{15}$$

$$B_i(\tau) = E\left(\frac{M_{t+\tau}}{M_t} \cdot \frac{Q_t}{Q_{t+\tau}} | \nu_t = \nu_i\right) = \left[\sum_{i=1}^{N} \Omega_i \, e^{\omega_i \, \tau}\right] \cdot (\Omega^{-1} \mathbf{1}_N), \tag{16}$$

where Ω_i and ω_i, $i = 1, \cdots, N$, are the i^{th} eigenvector and i^{th} eigenvalue, respectively, of the matrix $\hat{\Lambda} = \Lambda - Diag(r_1, r_2, \cdots, r_N)$.

The proof follows from a simple extension of the proofs for stock and bond prices in Veronesi (2000) and Veronesi and Yared (1999), respectively.

In (a), each constant C_i represents investors' P/E valuation of stocks conditional on the state being ν_i today. As in the classic Gordon model, the valuation depends on the expected growth of earnings, the real rate, and the equity premium. Since investors do not observe the state ν_i, they weight each C_i by its conditional probability π_i thereby obtaining (13). Notice in particular that the form of the constant vector C suggests that: (i) if $\alpha_\theta < 1$, a higher growth rate of earnings implies a higher P/E, (ii) if $\alpha_\beta > 0$, a higher inflation state implies a lower P/E, which is the *real rate effect* of inflation, and (iii) in addition, the P/E ratio in a given state of growth depends on the future sustainability of the growth rate and is determined by the transition probabilities λ_{ij} as shown in the solution to the N-equations in (a).

Similarly, the bond price is a weighted average of the nominal bond prices that would prevail in each state ν_i. Since investors do not actually observe the current state, they price the bond as a weighted average. Both higher inflation and higher growth rate of earnings lead to lower long term bond prices when $\alpha_\beta > -1$ and $\alpha_\theta > 0$. It is useful to notice that all asset prices follow continuous paths even though the drift rates for earnings and inflation jump between a discrete set of states. This results from the continuous updating process.

Let $P_t^N = P_t \cdot Q_t$ be the nominal value of stock, where P_t is the real value of stocks in Proposition 1. Using the dynamics of the inflation and earnings processes under the observed filtration, we now formulate the nominal return processes for stocks and bonds.

Proposition 2.

(a) The nominal stock return process under the investor's filtration is given by

$$\frac{dP_t^N}{P_t^N}(\pi_t) \;=\; (\mu^N(\pi_t) - \delta(\pi_t))\, dt \;+\; \sigma^N(\pi_t)\, d\tilde{W}_t,$$

where $\delta(\pi)$ is the dividend yield, and the nominal stock price volatility is

$$\sigma^N(\pi_t) \;=\; \sigma_E \,+\, \sigma_Q \,+\, \frac{\sum_{i=1}^{N} C_i \, \pi_{it} \, (\nu_i - \overline{\nu}(\pi_t))'(\Sigma')^{-1}}{\sum_{i=1}^{N} C_i \, \pi_{it}}. \tag{17}$$

(b) The nominal zero-coupon bond return process is

$$\frac{dB_t(\pi_t, \tau)}{B_t(\pi_t, \tau)} \;=\; \mu^B(\pi_t, \tau)\, dt \;+\; \sigma^B(\pi_t, \tau)\, d\tilde{W}_t,$$

where the nominal stock price volatility is

$$\sigma^B(\pi_t, \tau) \;=\; \frac{\sum_{i=1}^N \, B_i(\tau)\, \pi_{it}\, (\nu_i - \overline{\nu}(\pi_t))'(\Sigma')^{-1}}{\sum_{i=1}^N \, B_i(\tau)\, \pi_{it}}. \qquad (18)$$

Proof. See Veronesi (2000) and Veronesi and Yared (1999).

Stock price volatility has an exogenous component due to noise in the fundamental process and a learning-based endogenous component, which depends on the volatility of each state probability π_i. The volatility of each state probability depends on the fundamental uncertainties as discussed in comment (ii) below Lemma 1. However, the volatility of stock prices depends additionally on the valuation of stocks in each state as measured by the P/E vector, C. For a given news content, *states* in which valuations are higher contribute more to the total volatility. Since the valuations vary with both expected earnings growth and discount rates, stock price volatility in our model will endogenously load differently on news about these components in different stages of the business cycle as found in Boyd, Hu, and Jagannathan (2005). If, for example, in state 5 the earnings drift is far from its value in other states, so that the volatility of the probability of this state is most reactive to earnings news, then stock volatility will give a larger (smaller) weight to this earnings news if the P/E ratio is high (low) in this state. We will discuss this point further in Section 2 for our calibrated model.

The form of the bond volatility equation is very similar to the stock volatility equation, except that the P/E ratio in any state is replaced by the bond price in that state. In addition, there is no exogenous fundamental component to bond volatility. The two volatility equations (17) and (18) are the heart of this paper. From these, covariances between stocks and bonds, and bonds of different maturities, are derived straightforwardly. For example, the former is

$$\begin{aligned} \text{Cov}\left(\frac{dP_t^N(\pi_t)}{P_t^N(\pi_t)}, \frac{dB_t(\pi_t,\tau)}{B_t(\pi_t,\tau)}\right) \;&=\; (\sigma_E + \sigma_Q)\, \sigma^B(\pi_t,\tau)' \qquad (19) \\ &+\; \frac{\sum_{i=1}^N \sum_{j=1}^N \pi_{it}\, \pi_{jt}\, C_i\, B_j(\tau)(\nu_i - \bar{\nu}(\pi_t))'\,(\Sigma\Sigma')^{-1}\,(\nu_i - \bar{\nu}(\pi_t))}{\sum_{i=1}^N \sum_{j=1}^N \pi_{it}\, \pi_{jt}\, C_i\, B_j(\tau)}. \end{aligned}$$

2. Estimation

In this section we provide a brief description of the estimation methodology and the parameter estimates of our model.

2.1 Estimation Methodology

We estimate the model by using information in both fundamentals and first and second moments of financial variables to obtain the time series of investors' beliefs over fundamental states, as well as the underlying parameters using a Simulated Method of Moments (SMM) method as described below.

Let Ψ denote the set of structural parameters in the fundamental processes of inflation, earnings, earnings signals, and the pricing kernel in equations (2), (3), (4), and (5), respectively. Let the likelihood function for the fundamentals data observed at discrete points of time (quarterly) be $\mathcal{L}$. To extract information about investors' beliefs from financial variables and their volatilities, we use the pricing formulae for the P/E ratio and Treasury bond prices in Proposition 1 and their volatilities in equations (17) and (18) to generate model-determined moments. Let $\{e(t)\}$ denote the errors of the pricing and volatility variables, and define $\epsilon(t) = \left(e(t)', \frac{\partial \mathfrak{L}}{\partial \Psi}(t)'\right)'$, where the second term is the score of the likelihood function of fundamentals with respect to Ψ. We now form the SMM objective:

$$c = \left(\frac{1}{T}\sum_{t=1}^{T}\epsilon_{\mathbf{t}}\right)' \cdot \Omega^{-1} \cdot \left(\frac{1}{T}\sum_{t=1}^{T}\epsilon_{\mathbf{t}}\right). \tag{20}$$

The details of the procedure are in the Appendix.

It is worth emphasizing three key points about our choice of the SMM method of estimation. First, a simulation-based approach is necessary in our case since the likelihood function for the fundamental data observed at discrete points in time is not available in closed-form. In addition, series-based approximations of the likelihood function as in Ait-Sahalia (2002) are cumbersome given the high dimensionality of our state place (five dimensions for the beliefs and two for fundamentals). Second, our SMM approach allows for the fact that the econometrician observes data only on fundamentals while investors in addition observe their pricing kernel and signals about earnings, and hence update their beliefs about fundamental drifts based on a finer information filtration. Finally, the procedure combines information in asset price and volatility moments with the information in fundamental data so that the extracted investors' beliefs are potentially quite different from estimation methods that rely only on fundamental information [see, for example, Hamilton (1989)].

2.2 Estimation Results for the Regime Switching Model

In this subsection, we briefly describe the results of the estimation of our model. We start with the description of the data series used. Our data sample runs from 1960 to 2006. Aggregate earnings for the economy are approximated as the operating earnings of S&P 500 firms, and these data are obtained from Standard and Poor's. Similarly, the aggregate P/E ratio is estimated as the equity value of these firms divided by their operating earnings. Dividends for these firms, also obtained from Standard and Poor's, are used with the prices to compute returns. We use the Consumer Price Index (CPI), obtained from the Federal Reserve Bank of St. Louis, as our inflation series, which is used both for discounting nominal earnings, as well as for forecasting future real earnings using our joint regime switching model of fundamentals. The time series of zero-coupon yields and returns on bonds of different maturities are obtained from the Fama-Bliss data set available at the University of Chicago. Finally, the realized volatilities of stocks and bonds are estimated as squared average of daily returns in any given quarter. Returns are not demeaned, although the demeaning of the series does not significantly affect our results. Use of such daily averages has a long tradition in finance [see, for example, Schwert (1989)]. More recently, authors such as Anderson, Bollerslev, Diebold, and Labys (2003) use higher frequency intradaily data to estimate realized volatilities; however, such data are not available for the long sample that we use.

We estimate a model with three regimes each for inflation characterized by the states $\beta_1 < \beta_2 < \beta_3$ and earnings growth $\theta_1 < \theta_2 < \theta_3$, which lead to nine composite states overall. In our estimation, we find that the unconstrained estimates of the transition matrix led to several zero elements, leading to a more parsimonious five state model with the following states: $\{(\beta_1, \theta_2), (\beta_2, \theta_1), (\beta_2, \theta_2), (\beta_3, \theta_1), (\beta_1, \theta_3)\}$. We find that the remaining four states have close to zero probability of occurring in the sample. Overall, the five-state and nine-state models lead to almost the same value for the SMM objective function. Gray (1996) and Bansal and Zhou (2002) use a similar criterion for the choice among alternative regime specifications.

The top three panels of Table 1 reports estimates of the drifts and volatilities of fundamentals and signals. The fourth panel reports the transition probability (generator) matrix, while the fifth panel reports the estimates of the parameters of investors' pricing kernel. We estimate that inflation averages 2.1%, 4.1%, and 8.4% in the three states, which we shall refer to as low (LI), medium (MI), and high (HI) states of inflation. Earnings growth averages -3.5%, 2.5%, and 5.4% which we classify as regular low (LG) and high (HG) growth rate states, and the "new economy" (NG) growth rate state, respectively. We provide further motivation for the names given to these states by looking at the implied quarterly and five-year transition probability matrices in the top two panels of Table 2. These matrices are derived solely from the elements of the generator elements in Table 1. We provide some summary descriptions next.

Notice first that the regular high growth rate of earnings, θ_2, is far more persistent in the low inflation state: from the (LI-HG) state, there is a 99% chance of returning to this state and a 1% chance of transitioning to the (MI-HG) state in a quarter, and thus there is almost a zero chance of growth slowing in a quarter in which there is also low inflation. From the (MI-HG) state, on the other hand, there is an 2.2% chance of a transition to the (MI-LG) state in the following quarter. This is the signaling role of inflation – it provides an early warning signal of an unsustainable high growth rate of fundamentals. Second, we see from the bottom panel of Table 2 that, by our model estimates, regular high growth rates (states 1 and 3) would occur about 70% of the time and regular low growth states would occur about 27.5% of the time. In fact, in a model with a similar structure, David (2006) shows that these four states provide a good fit for fundamentals and credit spreads. However, including short-term (quarterly) stock volatility as one of the overidentifying conditions leads to a rejection of the four-state model. We will see shortly that this failure is largely due to the spectacular rise of stock volatility in the late 1990s and its equally spectacular fall in the first half of the current decade. These massive swings seem to be unrelated to the transitions of the economy between the four basic macro states as seen in the time series of implied investors' state probabilities that are shown in Figure 1. Overall, as seen, investors have held fairly strong views that the U.S. economy has remained in the LI-HG states. For most of the decade of the 1960s this probability hovered around 85%, while in the 1990s it was lower at around 70%. There were several switches within the medium inflation states in the 1970s, with two bouts of high inflation and low growth in the mid-1970s and early 1980s. The mid-to-late 1980s were characterized by high (nearly 40%) probabilities of the MI-HG state, which steadily declined through the end of the 1990s, although the probabilities of this state rose substantially to about 35% before the recession in 1990. This trend decline in expected inflation has also been noted by other authors [see, for example Sargent, Williams, and Zha (2006)]. In the current decade, the probability of the MI-HG state has slowly trended up and has averaged about 5% from 2002 to 2005. Unlike all previous recessions, the biggest reassessment in investors' expectations in the 2001 recession related to earnings growth rather inflation, as investors sharply revised their beliefs that the economy was in the NG (fifth) state. We discuss the implications of this revision further below.

The fifth state, (LI-NG), has a stationary (long-run) probability of only about 2.5%, but its inclusion in our model helps to a large extent to explain the high P/Es and high stock market volatility in the late 1990s. During this period, earnings growth was far above its historical average, and led investors to conjecture that due to productivity increases, there was a "new economy growth rate." As we see from investors filtered probabilities in Figure 1, investors were very uncertain that the economy was in this state, which led to high stock market volatility. In this sense, the experience of the late 1990s was extraordinary: while earnings grew rapidly in the 1960s as well, the probability of the NG state only briefly touched 5%, while in 1999 it reached 25%. This channel of expectations of high growth rates causing the high P/Es and high volatility in the 1990s was also used by Pastor and Veronesi (2006). As in their model, we see in Proposition 1 that P/Es are convex in expected growth rates of earnings, which implies that in periods of higher uncertainty, ceteris paribus, P/Es are higher. However, their paper does not analyze the longer term relationship between uncertainty, P/Es, and volatility. Examining the time series of uncertainty however, we see that in all other periods of high uncertainty in our sample, P/Es were *low*, because in such periods investors feared a recession and had low expectations of earnings growth. Finally, it is also useful to note that by our model estimates, the new economy growth only occurs in low inflation states, so that a pickup in inflation lowers investors' assessed probability that the new economy growth will persist. As seen in Figure 1, the slight trend increase in the probability of the MI-HG state in the current decade has been accompanied by a steady decline in the probability of the LI-NG state.

The low probability of occurrence of the high drift rate of earnings lends some similarity of our model-estimated parameters to the recent work on "long-run risks" starting with the work of Bansal and Yaron (2004). This channel is used to generate a large equity premium in their paper. In our model, investors are confronted with the possibility that a low probability switch of fundamentals occurred in the late 1990s. As can be seen from the middle panel of Table 2, even at the five-year horizon there is only about a 1% chance of the economy entering the NG state from any other state. This low probability explains why investors' filtered probability of this state reached a maximum of only around 25%, raising P/Es, but due to their high uncertainty, also raising stock market volatility. This role of the small probability of entering the LI - NG state distinguishes it from the LI-HG state, even though the persistence of both states, as measured by the probabilities of returning to the respective states in the following quarter, are very similar. Indeed, the volatility of stocks remains contained when investors perceive moderately high growth and mainly increase their probabilities of the LI-HG state. The increases in volatility following strong growth in earnings growth in the early 1960s and late 1990s explains why the leverage effect is fairly weak in our sample. This effect documents that volatility increases mainly following bad news. We will discuss this more completely in the forecasting results in Section 4. These episodes also help explain why the power of interest rates in forecasting volatility for our full sample has been quite limited, since they were not preceded by rising rates. Glosten, Jagannathan, and Runkle (1993) point out that interest rates are useful forecasters of stock market volatility in alternative subsamples of stock market data.

We next turn to the kernel parameter estimates, the fifth set of parameters in Table 1. We notice immediately that α_θ is very close to zero, which implies that the real rate does not depend on the state of real fundamentals, and that α_β is significantly positive, which means that the real rate is higher in states of higher inflation. Ceteris paribus, these estimates imply lower P/E ratios and higher Treasury yields during periods with higher expected inflation.[4] These observations are confirmed in the bottom panel of Table 2, which reports the implicit parameters for the P/E and bond yields across the states using the pricing formulas for stocks and bonds in equations (13) and (15), respectively.

Using the time series of investors' state probabilities in Figure 1 and the estimated parameters we generate time series of model-implied expected fundamental growth and prices in Figure 2. Our SMM procedure also uses the moments of assets volatilities. Table 3 shows the fits of our model to expected fundamental growth and the overidentifying price and volatility variables. The first two lines of the table show that the regression of historical on expected growth give an R^2 of 46.7% and 9.5% for inflation and earnings growth, respectively. We note that our SMM procedure, which maximizes the likelihood of investors observing the historical fundamental processes, does not have an explicit prediction on the fitted *actual* fundamentals in each period, but instead characterizes expected fundamental growth. Therefore, these fits reflect not simply the accuracy or our model, but in addition, investors' estimates on the fraction of variation in fundamental growth that is related to shifts in trend growth rates as opposed to purely idiosyncratic variation. Also note that the β coefficients in both expectations regressions are in excess of two, so that actual fundamentals are more than twice as volatile as their expectations.

In contrast to fundamental growth rates, the overidentifying price and volatility moments used in the estimation lead to explicit predictions of model-implied prices and volatilities in each period and are functions of investors beliefs of the states of the fundamentals. Indeed, our model fits the prices fairly accurately: the fit for the P/E ratio is about 72.7%, although notably, the model fails to fully fit the high valuations of the late 1990s, as investors were not fully convinced of the new economy growth state. In particular, the model correctly fits a P/E in the high teens for most of the 1960s and low teens for much of the 1970s and early 1980s. The fits for historical yield series have R^2s of between 61% and 67%, with the better fits for the longer maturities. In strong support of our model, the α coefficient in each of the regressions for the financial variables is not significantly different from zero, and the β coefficients of each financial variable are significant at the 1% level

[4]Some authors have called this inverse relationship between P/Es and Treasury yields the "Fed Model," a term that has been attributed to Prudential Securities strategist Ed Yardeni. The relationship seems to hold in several countries across varying time samples [see, for example Aubert and Giot (2007)].

and close to one. The model fits are not perfect though: the largest errors occur for the shorter maturity bonds following recessions. As seen in the figure, after each of the past two recessions, the Federal Reserve effectively lowered short-term rates dramatically to levels that cannot be justified by our purely fundamental-based model. The pricing errors decrease in the maturity of the bonds, as long bond yields did not decline as much in these periods.

The final components of our SMM error term are the volatilities of stocks, and one- and five-year Treasury bonds whose results are also shown in Table 3. As discussed in the introduction, consistent with the growing literature on realized volatility, we treat volatility as an observed variable and choose parameter values so that our model volatility series are close to the historical series. The model is able to explain a large proportion of the variation in these volatilities over the 45-year sample. For stocks the R^2 of the fit of realized volatility on model volatility is 31% for the full sample, and 44% if the fourth quarter of 1987 is excluded. Several authors have noted that the extreme volatility in this quarter was largely related to a breakdown in trading mechanisms rather than fundamental shocks [see, for example, Kyrillos and Tufano (1995)]. The time series of the cumulative four-quarter volatilities and their forecasts based on model inputs with a four-quarter lag are shown in Figure 3, and its top-left panel shows that besides this huge outlier, our model volatility is consistent with nearly all other episodes of historical volatility. The results for both bond volatilities are strong as well, with the model explaining 74% and 45% of the variation of the historical series for the two maturities, respectively. The plots show very good fits for all periods in the sample. We will discuss the forecasting performance of our model for the four-quarter cumulative volatilities in Section 3. We will also further discuss the implications of our model for the joint relationship between fundamental uncertainties, P/E ratios, and volatility.

Using the scores of the likelihood function and the errors of the price and volatility variables, we evaluate the SMM objective function, which serves as an omnibus test statistic [see for example Gray (1996) and Bansal and Zhou (2002)]. The overall SMM objective function value, which has a chi-squared distribution with 7 degrees of freedom, is 11.34, implying a p-value larger than 12%, so we fail to reject our model.

3 Asset Volatility Forecasts

This section contains our main results. We use the dynamics of the fundamentals, signals, pricing kernel and beliefs, and the derived closed-form expression for stock and bond volatilities to formulate optimal forecasts of volatilities and covariances over a finite horizon. Optimal forecasts are essentially the expected quadratic variations over the forecast interval of interest. In particular for a volatility forecast of asset A the optimal forecast of volatility between quarters T_1 and T_2 given the information that investors have at time t is

$$V^*(T_1, T_2; t) = \sqrt{E\left[\int_{T_1}^{T_2} \sigma^A(\pi_s)\sigma^A(\pi_s)'ds|\mathcal{F}_t\right]}. \tag{21}$$

Similarly, the optimal forecast of covariance of returns of assets A and B is given by

$$C^*(T_1, T_2; t) = E\left[\int_{T_1}^{T_2} \sigma^A(\pi_s)\sigma^B(\pi_s)'ds|\mathcal{F}_t\right]. \tag{22}$$

We approximate the expectations by Monte Carlo simulations sampling several paths of the state variables at small discrete intervals. Details are provided in the Appendix.

3.1 Volatility Forecasts

Tables 4, 5, and, 6 report the results of the time-series forecast regressions

$$\text{Vol}\,(t+1, t+k) = \beta_0 + \beta_1\,\text{Vol}\,(t-k, t) + \beta_2 V^*\,(t+1, t+k; t) + \beta_3\,\mathbf{X}\,(t) + \varepsilon\,(t+1, t+k) \tag{23}$$

for $k = 4$. The dependent variable, Vol$(t+1, t+k)$, is the realized volatility between quarters $t+1$ and $t+k$, Vol$(t, t-k+1)$ is the realized volatility of the current and past $k-1$ quarters, and $V^*(t+1, t+k; t)$ is the optimal forecast of future volatility in the following k quarters, conditional on investors' beliefs at t. The vector $\mathbf{X}\,(t)$ contains the following set of macroeconomic control variables:

1. A business cycle dummy variable taking value one during expansions, as defined by the NBER, NBER(t).
2. Current returns on the asset in periods when that return is negative, $R_A^{(-)}(t)$, where A is the asset under consideration (S for stocks; $1y$ and $5y$ are for one- and five-year bonds).
3. Term structure variables that include the short (three-month) Treasury Bill rate, $r(t)$, and the slope of the yield curve measure as the difference between the five-year and one-year Treasury yields, $term(t)$.
4. The current volatilities of inflation and earnings growth computed by fitting a GARCH(1,1) model to inflation or earnings growth, $\sigma_I(t)$ and $\sigma_E(t)$, respectively.
5. The dispersion of inflation and earnings growth forecasts from the Survey of Professional Forecasters, $\sigma_{PF}^I(t)$ and $\sigma_{PF}^E(t)$, respectively. These forecasts are obtained from the Federal

Reserve Bank of Philadelphia. Details about the construction of dispersion measures are in the Appendix.

Including lag volatility improves the $\bar{R}^2$ of volatility forecasts, but begs the question as to what causes volatility. The effects of persistent explanatory variables will result in the lag having a significant coefficient without increasing our understanding of the economic forces driving volatility. We therefore present results of regressions with and without volatility lags. In our regressions with controls we leave out lagged volatility to see which economic variables best explain the dynamics of volatilities.

The historical series and their forecast values with one year lagged data are shown in Figure 3. The forecasted value is the fitted value of the regression in (23) with β_1 and all elements of β_3 set equal to zero so that the forecast is based only on the optimal model-based forecast.

Our strongest results are for the volatility of stocks that are shown in Table 4. As seen in the table, the model-based forecast is the only variable that is able to improve on pure lag-based volatility forecasts, and even makes lagged volatility insignificant in a joint regression. Lagged volatility explains about 18% of the variation in future volatility, while the model explains about 38%. Excluding the fourth quarter of 1987 increases the $\bar{R}^2$ of the model forecast to nearly 47%, but has little affect on the $\bar{R}^2$ of the lag-only forecast. We have a single measure of cumulated four-quarter lagged volatility, but using several other combinations of lagged volatility fail to change the results significantly. Line 5 shows that the six macroeconomic variables can jointly forecast only 4% of the variation in future volatilities. The control list includes the popular term-structure variables, the short rate and the slope that are used in past studies. The former proxies for inflation risk that leads to macroeconomic instability and higher volatility in the future [see Glosten, Jagannathan, and Runkle (1993)], while the latter is known to proxy for risk premiums in the economy. In particular, note from Figures 2 and 3 that the episode of high stock volatility that started around 1996 was not immediately preceded by high rates. Line 6 of the table shows that using all the control variables and the model-based forecast increases the $\bar{R}^2$ to 44%, an improvement of only about six percentage points. The only variable that comes in significant in addition to the optimal forecast is the term slope. It is noteworthy that lagged stock returns are insignificant in the multivariate regression even though the coefficient is negative and significant if we do not include the model-based forecast. Therefore, any bad news that increases volatility is captured in the model-based forecast, which therefore provides a rationale for the leverage effect in the literature. All other macroeconomic variables, including the NBER recession indicator [found highly significant in Schwert (1989)] and fundamental volatilities are also insignificant in the joint regression. Line 7 shows that survey-based dispersion measures of inflation and earnings can together explain only 1% of the variation in future volatility over the shorter sample in which they are available. In the presence of our model-based forecast, their forecasting power is insignificant (line 8).

Therefore, somewhat surprisingly, the range of macroeconomic control variables cannot forecast stock market volatility at the four-quarter horizon while our model-based forecast has much greater success. We find similar results for shorter horizons of one to three quarters. While our model is based on macroeconomic variables similar to those included in the linear regression, we see three major improvements of our methodology over simply including macroeconomic factors in a forecasting equation. First, our model is forward-looking as it systematically extracts investors' forecasts of the future values of fundamentals from asset prices. Second, it appropriately weights the impact of real and inflation news on stock market volatility as summarized in our comments below Proposition 2. For example, the reaction of stock market volatility to interest rates depends on the stage of the business cycle and stock valuations at that point of time. Finally, in our estimated model investors' uncertainties are based on a finer filtration than the simple boom/bust states of a macroeconomic business cycle. The spectacular rise in stock market volatility in the late 1990s (and to a smaller extent in the 1960s), and its equally spectacular fall in the current decade, was in large part unrelated to recession concerns but instead to the likelihood that the U.S. economy had entered a new economy growth rate. Incorporating each of these features increases the accuracy of forecasted volatility.

We discuss our results for forecasts of volatility for one-year and five-year Treasury bonds in Tables 5 and 6 together. The model-based forecasts explain 70% and 66% of the variation in future volatilities for the two maturities, respectively. In both cases the model-based forecast improves on the lag-based forecast, although the improvement at eight to ten percentage points is smaller than for stocks. For one-year bonds, the six macroeconomic control variables can together explain an additional 12 percentage points of future variation, while for five-year bonds including all the controls does not lead to any improvement in the adjusted-R^2 relative to the model-only regression. For the one-year bond volatility regression, we find that the lagged short rate as well as the historical volatility of inflation have statistically significant coefficients (line 4), but only the latter remains significant once our model forecast is included (line 5). Therefore, our model fails to fully capture the effects of inflation volatility on short maturity bond volatility. We comment on this further below. For five-year bonds, the model forecast in line 2 is more accurate than the controls-only forecast in line 4.

Overall, the results of the three tables can be summarized as follows. For stock volatility our model is able to provide superior forecasts compared to lagged volatility (a proxy for all persistent explanatory variables) and a range of macroeconomic variables in forecasting future volatility. For five-year bonds, our model provides slightly better forecasts of future volatility compared to the controls-only forecast. At the very least, our model forecast is a good aggregator of this macroeconomic information, which includes real and nominal variables, information in the term structure, and surveys of forecasters. The improvement obtained by explicitly formulating the analytical form of bond volatilities and using these in forecasts does not help forecasts as much as for stocks, since the valuation effects discussed (that change the weighting given to alternative shocks) are less important for bonds. Indeed, as seen in our sample in Figure 2, the P/E ratio is twice as volatile as bond yields. For one-year Treasury bonds, our model explains 70% of the variation in future volatility, but fails to fully incorporate all macroeconomic information. We conjecture that this failure is related to the inability to explain the dramatic dips in shorter maturity bond yields in the recent recessions. Indeed, excluding the 2001 to 2005 period when our model could not fit the low rates on short maturity bonds, we find that the macroeconomic controls can provide no improvement in forecasting over and above our model forecast. In this sense, the low rates and high maturity bond volatility in this period were affected by macroeconomic / policy events to a larger extent than our fundamentals-based model would imply.

3.2 Covariance Forecasts

As for volatilities of different asset returns, the covariances of their returns also exhibit strong persistence. The fundamental uncertainties modeled in this paper are natural candidates for explaining this persistence: during periods of of high uncertainty, the faster updating of beliefs leads to higher volatility of stocks and bonds of all maturities (see the expressions in Proposition 2) and since investors' beliefs are common factors in all returns, this faster updating has a direct impact on their covariances. The predictability of the duration of the episodes of heightened uncertainty therefore leads to predictability in covariances. We once again use our structural form approach to forecasting realized covariation among the asset returns, which we describe next.

Our estimation procedure does not use the covariance of asset returns in our set of overidentifying moments. However, we can first check the cross-sectional out-of-sample ability of our model to match current-period covariances from our parameter estimates. Using investors' implied belief series we formulate model covariances and report the results in the last three lines of Table 3. The

R^2s for the covariances of stocks and bonds of the two maturities are 32.5% and 25.5%, which are lower than the R^2 for the volatility equations, and the beta coefficients are also small at about 0.5. The model fit for the covariance of one- and five-year Treasury bonds is considerably better with an R^2 of 66.3% and a beta coefficient very close to one. We will comment on the periods in which the model performs well and poorly below when we discuss the forecasts of covariance.

As in the previous subsection, we run the following forecasting regressions:

$$\text{Cov}\,(t+1,t+4) = \beta_0 + \beta_1\,\text{Cov}\,(t-4,t) + \beta_2\,C^*\,(t+1,t+4;t) + \beta_3\mathbf{X}\,(t) + \varepsilon\,(t+1,t+4)\,, \tag{24}$$

where Cov$(t+1,t+4)$ is the realized covariance between the two return series between quarters $t+1$ and $t+4$ calculated from daily returns, and the other variables are as defined below equation (23). The historical series and their forecast values based on the optimal model-based forecast are shown in Figure 3. We discuss the analogous set of regressions and continue to use the $\bar{R}^2$ as the metric of forecastability. The regressions are reported in Tables 7, 8, and 9, respectively.

We start with the description of the results for forecasts of covariances of stocks and bonds of each maturity. Both covariances are quite persistent and the lagged covariance in itself can forecast nearly 50% of the future variation (line 1). Our model forecast is much better for the one-year bonds explaining 59% of the future variation, while its improvement over the lag-based forecast is marginal for five-year bonds (line 2). In both cases, including both the lag and our model forecast increases the forecast $\bar{R}^2$ significantly (line 3). Next, using the seven macroeconomic control variables does not lead to much improvement in the forecast $\bar{R}^2$ in either case, although three of the regressors are highly significant for the one-year bond, and only the short rate is significant for the five-year bond (line 4). Finally, including our model forecast along with the macroeconomic controls does not improve the $\bar{R}^2$ much, although our model forecast remains highly significant in each case, while some of the controls lose their significance (line 5). Overall, our model forecast and the lagged covariance seem to be reflecting slightly different pieces of information, while they jointly have a forecasting accuracy almost as high as all the macroeconomic controls together. As for the bond volatilities, we find that once we exclude the 2001 to 2005 period, lagged covariance offers no improvement over our model-based forecast (results not shown). Thus, the incremental predictive power in the lagged covariance appears to be picking up the model's persistent overestimation of the covariance in this subsample. Lines 6 and 7 show that the survey measures of inflation and earnings dispersion have low forecasting power, and their significance vanishes when we include our model-based forecasts. We look at the conditional performance of our model in different periods next.

As seen from Figure 3, the covariance forecasts of the model are fairly accurate for most of our 45 year sample, with the notable exception of the 2001 to 2005 period. In this period, realized covariances are very negative, while our model covariances, though negative, are much closer to zero. We recall from the previous section that in this period our model failed to match the extremely low yields on bonds as well. Essentially, in this period the Federal Reserve cut interest rates in an effort to stimulate the economy as incoming bad economic news led to sharp declines in stock prices and thus bond returns were very positive and stock returns were very negative. The failure of the model to fit the covariance in this period is entirely due to its inability to explain the dramatic lowering of short-term interest rates in this period. In related work, other authors have similar findings. Viceira (2007) finds that measures of dispersion from the Survey of Professional forecasts are unable to explain the time series of the covariance and Baele, Bekaert, and Inghelbrecht (2007) are unable to explain the covariance using a broader set of macroeconomic variables than our paper but with a different empirical methodology that does not explicitly extract information from asset prices.[5] Instead, following the work of Connolly, Sun, and Stivers (2005) these authors find that the implied volatility of at-the-money (ATM) options provides incremental explanatory power for the covariance. The logic is that the ATM volatility proxies for economic uncertainty during crisis periods during which investors are driven from stocks to bonds in a "flight-to-safety" phenomenon that induces the negative covariance between stock and bond returns. One may similarly argue that there may be information about "flight-to-liquidity" episodes in measures of stock and bond market illiquidity. We follow up these lines of reasoning next to assess whether these phenomena could explain the extremely negative covariance in the 2001 to 2005 period.

To test the flight-to-safety hypothesis, we include lagged implied volatility as one of the X variables in the forecasting equation (24) for the covariance between stocks and five-year bonds. If implied volatility is a forecaster of future uncertainty, under this hypothesis, it should forecast the covariance of stocks and bonds as well. The results are provided in Table 10 and it is important to note that the sample for the regression is shorter, starting only in 1986 when options data on the S&P 500 index became available. The coefficient on ATM is negative as hypothesized, and the variable is able to forecast 19.5% and 10.8% of the future variation in the covariance of stocks and one- and five-year bonds, respectively. Since investors are more likely to flee to shorter maturity bonds, the forecasting powers are in support of this hypothesis. Our model forecast of covariance, which arises purely from valuation effects, explains 38% and 45% of the variation, and in a joint

[5] In addition to the control variables used in this paper, their paper uses information in the output gap and a measure of investor' risk aversion.

regression, both variables are significant for each maturity. However, the incremental explanatory power of implied volatility at 12% and 5% is small relative to our model. Finally, in the top-left panels of Figure 6 we notice that while the model with both variables is able to fit the covariance over most of the sample very well, it still fails to explain the dramatic drop in the covariance in the 2001 to 2005 period. The top-right panel shows that the implied volatility in this period was indeed higher than in the preceding period, but did not rise high enough to justify the dramatic drop in the covariance.

We similarly test whether the flight-to-liquidity phenomenon can explain the negative covariance of stocks and bond returns. Our bond illiquidity measure is the average bid-ask spread of off-the-run bonds as in Goyenko (2007), while our stock market illiquidity measure is from Amihud (2002).[6] We also construct a measure of cross-illiquidity as the interaction of the stock and bond market illiquidity measures. We find that stock illiquidity in and of itself has negligible forecasting power (untabulated), while the bond and cross-measures do have significant coefficients but with opposite signs (lines 7 and 10 of Table 10). Bond illiquidity has a positive coefficient, since as seen in the left panel of Figure 6, by this measure bonds were most illiquid in the 1970s (a period of large positive covariance between stocks and bonds) and illiquidity has trended lower since then.[7] We interpret this finding as price reactions in the bond market being larger in periods of greater illiquidity leading to larger covariance. Cross-illiquidity has a negative coefficient as in Baele, Bekaert, and Inghelbrecht (2007), perhaps better capturing the common liquidity problems in stock and bond markets that lead to flight-to-liquidity and a negative covariance between stocks and bonds. These results hold when our model forecast is not included in the regressions. Once we include the model forecast, cross illiquidity loses significance for the covariance of stocks and one-year bonds and bond illiquidity loses its significance for the covariance of stocks and five-year bonds (lines 9 and 12). In each case the improvement in the $\bar{R}^2$ relative to the model-only forecast is small of between four and six percentage points. Therefore, if at all, the flight-to-quality seems to be a short-term phenomenon. Finally, the right panels of Figure 6 show that the fitted values with the illiquidity measures included fail to explain the large negative covariance between stocks and bonds in the 2001 to 2005 period.

[6]The inverse of the stock liquidity measure of Pastor and Stambaugh (2003) provides similar results, while the measure of Sadka (2006) does not provide significant results.

[7]We also attempt several definitions of innovations in this measure by using changes as well as residuals from fitting AR processes of different orders, but do not obtain significant results.

4 Relationship Between Fundamental Uncertainties, Price-Earnings Ratios, and Stock Price Volatility

In this final section we pursue further the longer term relation between fundamental macroeconomic uncertainties, P/E ratios, and stock market volatility over our full sample. As noted above, Pastor and Veronesi (2006) show that for NASDAQ stocks, earnings uncertainty was very high in the late 1990s, which is one possible explanation for their high prices and volatilities. Similarly, Pastor and Veronesi (2003) show that high uncertainty – proxied by a firm's age – increases the valuation and volatility of individual stocks, even after controlling for expected future returns and expected future profitability. In this section we investigate whether such a relation between these three variables also holds on average for the aggregate index.

We start by constructing model-based measures of fundamental uncertainty. Given investors' beliefs, π_t, at any point of time we can construct model-based measures of inflation and earnings uncertainty as

$$\text{RMSE}_I(t) = \sqrt{\sum_{i=1}^{N} \pi_{it} \left(\beta_i - \overline{\beta}(\pi_t)\right)^2} \quad \text{and} \quad \text{RMSE}_E(t) = \sqrt{\sum_{i=1}^{N} \pi_{it} \left(\theta_i - \overline{\theta}(\pi_t)\right)^2}, \tag{25}$$

respectively. It is evident that both inflation and earnings uncertainties enter into the expression for stock market volatility in (17). However, as discussed below that equation, the uncertainty components in each state are weighted by stock valuations in these different states, so that stock volatility is a composite and nonlinear function of these variables. As such, it may well be the case that our model fits stock volatilities well, but neither of these measures can by itself, nor in any linear combination, explain the time series of stock market volatility. Indeed, for our estimated parameters the two uncertainty measures explain only 3% and 4% of the variation in historical volatility, while the model volatility explains 37%. This is of course the strength of our structural model-based approach to understanding volatility, since it explicitly builds in pricing relationships to fundamental-based variables and thus is better able to fit price volatility.

In Figure 4 we display the time series of our model based fundamental uncertainty measures as well the survey-based measures briefly introduced earlier. The two sets of measures are closely related. It is evident that both measures increase in and around NBER-dated recessions. It is also evident that earnings uncertainty was higher in late 2001 than in the late 1990s, while in Figure 2 we see that the model P/E ratio did peak in the late 1990s, around the time of the historical peak. Figure 3 shows that our model-based four-quarter-ahead volatility forecast was also highest at the end of

2001. Finally, note that our model-based earnings uncertainty was higher in the past recessions in our sample.

We next examine whether the overall relation between the three variables (uncertainty, P/E ratio, and volatility) has been generally positive. As noted below Proposition 1, P/E ratios in our model will increase with greater earnings uncertainty, but this is a partial effect. If during periods of high uncertainty, expectations of future growth in earnings is low, then high uncertainty could well be accompanied by low P/E ratios. In Figure 5 we show cross-plots of fundamental uncertainties, P/E ratios, and stock volatility to study the longer term relationship between these variables. The top-left panel shows that the relationship between the model-based inflation uncertainty measure and historical P/E ratios has been strongly negative, with the correlation between the two series being -0.69. The right panel shows that the same sign relationship holds for the model-based inflation uncertainty and model P/E ratios, although the relationship is stronger, with a correlation between the two measures being around -0.84. The middle panels show analogous plots for earnings uncertainty and P/E ratios, and once again the relationship is strongly negative. Finally, the bottom plots show the relationship between stock volatility and P/E ratios, the left panel showing the relation for the data series and the right for our model. In both, the correlation is almost zero. Due to the huge outliers in realized volatility in 1987 and 1998, in the data panel the points seemed to be bunched closer together, although this is not true. The zero unconditional correlation of course does not mean that there is no conditional relationship. Indeed, during the late 1990s, there was a positive relationship, which is masked by other periods when the relationship was negative. This conditional relationship can be best seen in Figure 2 (middle-left panel) and Figure 3 (top-left panel). In this period, the positive uncertainty-convexity effect is reinforced by the higher expectation effect as investors grew increasingly optimistic about a new economy growth rate.

In summary, our results reveal a negative unconditional relation between earnings uncertainty and P/E ratios, and no unconditional relation between stock volatility and P/E ratios. This is not inconsistent with the findings in Pastor and Veronesi (2006) and Pastor and Veronesi (2003). Indeed, the former paper only documents a positive *conditional* relation between the three variables in the late 1990s, which we find in our estimated model as well.[8] Pastor and Veronesi (2003) instead document the effect of "idiosyncratic" uncertainty, as it is firm specific and thus not priced. Our evidence thus provides a complementary view of the complex interaction between fundamental

[8]In addition, Pastor and Veronesi (2006) report results for NASDAQ firms for which the convexity effect on P/E ratios was likely greater in this period due to the greater uncertainty surrounding their earnings growth relative to the larger capitalization and more mature firms in the S&P 500 considered in this paper.

uncertainty, prices, and volatility for the aggregate market.

5 Conclusion

In this paper we propose a new structural form methodology for understanding the fluctuations and predictability of volatilities and covariances of asset returns. The methodology tests a particular economic mechanism by formulating exact functional forms of the second moment matrix on these asset returns and it thus explicitly quantifies the effects generated by the economic model. In particular, we apply the methodology to an equilibrium model in which the representative agent learns about the joint movements in inflation and real fundamental drifts through business cycles. An important feature of our methodology is that the information set of the econometrician is smaller than that of investors in the economy, but the econometrician uses the information aggregation property of asset prices to back out the information of investors. Our results help provide a new perspective to the huge literature on modeling volatility and addresses the three questions that we raised in the introduction, which we discuss next.

A significant contribution of our paper is in providing an understanding of the major moves in stock market volatility over the past 45 years. Important as this statistic is, it appears that stock market volatility has a life of its own, and cannot be understood with simple regressions on macroeconomic variables, which is perhaps why the literature using lags has had such tremendous success. Our results indicate that once we use our model-based, optimally constructed volatility forecast, lagged volatility no longer matters. On its own, lagged volatility picks up some of the information that causes investors' uncertainty. The latter is in itself a persistent process since fundamental information is noisy and investors' rational learning of underlying trend shifts is gradual. By fitting our structural form model and extracting investors' beliefs, we find that one of the largest stock market volatility episodes in the past 45 years was not related purely to interest rate movements and the prediction of the recession/boom events, but instead to the uncertainty of the U.S. economy having entered a new economic growth rate state. In the late 1990s the uncertainty of this state was strong enough to cause a conditional positive relationship between uncertainty, volatility, and price-earnings ratios, while for most of the remainder of our sample, these variables were negatively related.

For the volatility of bonds and the covariances of stocks and bond, lags and a set of macroeconomic control variables provide similar levels of forecasting power, while our model forecasts are a little less precise. In particular, our model has difficulty matching the very low interest rates experienced in the first half of the current decade, and the resulting extremely negative covariance. We discuss this issue further below. Outside of this episode, our learning model forecasts fairly accurately all major movements in bond volatilities and covariances and captures all the information in lags and macroeconomic control variables over the preceding 40 years. Our analysis also reveals why the performance of our model relative to lags and macroeconomic controls is different for stocks and bonds. In particular, our derived volatility equation shows how investors will optimally change the relative weights they give to interest rate and real fundamental growth news over time in the formulation of forecasts. Investors' optimal weights are shown to be related to the valuation ratios of stocks in different states of the macroeconomic cycle. For bond volatilities, our model shows that the valuation changes are smaller and hence interest rate measures do indeed capture most of the information that investors would use in volatility forecasts.

We conclude by noting that our model is just one example of a broader structural form approach that explicitly incorporates the impact of a particular economic channel on asset volatilities and their covariances. Indeed, our results suggest that the line of research can be extended to incorporate the impact of additional economic channels. Our analysis based on the valuation effects of investors' learning is not sufficient to understand the extremely negative covariance between stocks and bonds in the first half of the current decade. While movements in proxies for flight-to-safety and flight-to-liquidity in this period also do not seem sufficient to justify the extreme covariance, the regression analysis does not formulate potential nonlinearities between the economic variables considered. Explicitly building equilibrium models of flight-to-quality and flight-to-liquidity and structurally estimating them would be more intellectually satisfying, and would perhaps uncover new insights, but is considerably beyond the scope of our paper.

Appendix

1. SMM Estimation of the Regime Switching Model

We provide here the details of the SMM estimation procedure. Since fundamentals are stationary in growth rates, we start by defining logs of variables: $y_t = \log(Y_t)$, $s_t = \log(S_t)$, and $m_t = \log(M_t)$. Using (11), (4), and (5) we can write

$$dy_t = (\bar{\varrho}(\pi_t) - \frac{1}{2}(\sigma_Q\sigma_Q', \sigma_E\sigma_E')')dt + \Sigma_2\, d\tilde{W}_t, \tag{26}$$

$$ds_t = (\bar{\theta}(\pi_t) - \frac{1}{2}\sigma_E\sigma_E')dt + \sigma_S\, d\tilde{W}, \tag{27}$$

$$dm_t = (-\bar{k}(\pi_t) - \frac{1}{2}\sigma_M\sigma_M')dt - \sigma_M\, d\tilde{W}_t. \tag{28}$$

It is immediate that investors' beliefs π_t completely capture the state of the system (y_t, s_t, m_t)

for forecasting future growth rates. The specification of the system is completed with the belief dynamics in (8).

The econometrician has data series $\{y_{t_1}, y_{t_2}, \cdots, y_{t_K}\}$. Let Ψ be the set of parameters of the model. We start by specifying the likelihood function over data on fundamentals observed discretely using the procedure in the SML methodology of Brandt and Santa-Clara (2002). See also Duffie and Singleton (1993). Adapting their notation, let

$$\mathcal{L}(\Psi) \equiv p(y_{t_1}, \cdots, y_{t_K}; \Psi) \ = \ p(\pi_{t_0}; \Psi) \prod_{k=1}^{K} p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi),$$

where $p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi)$ is the marginal density of fundamentals at time t_{k+1} conditional on investors' beliefs at time t_k. Since $\{\pi_{t_k}\}$ for $k = 1, \cdots, K$ is not observed by the econometrician, we maximize

$$E[\mathcal{L}(\Psi)] = \int \cdots \int \mathcal{L}(\Psi) f(\pi_{t_1}, \pi_{t_2}, \cdots, \pi_{t_K}) d\pi_{t_1}, d\pi_{t_2}, \cdots, d\pi_{t_K}, \tag{29}$$

where the expectation is over all continuous sample paths for the fundamentals, $\tilde{y}_t$, such that $\tilde{y}_{t_k} = y_{t_k}$, $k = 1, \cdots, K$. In general, along each path, the sequence of beliefs $\{\pi_{t_k}\}$ will be different.

As a first step, we need to calculate $p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi)$. Following Brandt and Santa-Clara (2002), we simulate paths of the state variables over smaller discrete units of time using the Euler discretization scheme (see also Kloeden and Platen 1992):

$$\begin{aligned}
\tilde{y}_{t+h} - \tilde{y}_t &= (\bar{\varrho}(\pi_t) - \frac{1}{2}(\sigma_Q \sigma_Q', \sigma_E \sigma_E')')\, h + \Sigma_2 \ \sqrt{h}\tilde{\epsilon}_t; && (30)\\
s_{t+h} - s_t &= (-\bar{\theta}(\pi_t) - \frac{1}{2}\sigma_S\, \sigma_S')\, h + \sigma_S \ \sqrt{h}\tilde{\epsilon}_t; && (31)\\
m_{t+h} - m_t &= (-\bar{k}(\pi_t) - \frac{1}{2}\sigma_M\, \sigma_M')\, h + \sigma_M \ \sqrt{h}\tilde{\epsilon}_t; && (32)\\
\pi_{t+h} - \pi_t &= \mu(\pi_t)\, h + \sigma(\pi_t)\sqrt{h}\tilde{\epsilon}_t && (33)
\end{aligned}$$

where $\tilde{\epsilon}$ is a 4×1 vector of standard normal variables, and $h = 1/M$ is the discretization interval. The Euler scheme implies that the density of the 2×1 fundamental growth vector y_t over h is bivariate (since $\sigma_{Q,3} = \sigma_{Q,4} = \sigma_{E,3} = \sigma_{E,4} = 0$) normal.

We approximate $p(\cdot|\cdot)$ with the density $p_M(\cdot|\cdot)$, which obtains when the state variables are discretized over M subintervals. Since the drift and volatility coefficients of the state variables in (8), and (26) to (28) are infinitely differentiable, and $\Sigma\Sigma'$ is positive definite, Lemma 1 in Brandt and Santa-Clara (2002) implies that $p_M(\cdot|\cdot) \rightarrow p(\cdot|\cdot)$ as $M \rightarrow \infty$.

The Chapman-Kolmogorov equation implies that the density over the interval (t_k, t_{k+1}) with M subintervals satisfies $p_M(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi) \ =$

$$\int \int \phi\left(y_{t_{k+1}} - y; \varrho(\pi)\, h, \Sigma_2 \Sigma_2'\, h; \Psi\right) \times p_M\left(y - y_{t_k}, \pi, m, t_k + (M-1)h | \pi_{t_k}, t_k\right)\, d\pi\, dy, \tag{34}$$

where $\phi(y; \text{mean}, \text{variance})$, denotes a bivariate normal density. Now $p_M(\cdot|\cdot)$ can be approximated by simulating L paths of the state variables in the interval $(t_k, t_k + (M-1)h)$ and computing the average

$$\hat{p}_M\left(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi\right) = \frac{1}{L}\sum_{l=1}^{L} \phi\left(y_{t_{k+1}} - y^{(l)}; \varrho(\pi^{(l)})\, h, \Sigma_2 \Sigma_2'\, h; \Psi\right). \qquad (35)$$

The Strong Law of Large Numbers (SLLN) implies that $\hat{p}_M \to p_M$ as $L \to \infty$.

To compute the expectation in (29), we simulate S paths of the system (30) to (33) "through" the full time series of fundamentals. Each path is started with an initial belief, $\pi_{t_0} = \pi^*$, the stationary beliefs implied by the generator matrix Λ. In each time interval (t_k, t_{k+1}) we simulate (M-1) successive values of $\tilde{y}_t^{(s)}$ using the discrete scheme in (30), and set $\tilde{y}_{t_k}^{(s)} = y_{t_k}$. The results in the paper use $M = 90$ for quarterly data, so that shocks are approximated at roughly a daily frequency. The pricing kernel and beliefs along the entire path of the s^{th} simulation are obtained by iterating on (32) and (33). We approximate the expected likelihood as

$$\hat{\mathcal{L}}^{(S)}(\Psi) = \frac{1}{S}\sum_{s=1}^{S}\prod_{k=0}^{K-1} \hat{p}_M(y_{t_{k+1}}^{(s)} - y_{t_k}^{(s)}, t_{k+1} | \pi_{t_k}^{(s)}, t_k; \Psi), \qquad (36)$$

where $\hat{p}_M(\cdot|\cdot)$ is the density approximated in (35). The SLLN implies that $\hat{\mathcal{L}}^{(S)}(\Psi) \to E[\mathcal{L}(\Psi)]$ as $S \to \infty$. We often report $\bar{\pi}_{t_k} = 1/S \sum_{s=1}^{S} \pi_{t_k}^{(s)}$, which is the econometrician's expectation of investors' belief at t_k.

To extract investors' beliefs from data on price levels and volatilities in addition to fundamentals we add overidentifying moments to the SML method above. From Proposition 1, we can compute the time series of model-implied price-earning ratios and bond yields at the discrete data points t_k, $k = 1, \cdots, K$ as

$$\widehat{\text{P/E}}_{t_k} = C \cdot \bar{\pi}_{t_k}, \qquad \widehat{i}_{t_k}(\tau) = -\frac{1}{\tau}\log\left(B(\tau) \cdot \bar{\pi}_{t_k}\right).$$

We note that the constants Cs and the functions $B(\tau)$ both depend on the parameters of the fundamental processes, Ψ. Hence, we let the pricing errors be denoted

$$e_{t_k}^{P} = \left(\widehat{\text{P/E}}_{t_k} - \text{P/E}_{t_k}, \widehat{i}_{t_k}(0.25) - i_{t_k}(0.25), \widehat{i}_{t_k}(1) - i_{t_k}(1), \widehat{i}_{t_k}(5) - i_{t_k}(5)\right).$$

Also note that since the pricing formulas are linear in beliefs, $1/S \sum_{s=1}^{S} C \cdot \pi_{t_k}^{(s)} = C \cdot \bar{\pi}_{t_k}$ (and similarly for the bond yields) and no information is lost by simply evaluating the errors at the econometrician's conditional mean of beliefs. We similarly formulate the volatility errors as

$$e_{t_k}^{V} = \left(\widehat{\sigma_{t_k}^{N}} - \sigma_{t_k}^{N}, \widehat{\sigma_{t_k}^{B}(1)} - \sigma_{t_k}^{B}(1), \widehat{\sigma_{t_k}^{B}(5)} - \sigma_{t_k}^{B}(5)\right),$$

where the model-implied nominal stock volatility is obtained from the derived expression $\sigma^N(\pi)$ in (17) and averaged over the simulations as $\widehat{\sigma_{t_k}^N} = \frac{1}{S}\sum_{s=1}^S \sigma^N(\pi_{t_k}^{(s)})$. Similarly, the model-implied nominal bond volatility is obtained from the derived expression $\sigma^B(\pi,\tau)$ in (18) and averaged over the simulations as $\widehat{\sigma_{t_k}^B}(\tau) = \frac{1}{S}\sum_{s=1}^S \sigma^B(\pi_{t_k}^{(s)},\tau)$, for $\tau = 1, 5$.

To estimate Ψ from data on fundamentals as well as financial variables, we form the overidentified SMM objective function as in (20). The moments used are the scores of the log likelihood function from fundamentals, the pricing errors from financial variables, and their volatilities. Since the number of scores in $\frac{\partial \log(\hat{\mathfrak{L}})}{\partial \Psi}(t_k)$ equals the number of parameters driving the fundamental processes in Ψ, the number of pricing errors is four, and the number of volatility errors is three, the statistic c in (20) has a chi-squared distribution with seven degrees of freedom. We correct the variance covariance matrix for autocorrelation and heteroskedasticity using the Newey-West method [see, for example, Hamilton (1994) equation 14.1.19] using a lag length of $q = 24$. A long lag length is chosen since interest rates and P/E ratios used in the error terms are highly persistent processes.

2. Optimal Volatility and Covariance Forecasts

As for the likelihood function we formulate the expected quadratic variations in equation (21) and (22) across sample paths by Monte Carlo simulation while discretizing the dynamics of the state variables of our system as in (30) to (33). We divide each quarter into M equally sized intervals of length $h = 1/M$. Then the volatility forecast for asset A is approximated as

$$V^{M*}(T_1, T_2, t) = \sqrt{\frac{1}{S}\sum_{j=1}^{(T_2-T_1)\,M} \sigma^A(\pi_{T_1+j\,h}^{(s)})\,\sigma^A(\pi_{T_1+j\,h}^{(s)})' h},$$

where on each sample the process for the state variables is simulated starting with $\pi_t^{(s)} = \pi_t$, the assumed beliefs of investors at time t. The covariance forecasts in (22) are analogously formulated. We report forecasts for $M = 90$.

3. Survey-Based Measures of Uncertainty

We obtain survey data from the Survey of Professional Forecasters, available at the Federal Reserve Bank of Philadelphia. The data are available since 1968, but we drop the first three years in our analysis since we find several missing entries, therefore restricting attention to the sample from 1971 to 2006. Forecasts are for horizons of $\tau = 0, 1, .., 4$ quarters ahead, where $\tau = 0$ indicates the forecast for the current quarter, which typically ends 1.5 months after the deadline to submit the questionnaire. The number of forecasters varies between 75 and 9 (for one quarter), and the mean number of forecasters is about 34.

We use the cross-sectional dispersion of the percentage inflation and earnings growth of individual forecasts as a measure of forecasters' uncertainty. Specifically, for each quarter t, let $FI_i(t,\tau)$ be the forecast of individual i of the price index level at time $t+\tau$, where τ is the horizon, and let $I(t)$ be its current level (made available to the forecaster). If n_t is the number of individuals at time t, we then define the time t "uncertainty" on the inflation at time $t+\tau$ as

$$\sigma_{PF}^{E}(t,\tau)=\sqrt{\frac{1}{n_t-1}\sum_{i=1}^{n_t}\left(\left(\frac{FI_i(t,\tau)}{I(t)}\right)-\frac{1}{n_t}\sum_{i=1}^{n_t}\left(\frac{FI_i(t,\tau)}{I(t)}\right)\right)^2}. \tag{37}$$

To safeguard against typos and mistakes, we delete observations for $FI_i(t,\tau)/I(t)$ that are four standard deviations away from the mean forecast.

A similar procedure is used for the forecast of real future corporate profits. Let $FD_i(t,\tau)$ be the forecast of individual i about the level of corporate profits at time $t+\tau$ and let $D(t)$ be its current level (again, provided to the forecaster). We then define $FRD_i(t,\tau)=FD_i(t,\tau)/FI_i(t,\tau)$ as a measure of the forecasted real future earnings by individual i and $RD(t)=D(t)/I(t)$ as the current real earnings.[9] Then the empirical measure of uncertainty for profits (earnings) growth is

$$\sigma_{PF}^{E}(t,\tau)=\sqrt{\frac{1}{n_t-1}\sum_{i=1}^{n_t}\left(\left(\frac{FRD_i(t,\tau)}{RD(t)}\right)-\frac{1}{n_t}\sum_{i=1}^{n_t}\left(\frac{FRD_i(t,\tau)}{RD(t)}\right)\right)^2}, \tag{38}$$

where again, we eliminate the observations of individuals that are more than four standard deviations away from the mean.

The two measures $\sigma_{PF}^{I}(t,\tau)$ and $\sigma_{PF}^{E}(t,\tau)$ introduced above are really measures of *dispersion* rather than *uncertainty*. However, there is an a priori reason to believe the two measures should be highly correlated: if all forecasters are uncertain about the future value of a macroeconomic variable, then it is also likely that their point forecasts have greater relative dispersion. Zarnowitz and Lambros (1987) find a positive relationship between similar disagreement and uncertainty measures.

We compare the four-quarters-ahead survey-based measures of uncertainty with the asset-pricing-based measures of uncertainty obtained in Section 4. Figure 4 plots the survey-based measures

[9] It should be noted that the measure of forecasted real earnings defined as $FRD_i(t,\tau)=FD_i(t,\tau)/FI_i(t,\tau)$ effectively uses the formula $E_t[D(t+\tau)]/E_t[I(t+\tau)]$, which is biased compared to the correct measure $E_t[D(t+\tau)/I(t+\tau)]$ due to Jensen's inequality. However, since we are interested in the cross-sectional standard deviation of the forecasts, if the bias is reasonably constant across individuals, it is likely it will affect the measured "uncertainty" only very marginally.

alongside the model-based measures. In simple regressions the model-based uncertainty explains 58% and 24% of the variation in the survey-based measures. It must be noted that earnings growth is about four times more volatile as inflation, which leads to a lower R^2 for the earnings series. The results are remarkable since we make no attempt to fit our model parameters to the survey measures, and hence the results are out-of-sample implications of our model.

Table 1: Five-State Model Estimation

Fundamental Drifts	β_1	β_2	β_3	θ_1	θ_2	θ_3
	0.021	0.041	0.084	-0.035	0.025	0.054
	(0.005)	(0.004)	(0.001)	(0.001)	(0.015)	(.000)
Fundamental Volatilities	$\sigma_{Q,1}$	$\sigma_{Q,2}$	$\sigma_{E,2}$			
	0.022	-0.000	0.092			
	(0.001)	(0.000)	(0.001)			
Signal Volatilities	$\sigma_{S,1}$	$\sigma_{S,2}$	$\sigma_{S,3}$	$\sigma_{S,4}$		
	0.001	0.050	0.062	0.053		
	(0.050)	(0.020)	(0.081)	(0.082)		
Generator Elements	λ_{13}	λ_{15}	λ_{21}	λ_{23}	λ_{24}	
	0.033	0.001	0.086	0.124	0.264	
	(0.015)	(0.029)	(0.079)	(0.003)	(0.024)	
Generator Elements	λ_{32}	λ_{42}	λ_{51}	λ_{52}	λ_{53}	
	0.092	0.300	0.005	0.010	0.005	
	(0.004)	(0.061)	(0.062)	(0.050)	(0.021)	
Pricing Kernel (Real Rate)	α_0	α_β	α_θ			
	0.002	1.072	0.000			
	(0.002)	(0.087)	(0.013)			
Pricing Kernel (Prices of Risk)	$\sigma_{M,1}$	$\sigma_{M,2}$	$\sigma_{M,3}$	$\sigma_{M,4}$		
	0.177	0.508	-0.158	0.001		
	(0.063)	(0.026)	(0.176)	(0.050)		
SMM Error Value ($\chi^2(7)$):	11.343	P-Value: 0.124				

The table reports SMM estimates of the following model for CPI, Q_t, real earnings, E_t, earnings signals, S_t, and the real pricing kernel, M_t :

$$\begin{aligned}
\frac{dQ_t}{Q_t} &= \beta_t\,dt + \sigma_Q\,dW_t, \\
\frac{dE_t}{E_t} &= \theta_t\,dt + \sigma_E\,dW_t, \\
\frac{dM_t}{M_t} &= -k_t dt - \sigma_M\,dW_t, \\
\frac{dS_t}{S_t} &= \theta_t dt + \sigma_S\,dW_t,
\end{aligned}$$

where $\sigma_Q = (\sigma_{Q,1}, \sigma_{Q,2}, 0)$, $\sigma_E = (0, \sigma_{E,2}, 0)$, $\sigma_M = (\sigma_{M,1}, \sigma_{M,2}, \sigma_{M,3})$, $\sigma_S = (\sigma_{S,1}, \sigma_{S,2}, \sigma_{S,3}, \sigma_{S,4})$, $k_t = \alpha_0 + \alpha_\theta\,\theta_t + \alpha_\beta\,\beta_t$, and the vector $\nu_t = (\beta_t, \theta_t, -k_t, \theta_t)'$, follows a five-state regime switching model with the generator matrix Λ whose non-zero diagonal elements are shown as $\lambda_{i,j}$. The pricing kernel, M_t, and the signal S_t, is observed by investors but not by the econometrician. Assets are priced using the formulas in Proposition 1. Estimates are obtained from data on the fundamentals as well six price levels and three price volatilities using the SMM methodology described in Appendix 2. Standard errors are in parentheses.

Table 2: Model Implied Transition Probabilities, Investor Expectations, and Prices

Implied Quarterly Transition Probability Matrix

	(LI-HG)	(MI-LG)	(MI-HG)	(HI-LG)	(LI-NG)
(LI-HG)	0.991	0.000	0.008	0.000	0.000
(MI-LG)	0.020	0.891	0.029	0.060	0.000
(MI-HG)	0.000	0.022	0.978	0.001	0.000
(HI-LG)	0.001	0.068	0.001	0.930	0.000
(LI-NG)	0.001	0.002	0.001	0.000	0.995

Implied Five-Year Transition Probability Matrix

	(LI-HG)	(MI-LG)	(MI-HG)	(HI-LG)	(LI-NG)
(LI-HG)	0.838	0.018	0.128	0.006	0.008
(MI-LG)	0.184	0.296	0.242	0.277	0.001
(MI-HG)	0.046	0.170	0.689	0.093	0.000
(HI-LG)	0.107	0.314	0.146	0.431	0.000
(LI-NG)	0.017	0.031	0.027	0.015	0.909

Implied Stationary Probabilities, P/E Ratio, and Treasury Yields

State	π^*	C	$i_{0.25}$	i_1	i_5
(LI-HG)	0.372	18.021	0.042	0.043	0.045
(MI-LG)	0.146	10.693	0.086	0.091	0.097
(MI-HG)	0.329	12.396	0.084	0.084	0.087
(HI-LG)	0.128	8.893	0.169	0.161	0.134
(LI-NG)	0.025	34.225	0.042	0.042	0.044

The **top** and **middle panels** report the quarterly and five-year *implied* transition probability matrix between the five states implied from the generator matrix elements displayed in Table 1. Rows may not sum to one due to rounding. The **bottom panel** report the implied stationary probababilities and implied prices of the variables used in the SMM estimation procedure in the five states. C is the P/E ratio and i_T is the Treasury yield with maturity T. The V/E ratio and bond yields are computed as shown in Proposition 1.

Table 3: Model Fits for Expected Fundamental Growth, Asset Prices, and Asset Volatilities from SMM Procedure

Variable	α	β	R^2
Inflation	-0.002	2.135	0.467
	[-2.978]	[7.230]	
Earnings	-0.004	2.693	0.095
	[-1.601]	[3.046]	
P/E Ratio	-0.007	1.444	0.727
	[-3.503]	[9.871]	
Three-Month Yield	-0.008	1.062	0.618
	[-1.043]	[7.888]	
One-Year Yield	-0.008	1.143	0.639
	[-0.958]	[7.714]	
Five-Year Yield	-0.007	1.206	0.670
	[-0.705]	[6.270]	
Stock Volatility	$-1 \cdot 10^{-4}$	1.048	0.315
	[-0.875]	[9.683]	
Stock Volatility (EC)	$-1 \cdot 10^{-4}$	1.029	0.441
	[-0.853]	[9.611]	
One-Year T. Bond Volatility	$2 \cdot 10^{-4}$	0.886	0.742
	[3.920]	[12.942]	
Five-Year T. Bond Volatility	$2 \cdot 10^{-4}$	0.742	0.453
	[1.667]	[4.409]	
Out-of-Sample Predictions:			
Covariance of Stocks and 1-Y T. Bond	$5 \cdot 10^{-6}$	0.383	0.325
	[0.351]	[7.589]	
Covariance of Stocks and 5-Y T. Bond	$3 \cdot 10^{-6}$	0.474	0.255
	[0.031]	[4.819]	
Covariance of 1-Y and 5-Y T. Bonds	0.002	1.061	0.663
	[1.082]	[9.324]	

We display the fits of the variables used in our SMM procedure: the fundamentals, and the 6 overidentifying conditions. For the two fundamentals we provide the regression results for the equation $x(t) = \alpha + \beta\, E[x|\mathcal{F}_t] + \epsilon(t)$, where $x(t)$ is the realized growth and $E[x|\mathcal{F}_t]$ is investors' conditional expected growth of the fundamental under consideration. The conditional expected growth is obtained from the filtered probabilities $\pi(t)$ displayed in Figure 1, and for earnings, for example, is given by $\sum_i^N \theta_i\, \pi_i(t)$. For the price (volatility) series, we present the regression results for the equation $p(t) = \alpha + \beta\, p(\pi(t)) + \epsilon(t)$, where $p(t)$ and $p(\pi(t))$ are the realized and model price (volatilities) conditional on investors' beliefs at t respectively. The three covariances are not used in the set of overidentifying conditions, and their out-of-sample fits are similarly reported. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The term EC stands for ex-crash to denote that the fourth quarter of 1987 is removed from the sample.

Table 4: Forecasts of Four-Quarter Cumulative Volatility of Stocks and Macroeconomic Controls

No.	Sample	Const.	Vol $(t-k,t)$	$V^*(t+1,t+4)$	NBER(t)	$R_S^{(-)}(t)$	$r(t)$	Term(t)	$\sigma_I(t)$	$\sigma_E(t)$	$\sigma_{PF}^I(t)$	$\sigma_{PF}^E(t)$	$\bar{R}^2$
1	1960–2006	0.075	0.432										0.181
		[4.040]*	[2.926]*										
2	1960–2006	-0.043		1.254									0.377
		[-2.418]		[5.956]*									
3	1960-2006 (EC)	-0.031		1.158									0.468
		[1.313]		[6.221]*									
4	1960–2006	-0.004	0.291	1.167									0.376
		[-1.372]	[1.149]	[3.921]*									
5	1960–2006	0.133			-0.016	-0.217	0.294	0.017	-2.978	0.04			0.041
		4.416			[-1.043]	[-2.113]	[0.929]	[0.018]	[-0.205]	[0.087]			
6	1960–2006	-0.02		1.507	-0.007	-0.065	-0.402	-2.312	-8.541	-0.184			0.441
		[-0.785]		[7.805]*	[-0.690]	[-0.855]	[-1.315]	[-3.693]*	[-0.745]	[-0.595]			
7	1971-2006	0.123									-1.830	1.021	0.012
		5.233									[-0.909]	[0.458]	
8	1971-2006	-0.084		1.456							0.003	0.056	0.326
		[-2.671]*		[7.056]*							[0.458]	[0.605]	

This table reports the time series regressions Vol $(t+1,t+4) = \beta_0 + \beta_1 \text{Vol}\,(t-k,t) + \beta_2 V^*\,(t+1,t+4;t) + \beta_3 \mathbf{X}\,(t) + \varepsilon\,(t+1,t+4)$, where Vol$(t+1,t+4)$ is the realized volatility between quarters $t+1$ and $t+4$, $V^*(t+1,t+4;t)$ is the optimal forecast of future volatility in the following four quarters in (21), and $\mathbf{X}\,(t)$ contains a vector of the following controls: NBER(t) is a business cycle dummy variable taking value = 1 during expansions as defined by the NBER; $R_S^{(-)}(t)$ is the return on the S&P 500 index in quarter t if it is negative and zero otherwise; $r(t)$ is the 3-month Treasury Bill rate; Term(t) is the slope of the term structure defined as the five-year Treasury yield less the one-year Treasury yield; $\sigma_I(t)$ and $\sigma_E(t)$ are the current volatilities of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth; and $\sigma_{PF}^I(t)$ and $\sigma_{PF}^E(t)$ are the dispersion of forecasts from the Survey of Professional Forecasters. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols * denotes significance at the 1% level. The term EC on line 3 stands for ex-crash to denote that the fourth quarter of 1987 is removed from the sample.

Table 5: Forecasts of Four-Quarter Cumulative Volatility of One-Year Treasury Bonds and Macroeconomic Controls

No.	Sample	Const.	Vol $(t-k,t)$	$V^*(t+1,t+4)$	NBER(t)	$R_{1y}^{(-)}(t)$	$r(t)$	Term(t)	$\sigma_I(t)$	$\sigma_E(t)$	$\sigma_{PF}^I(t)$	$\sigma_{PF}^E(t)$	$\bar{R}^2$
1	1960–2006	0.238	0.796										0.637
		[2.108]	[6.115]*										
2	1960–2006	-0.007		0.881									0.729
		[-0.048]		[5.917]*									
3	1960-2006	0.003	0.394	0.505									0.763
		[0.290]	[2.336]	[3.022]*									
4	1960–2006	-0.007			0.001	-0.525	0.167	0.071	9.949	-0.002			0.818
		[-2.626]			[0.930]	[-1.543]	[7.913]*	[1.175]	[4.076]*	[-0.099]			
5	1960-2006	-0.021		0.405	0.001	-0.506	0.052	-0.135	6.666	-0.007			0.841
		[-0.728]		[3.502]*	[0.657]	[-1.484]	[1.150]	[-1.682]	[2.859]*	[-0.271]			
6	1971-2006	-0.039									0.956	0.108	0.331
		[-1.121]									[2.625]*	[2.065]	
7	1971-2006	-0.001		0.989							-0.242	0.027	0.739
		[-0.887]		[5.275]*							[-1.120]	[1.059]	

This table reports the time series regressions

$$\text{Vol}\left(t+1,t+4\right)=\beta_0+\beta_1\text{Vol}\left(t-k,t\right)+\beta_2 V^*\left(t+1,t+4;t\right)+\beta_3\mathbf{X}\left(t\right)+\varepsilon\left(t+1,t+4\right),$$

where Vol$(t+1,t+4)$ is the realized volatility between quarters $t+1$ and $t+4$, $V^*(t+1,t+4;t)$ is the optimal forecast of future volatility in the following four quarters in (21), and $\mathbf{X}\left(t\right)$ contains a vector of the following controls: NBER(t) is a business cycle dummy variable taking value = 1 during expansions as defined by the NBER; $R_{1Y}^{(-)}(t)$ is the return on the one-year Treasury Bond in quarter t if it is negative and zero otherwise; $r(t)$ is the 3-month Treasury Bill rate; Term(t) is the slope of the term structure defined as the five-year Treasury yield less the one-year Treasury yield; $\sigma_I(t)$ and $\sigma_E(t)$ are the current volatilities of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth; and $\sigma_{PF}^I(t)$ and $\sigma_{PF}^E(t)$ are the dispersion of forecasts from the Survey of Professional Forecasters. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols * denotes significance at the 1% level.

Table 6: Forecasts of Four-Quarter Cumulative Volatility of Five-Year Treasury Bonds and Macroeconomic Controls

No.	Sample	Const.	Vol $(t-k,t)$	$V^*(t+1,t+4)$	NBER(t)	$R_{5Y}^{(-)}(t)$	$r(t)$	Term(t)	$\sigma_I(t)$	$\sigma_E(t)$	$\sigma_{PF}^I(t)$	$\sigma_{PF}^E(t)$	$\bar{R}^2$
1	1960–2006	0.013	0.758										0.593
		[2.618]*	[7.264]*										
2	1960–2006	0.001		0.74									0.659
		[1.948]		[4.250]*									
3	1960–2006	0.008	0.598	0.266									0.682
		[1.784]	[5.118]*	[2.735]*									
4	1960–2006	-0.012			0.007	-0.375	0.601	0.807	22.265	0.062			0.626
		[-0.985]			[0.748]	[-2.819]*	[5.561]*	[2.722]*	[2.167]	[0.508]			
5	1960–2006	-0.012		0.695	0.005	-0.176	0.647	0.966	20.351	0.006			0.679
		[-0.991]		[2.722]*	[0.609]	[-3.279]*	[3.174]*	[2.261]	[2.317]	[0.108]			
6	1971-2006	0.017									0.141	0.381	0.197
		[1.562]									[1.062]	[3.007]*	
7	1971-2006	0.013		0.681							-0.052	0.025	0.461
		[1.281]		[3.178]*							[-3.982]*	[1.672]	

This table reports the time series regressions

$$\text{Vol}\,(t+1,t+4) = \beta_0 + \beta_1 \text{Vol}\,(t-k,t) + \beta_2 V^*\,(t+1,t+4;t) + \beta_3 \mathbf{X}\,(t) + \varepsilon\,(t+1,t+4)\,,$$

where Vol$(t+1,t+4)$ is the realized volatility between quarters $t+1$ and $t+4$, $V^*(t+1,t+4;t)$ is the optimal forecast of future volatility in the following four quarters in (21), and $\mathbf{X}\,(t)$ contains a vector of the following controls: NBER(t) is a business cycle dummy variable taking value = 1 during expansions as defined by the NBER; $R_{5Y}^{(-)}(t)$ is the return on the five-year Treasury Bond in quarter t if it is negative and zero otherwise; $r(t)$ is the 3-month Treasury Bill rate; Term(t) is the slope of the term structure defined as the five-year Treasury yield less the one-year Treasury yield; $\sigma_I(t)$ and $\sigma_E(t)$ are the current volatilities of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth; and $\sigma_{PF}^I(t)$ and $\sigma_{PF}^E(t)$ are the dispersion of forecasts from the Survey of Professional Forecasters. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols * denotes significance at the 1% level.

Table 7: Forecasts of Four-Quarter Cumulative Covariance of Stocks and One-Year Treasury Bonds and Macroeconomic Controls

No.	Sample	Const.	Cov $(t-k,t)$	$C^*(t+1,t+4)$	NBER(t)	$R_S^{(-)}(t)$	$R_{1Y}^{(-)}(t)$	$r(t)$	Term(t)	$\sigma_I(t)$	$\sigma_E(t)$	$\sigma_{PF}^I(t)$	$\sigma_{PF}^E(t)$	$\bar{R}^2$
1	1960–2006	$7 \cdot 10^{-5}$	0.713											0.506
		[1.525]	[5.563]*											
2	1960–2006	$4 \cdot 10^{-5}$		0.381										0.592
		[-1.589]		[9.937]*										
3	1960–2006	$-5 \cdot 10^{-6}$	0.453	0.149										0.651
		[-0.866]	[3.475]*	[3.765]*										
4	1960–2006	$-7 \cdot 10^{-4}$			$3 \cdot 10^{-4}$	$5 \cdot 10^{-4}$	-0.028	0.001	$7 \cdot 10^{-4}$	0.518	-0.002			0.681
		[-2.331]			[2.724]*	[0.936]	[-0.222]	[5.715]*	[1.304]	[4.434]*	[-1.575]			
5	1960–2006	$-3 \cdot 10^{-4}$		0.142	$3 \cdot 10^{-4}$	$7 \cdot 10^{-4}$	-0.027	$7 \cdot 10^{-5}$	$-7 \cdot 10^{-6}$	0.452	-0.003			0.697
		[1.590]		[4.116]*	[2.447]	[1.132]	[-0.200]	[1.615]	[-0.081]	[3.260]*	[-1.785]			
6	1971-2006	$-5 \cdot 10^{-4}$										0.061	0.005	0.233
		[-2.598]*										[2.457]	[1.144]	
7	1971-2006	$-1 \cdot 10^{-4}$		0.339								-0.003	$-1 \cdot 10^{-4}$	0.550
		[-1.871]		[6.008]*								[-0.155]	[-0.106]	

This table reports the time series regressions

$$\text{Cov}\,(t+1,t+k) = \beta_0 + \beta_1 \text{Cov}\,(t-k,t) + \beta_2 C^*\,(t+1,t+4;t) + \beta_3 \mathbf{X}\,(t) + \varepsilon\,(t+1,t+4)\,,$$

where Cov$(t+1,t+k)$ is the realized covariance in between quarters $t+1$ and $t+k$, $C^*(t+1,t+4;t)$ is the optimal forecast of future covariance in the following four quarters in (22), and $\mathbf{X}\,(t)$ contains a vector of the following controls: NBER(t) is a business cycle dummy variable taking value = 1 during expansions as defined by the NBER; $R_{5Y}(t)$ is the return on the five-year Treasury Bond in quarter t; $R_S^{(-)}(t)$ is the return on the S&P 500 in quarter t if it is negative and zero otherwise; $R_{1y}^{(-)}(t)$ is the return on the one-year Treasury Bonds in quarter t if it is negative and zero otherwise; $r(t)$ is the 3-month Treasury Bill rate; Term(t) is the slope of the term structure defined as the five-year Treasury yield less the one-year Treasury yield; $\sigma_I(t)$ and $\sigma_E(t)$ are the current volatilities of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth; and $\sigma_{PF}^I(t)$ and $\sigma_{PF}^E(t)$ are the dispersion of forecasts from the Survey of Professional Forecasters. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols * denotes significance at the 1% level.

Table 8: Forecasts of Four-Quarter Cumulative Covariance of Stocks and Five-Year Treasury Bonds and Macroeconomic Controls

No.	Sample	Const.	Cov $(t-k,t)$	$C^{*}(t+1,t+4)$	NBER(t)	$R_{5Y}^{(-)}(t)$	$R_{SY}(t)$	$r(t)$	Term(t)	$\sigma_I(t)$	$\sigma_E(t)$	$\sigma_{PF}^{I}(t)$	$\sigma_{PF}^{E}(t)$	$\bar{R}^2$
1	1960–2006	$4\cdot10^{-4}$	0.697											0.485
		[1.048]	[4.514]*											
2	1960–2006	$-7\cdot10^{-4}$		0.3954										0.491
		[-1.350]		[5.699]*										
3	1960-2006	$-3\cdot10^{-4}$	0.453	0.277										0.585
		[-0.856]	[2.853]*	[4.330]*										
4	1960-2006	-0.003			$8\cdot10^{-4}$	0.009	-0.014	0.007	0.007	0.704	-0.032			0.545
		[-2.597]*			[1.137]	[1.522]	[-1.123]	[5.524]*	[1.969]	[0.893]	[-1.811]			
5	1960–2006	$2\cdot10^{-4}$		0.399	0.001	0.001	-0.018	0.016	-0.067	0.002	-0.038			0.604
		[0.154]		[3.693]*	[1.305]	[2.051]	[-1.320]	[0.657]	[-1.015]	[0.024]	[-2.330]			
6	1971-2006	-0.001										0.2817	0.008	0.122
		[-1.382]										[1.988]	[0.403]	
7	1971-2006	$8\cdot10^{-4}$		0.498								-0.024	-0.018	0.453
		[0.082]		[5.250]*								[-0.281]	[-1.069]	

This table reports the time series regressions

$$\text{Cov}\,(t+1,t+k)=\beta_0+\beta_1\text{Cov}\,(t-k,t)+\beta_2 C^{*}\,(t+1,t+4;t)+\beta_3\mathbf{X}\,(t)+\varepsilon\,(t+1,t+4)\,,$$

where Cov$(t+1,t+k)$ is the realized covariance in between quarters $t+1$ and $t+k$, $C^{*}(t+1,t+4;t)$ is the optimal forecast of future covariance in the following four quarters in (22), and $\mathbf{X}\,(t)$ contains a vector of the following controls: NBER(t) is a business cycle dummy variable taking value = 1 during expansions as defined by the NBER; $R_{5Y}(t)$ is the return on the five-year Treasury Bond in quarter t; $R_{S}^{(-)}(t)$ is the return on the S&P 500 in quarter t if it is negative and zero otherwise; $R_{5y}^{(-)}(t)$ is the return on the five-year Treasury Bonds in quarter t if it is negative and zero otherwise; $r(t)$ is the 3-month Treasury Bill rate; Term(t) is the slope of the term structure defined as the five-year Treasury yield less the one-year Treasury yield; $\sigma_I(t)$ and $\sigma_E(t)$ are the current volatilities of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth; and $\sigma_{PF}^{I}(t)$ and $\sigma_{PF}^{E}(t)$ are the dispersion of forecasts from the Survey of Professional Forecasters. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols * denotes significance at the 1% level.

Table 9: Forecasts of Four-Quarter Cumulative Covariance of One-year and Five-Year Treasury Bonds and Macroeconomic Controls

No.	Sample	Const.	$\mathrm{Cov}\,(t-k,t)$	$C^{*}(t+1,t+4)$	NBER(t)	$R_{1Y}^{(-)}(t)$	$R_{5Y}^{(-)}(t)$	$r(t)$	Term(t)	$\sigma_I(t)$	$\sigma_E(t)$	$\sigma_{PF}^{I}(t)$	$\sigma_{PF}^{E}(t)$	$\bar{R}^2$
1	1960–2006	$5\cdot 10^{-4}$	0.752											0.565
		[-1.617]	[4.543]*											
2	1960–2006	$-2\cdot 10^{-4}$		0.981										0.622
		[-1.617]		[4.501]*										
3	1960–2006	-0.0001	0.392	0.572										0.691
		[-1.105]	[2.0094]	[2.551]*										
4	1960–2006	$-4\cdot 10^{-4}$			$4\cdot 10^{-4}$	0.015	-0.02	0.017	0.002	1.3	0.001			0.710
		[-0.753]			[1.450]	[0.221]	[-3.291]*	[5.191]*	[0.267]	[2.927]*	[0.246]			
5	1960–2006	$-4\cdot 10^{-4}$		0.729	$4\cdot 10^{-4}$	-0.013	-0.015	-0.003	-0.037	0.817	-0.001			0.753
		[-1.079]		[2.638]*	[1.203]	[-0.021]	[-0.2644]	[-0.469]	[-2.566]*	[1.843]	[-2.533]*			
6	1971-2006	$-9\cdot 10^{-4}$										-0.616	0.001	0.193
		[-2.019]										[-2.03]	[0.273]	
7	1971-2006	$-1\cdot 10^{-4}$		1.205								-0.569	$2\cdot 10^{-4}$	0.661
		[-0.999]		[4.827]*								[-2.369]	0.081	

This table reports the time series regressions

$$\mathrm{Cov}\,(t+1,t+k)=\beta_0+\beta_1\mathrm{Cov}\,(t-k,t)+\beta_2 C^{*}\,(t+1,t+4;t)+\beta_3\mathbf{X}\,(t)+\varepsilon\,(t+1,t+4)\,,$$

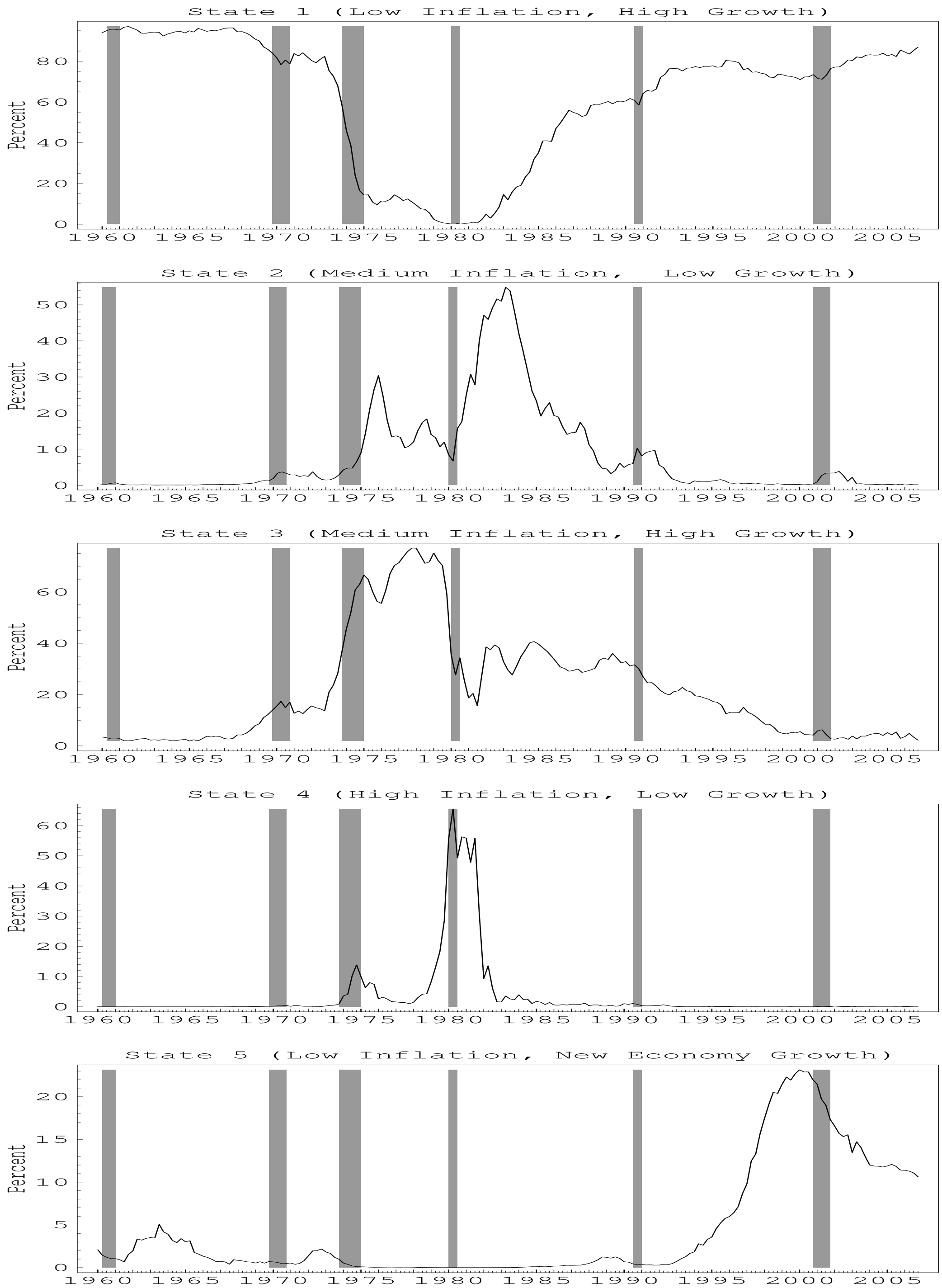
where Cov$(t+1,t+k)$ is the realized covariance between quarters $t+1$ and $t+k$, $C^{*}(t+1,t+4;t)$ is the optimal forecast of future covariance in the following four quarters in (22), and $\mathbf{X}\,(t)$ contains a vector of the following controls: NBER(t) is a business cycle dummy variable taking value = 1 during expansions as defined by the NBER; $R_{1Y}^{(-)}(t)$ is the return on the one-year Treasury Bond in quarter t if it is negative and zero otherwise; $R_{5Y}^{(-)}(t)$ is the return on the five-year Treasury Bond in quarter t if it is negative and zero otherwise; $r(t)$ is the 3-month Treasury Bill rate; Term(t) is the slope of the term structure defined as the five-year Treasury yield less the one-year Treasury yield; $\sigma_I(t)$ and $\sigma_E(t)$ are the current volatilities of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth; and $\sigma_{PF}^{I}(t)$ and $\sigma_{PF}^{E}(t)$ are the dispersion of forecasts from the Survey of Professional Forecasters. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols * denotes significance at the 1% level.

Table 10: Forecasts of Four-Quarter Cumulative Covariance of One-Year and Five-Year Treasury Bonds with Flight-to-Safety and Liquidity Controls

No.	Sample	Const.	$C^{*}(t+1,t+4)$	ATM(t)	Bond Illiquidity(t)	Cross Illiquidity(t)	R^2
Implied Volatility Control							
	Covariance of Stocks and One-Year T. Bonds						
1	1986:Q2-2006	$6 \cdot 10^{-4}$		-0.003			0.195
		[3.967]*		[-3.107]*			
2	1986:Q2-2006	-$6 \cdot 10^{-4}$	0.313				0.381
		[-3.230]*	[4.410]*				
3	1986:Q2-2006	-$1 \cdot 10^{-4}$	0.305	-0.003			0.500
		[-1.217]	[5.420]*	[-3.632]*			
	Covariance of Stocks and Five-year T. Bonds						
4	1986:Q2-2006	0.0042		-0.02			0.108
		[2.738]*		[-1.852]			
5	1986:Q2-2006	-$1 \cdot 10^{-4}$	0.511				0.449
		[-1.311]	[4.413]*				
6	1986:Q2-2006	$4 \cdot 10^{-4}$	0.492	-0.003			0.494
		[3.166]*	[4.717]*	[-3.765]*			
Liquidity Controls							
	Covariance of Stocks and One-year T. Bonds						
7	1962:Q3–2003	$3 \cdot 10^{-4}$			0.420	-0.128	0.417
		[-2.441]			[3.958]*	[-2.211]	
8	1962:Q3–2003	-$1 \cdot 10^{-4}$	0.397				0.591
		[-1.590]	[5.301]*				
9	1962:Q3–2003	-$3 \cdot 10^{-4}$	0.208		0.239	-0.097	0.657
		[-2.852]*	[3.891]*		[3.015]*	[-2.458]	
	Covariance of Stocks and Five-Year T. Bonds						
10	1962:Q3–2003	-0.001			2.072	-0.894	0.331
		[-1.452]			[3.668]*	[-3.477]*	
11	1962:Q3–2003	-$8 \cdot 10^{-4}$	0.476				0.490
		[-1.285]	[6.228]*				
12	1962:Q3–2003	-0.002	0.301		1.149	-0.756	0.538
		[-1.751]	[5.234]*		[2.273]	[-4.174]*	

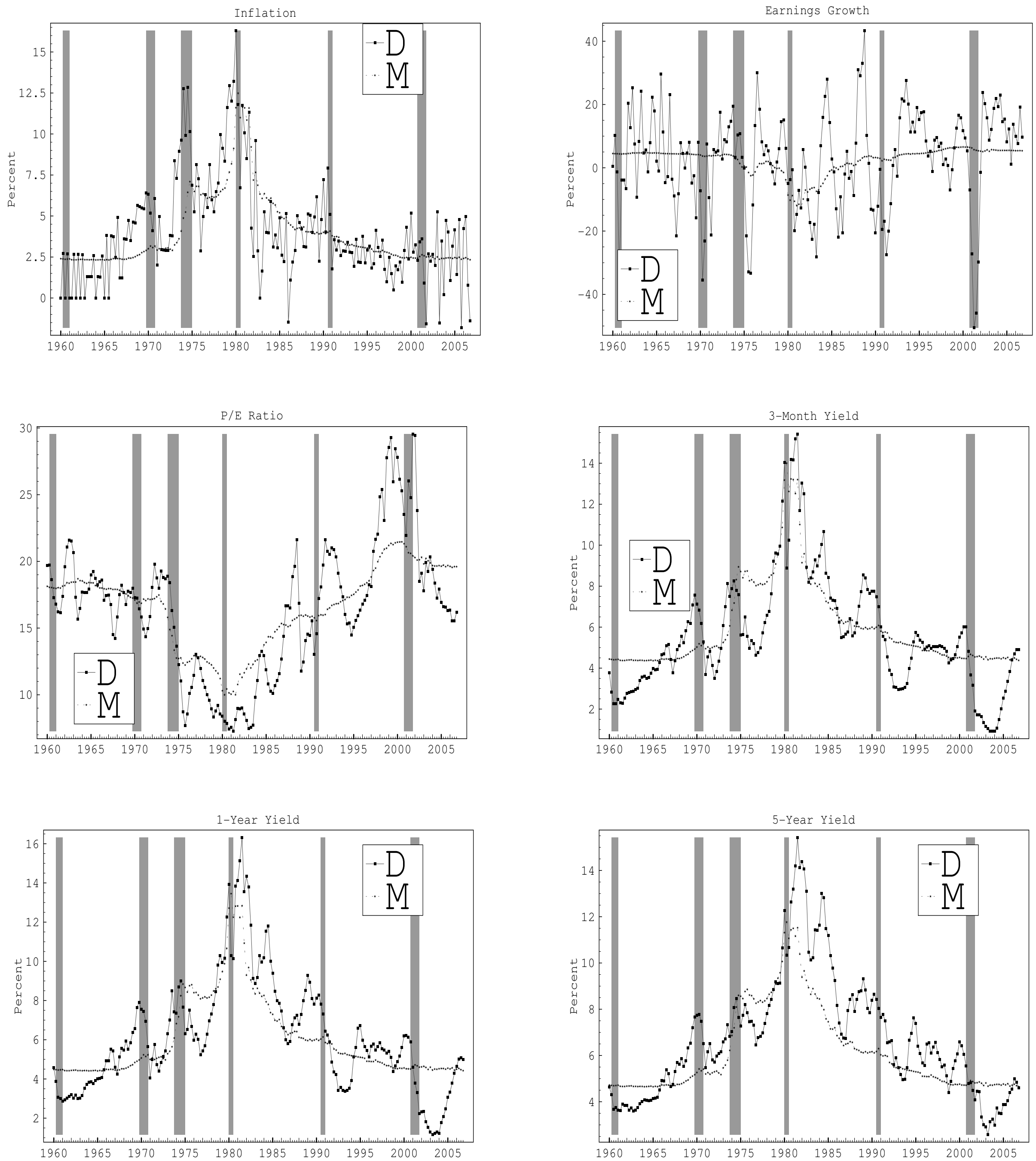
This table reports the time series regressions: Cov $(t+1,t+k) = \beta_0 + \beta_1 \text{Cov}\,(t-k,t) + \beta_2 C^{*}\,(t+1,t+4;t) + \beta_3 \mathbf{X}\,(t) + \varepsilon\,(t+1,t+4)$, where Cov$(t+1,t+k)$ is the realized covariance between quarters $t+1$ and $t+k$, $C^{*}(t+1,t+4;t)$ is the optimal forecast of future covariance in the following four quarters in (22), and $\mathbf{X}\,(t)$ contains a vector of the following controls: ATM(t) is the at-the-money implied volatility of S&P 500 index options; Bond Illiquidity(t) is the bond illiqudity series; Cross Illiquidity(t) is a measure of cross stock and bond market illiquidity. See Figure 6 for further description of these series. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols * denotes significance at the 1% level.

Figure 1: Conditional Probabilities of Five States (1960-2006)



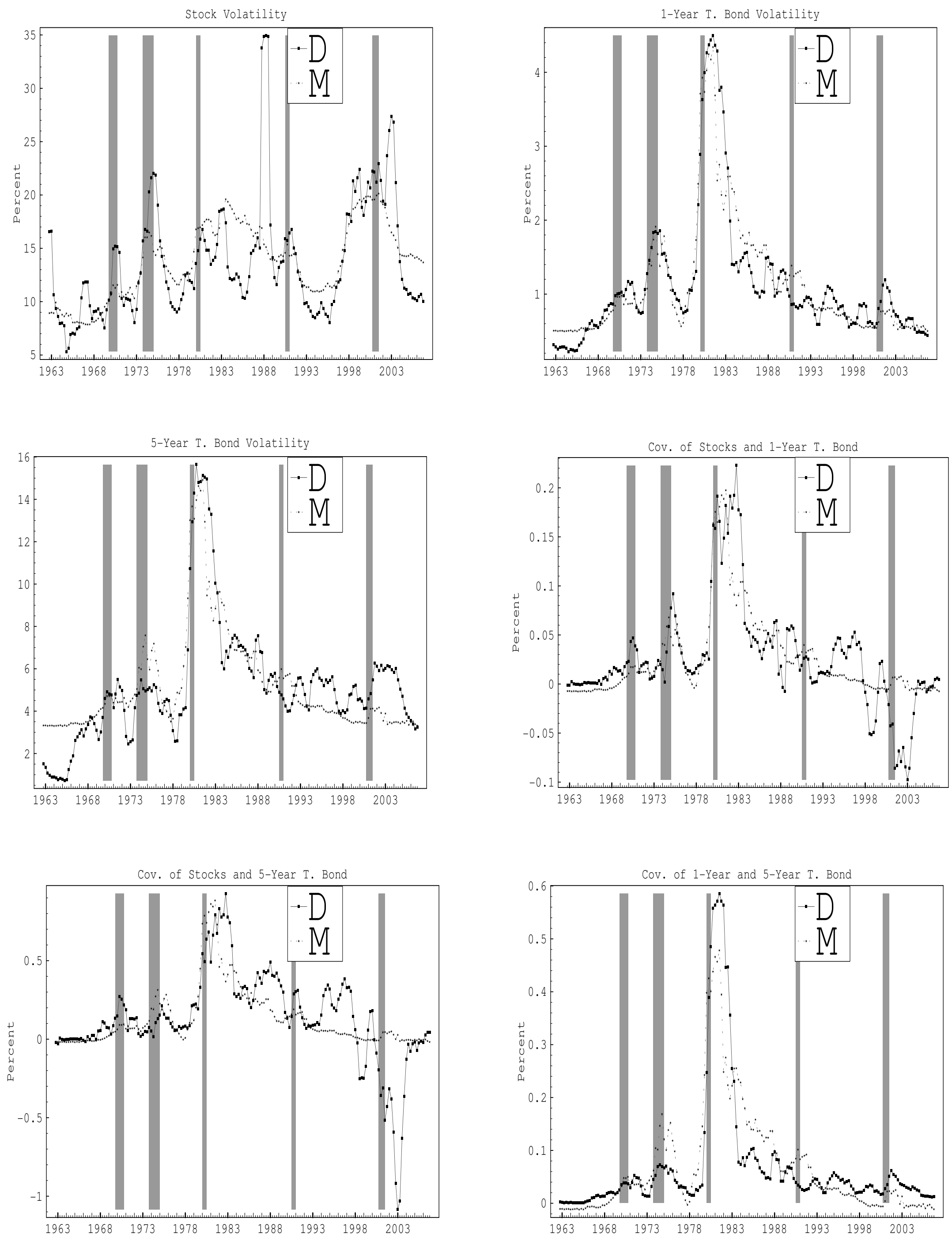
The states are numbered as (β_1, θ_2), (β_2, θ_1),(β_2, θ_2), (β_3, θ_1), and (β_1, θ_3), where β_i, $i = 1, 2, 3$ are the low, medium, and high states of inflation, and θ_1, θ_2, and θ_3, are the regular low and high states, and the "new economy" rates of earnings growth. The filtered beliefs are obtained from the SMM procedure in Appendix B. The calibrated values of the parameters are shown in Table 1. Shaded areas represent NBER-dated recessions.

Figure 2: Fundamental and Financial Variables: Empirical and Model Fitted (1960-2006)



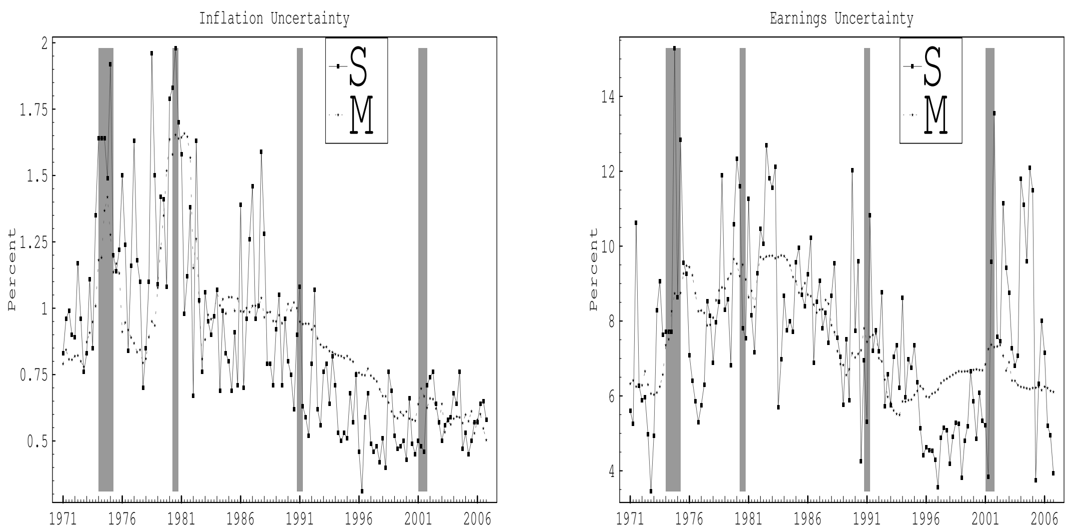
Historical values of financial and fundamental variables series (D) are in solid lines and their fitted values (M) from the SMM estimation procedure in Appendix B are in dashed lines. The calibrated values of the parameters are shown in Table 1. The filtered beliefs series of investors used to generate the fitted values are shown in Figure 1. Shaded areas represent NBER-dated recessions.

Figure 3: Optimal Forecasts of Cumulated Four-Quarter Volatilities and Covariance (1963 - 2006)



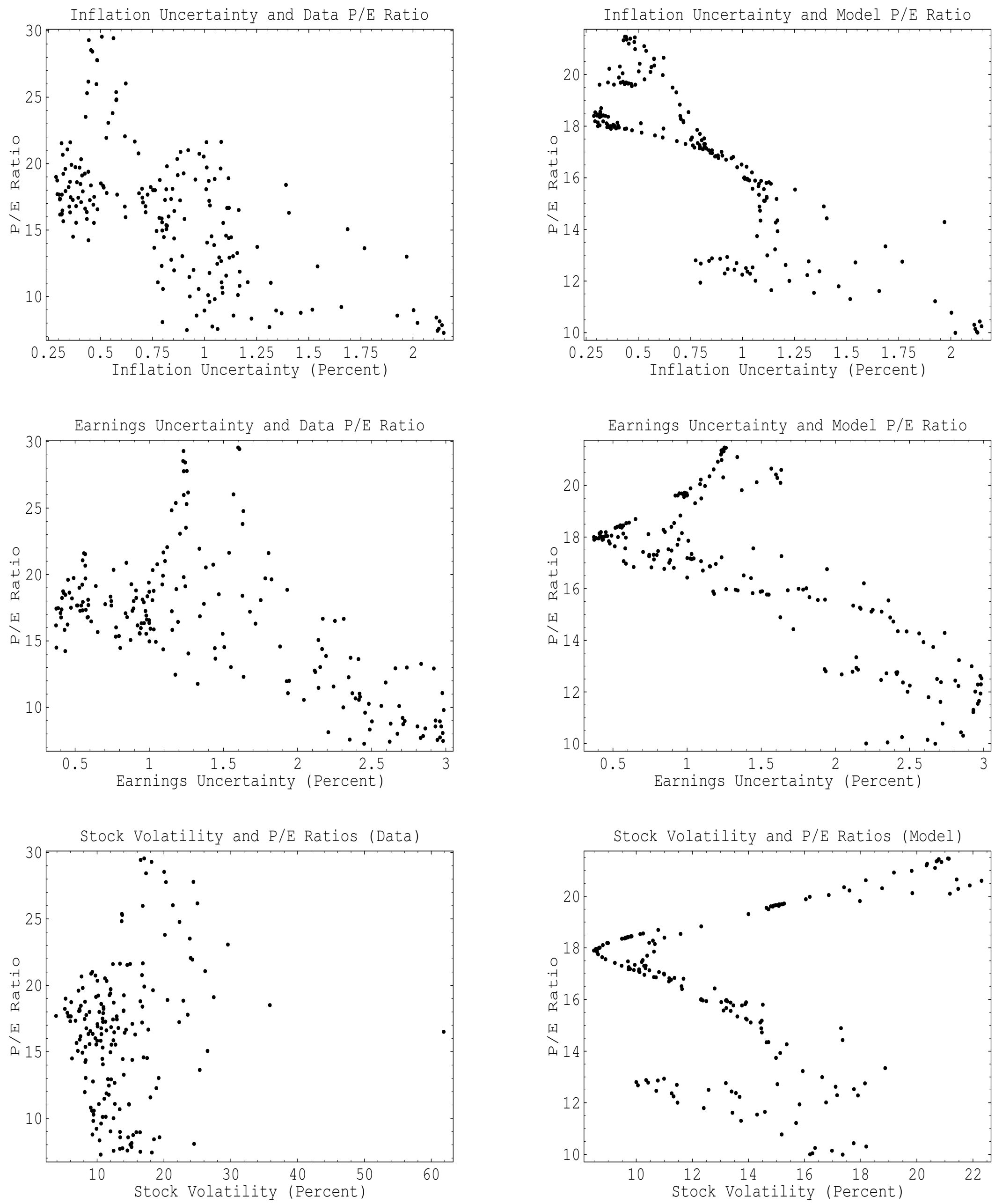
The volatility forecast for each variable is the fitted value from the regression: $\text{Vol}\,(t+1,t+4) = \beta_0 + \beta_2 V^*(t+1,t+4;t) + \varepsilon\,(t+1,t+4)$, where the optimal model-based forecast $V^*(t+1,t+4;t)$ is formulated as in (21). Similarly, the covariances are forecasted as the fitted values of the regression $\text{Cov}\,(t+1,t+k) = \beta_0 + \beta_2\, C^*(t+1,t+4;t) + \varepsilon\,(t+1,t+4)$, where the optimal covariance forecast is formulated as in (22). The calibrated values of the parameters are shown in Table 1. The filtered beliefs series of investors used to generate the fitted values are shown in Figure 1. Shaded areas represent NBER-dated recessions.

Figure 4: Survey and Model Based Uncertainty Measures Surrounding Inflation and Earnings Growth (1971–2006)



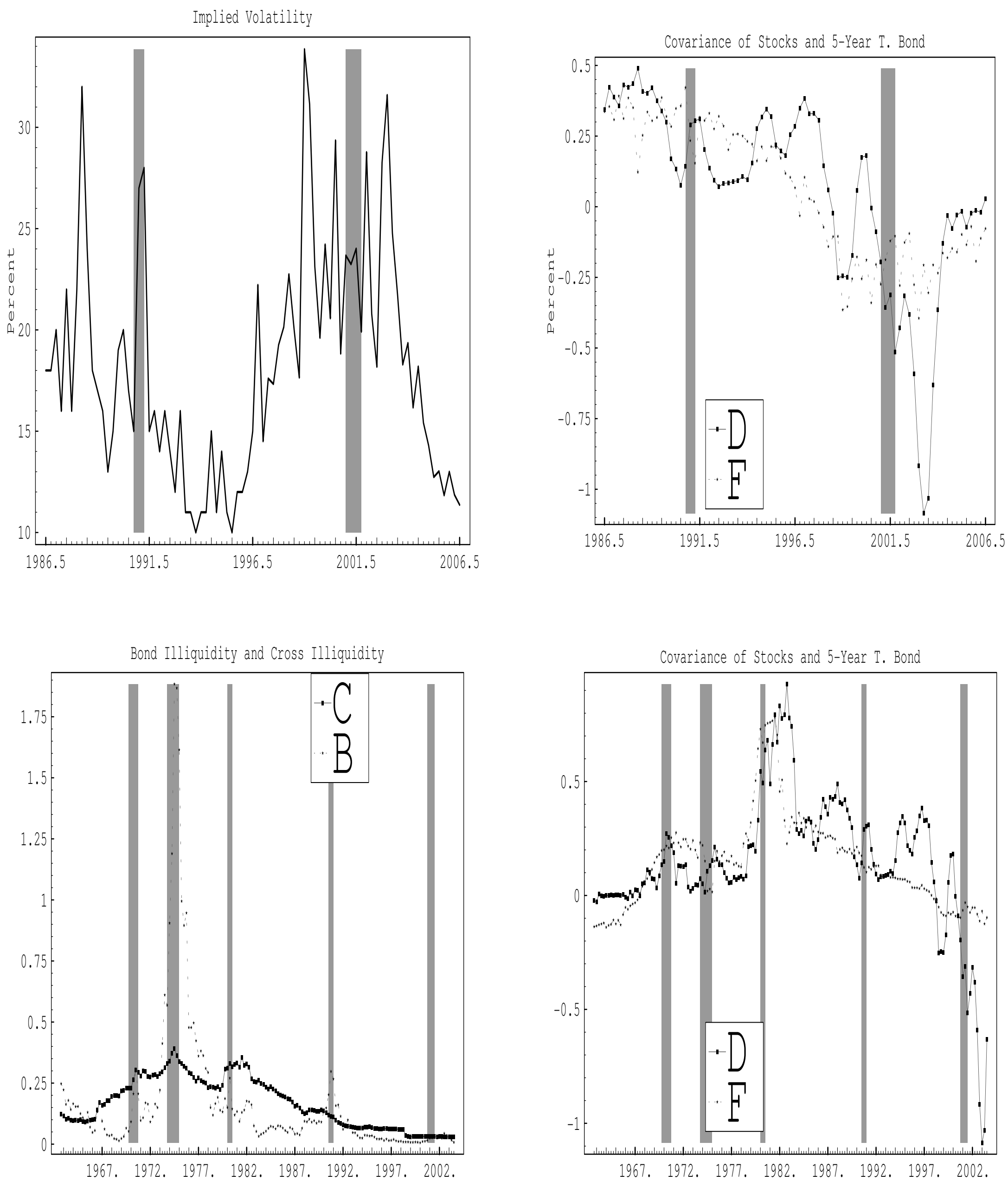
Survey based uncertainty measures (solid lines) are obtained from the Survey of Professional Forecasters data, maintained by the Federal Reserve Bank of Philadelphia. Inflation and earnings uncertainty (dispersion) in any given quarter is calculated using (37) and (38) respectively. We also provide the fitted values of these series (dashed lines) from a regression only on their RMSE counterparts. RMSE uncertainties are calculated as in (25) using the calibrated parameters as shown in Table 1 and investors' probabilities of underlying states as shown in Figure 1. The model-based measures explain 55 and 24 percent of the variation of the survey measure for inflation and earnings growth respectively. Shaded areas represent NBER-dated recessions.

Figure 5: Relationship Between Fundamental Uncertainties, Price-Earnings Ratios, and Stock Price Volatility (1962-2006)



Model-based inflation and earnings uncertainty measures are as displayed in Figure 4. Date and model P/Es are shown in Figure 2. Stock volatility is computed using (17), using the calibrated values of the parameters are shown in Table 1 and the filtered beliefs series of investors in Figure 1. The 1-year ahead forecasts of volatility are in Figure 3.

Figure 6: Model Based Covariance Forecasts Augmented by Implied Volatility (1986:Q2-2006:Q2) and Liquidity Measures (1962-2003)



"Implied Volatility" is the at-the-money implied volatility on S&P 500 options interpolated from data obtained from the CBOE (until 1996) and from Optionsmetrics (thereafter). "Bond Illiquidity" is the average bid-ask spread of off-the-run bonds as computed by Goyenko (2007), and "Cross Illiqudity" is the interaction of bond illiquidity and the stock market illiquidity measure of Amihud (2002). The latter has been scaled in the plot. Covariance is forecasted as the fitted values of the regression Cov $(t+1, t+k) = \beta_0 + \beta_2 C^* (t+1, t+4; t) + \beta_3 \mathbf{X}(t) + \varepsilon (t+1, t+4)$, where $\mathbf{X}$ contains either implied volatility or the illiquidity measures.

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