

Gravito-electromagnetism versus electromagnetism

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Abstract

The paper contains a discussion of the properties of the gravito-magnetic interaction in non stationary conditions. A direct deduction of the equivalent of Faraday-Henry law is given. A comparison is made between the gravito-magnetic and the electro-magnetic induction, and it is shown that there is no Meissner-like effect for superfluids in the field of massive spinning bodies. The impossibility of stationary motions in directions not along the lines of the gravito-magnetic field is found. Finally the results are discussed in relation with the behavior of superconductors.

1 Introduction

While teaching basic physics in universities both for science and for engineering curricula, the electromagnetic field and the gravitational field are still treated as completely separated topics. Even when the essentials of relativity are taught this separation is kept. Gravitation and electromagnetism are indeed different: the former is impied in the geometric properties of space time, the latter is a field living within that geometric environment on which it reacts back as any other field. It is however worth evidencing some strong analogies between the two theories, which inspired many a physicist in the late XIX century. Even today the similarity between the Maxwell equations on one side and the linearized Einstein equations on the other, is intriguing. It would be an error to overlook as well as to overestimate it. With the appropriate caveats the analogy could suggest interesting though extremely difficult experiments exploiting a 'gravitational' Faraday-Henry law. We think that, in general, students should have a glimpse to the interplay between classical electromagnetism and General Relativity. A glimpse

on the side of the students means attention and commitment on the side of the teacher. Here we shall try to summarize and underline the essentials of the correspondence between the two theories.

Since the early times of general relativity it is known that linearizing the Einstein field equations in vacuo leads to a form almost identical to the Maxwell equations of electromagnetism[1]. In practice in the linear approximation the gravitational interaction may be thought of as if it were the effect of a gravito-electromagnetic field. The gravito-electric field is the known Newtonian solution. The gravito-magnetic part is an unexpected contribution much similar to the magnetic field originated by electric charge currents. There have been many attempts to exploit the electromagnetic analogy in order to evidence gravito-magnetic effects. Observational or experimental activities have also been set up to directly reveal the effects[2],[3],[4]. What we would like to discuss here is to what extent the electromagnetic analogy can be exploited.

Once the analogy has been established there are many consequences one can draw, for instance a gravito-magnetic induction can be expected from the analogue of Faraday-Henry law

$$\vec{\nabla} \wedge \vec{E}_g = -\frac{1}{2} \frac{\partial \vec{B}_g}{\partial t} \quad (1)$$

Here \vec{E}_g is the gravito-electric part of the gravitational field, and \vec{B}_g its gravito-magnetic part. A time varying gravito-magnetic field will induce a gravito-electric field, and vice versa.

Of course there are some caveats: E_g is a (three)acceleration, B_g is the inverse of a time much like an angular velocity; the coupling parameter is now the mass, and for \vec{B}_g it doubles the value it has for \vec{E}_g (hence the 1/2 factor in eq. 1). Furthermore a peculiar difference in sign in the equivalent of the Ampère-Maxwell equation brings about some interesting consequences which develop in actual inconsistencies when inadvertently exceeding the limits of the basic approximation. The most important point to be advised of is that the linearized equations of gravito-electromagnetism are neither really gauge-invariant nor generally covariant[5].

Despite the necessary cautions, however, eq. (1) lends a principle possibility to reveal gravito-magnetism, so it is useful to analyze it better.

In this paper we shall deduce (1) not from the linearized Einstein equations, but directly from the equations of motion of a mass and from the metric tensor. The approximation we shall use is consequent to the hypothesis that all velocities will be small as compared with the speed of light, and

the field will be weak enough. We shall then discuss the equivalent of the Meissner effect for superconductors when a superfluid is taken into account. As we shall see, the behavior induced in matter by gravito-magnetism will be different from what happens with electro-magnetism in superconductors. Finally we shall discuss the implication that some kinds of stationary motions of a fluid in gravito-magnetic fields are untenable.

2 Direct deduction of the gravito-electromagnetic Faraday-Henry law

Let us consider an axially symmetric stationary space time. Its line element may be written:

$$ds^2 = g_{tt}dt^2 + g_{xx}dx^2 + g_{yy}dy^2 + g_{zz}dz^2 + 2g_{tx}dtdx + 2g_{ty}dtdy$$

If

$$\begin{aligned} g_{tx} &= -B_g y/2 \\ g_{ty} &= B_g x/2 \end{aligned}$$

the space-time contains a constant gravito-magnetic field B_g along the z axis of a Cartesian coordinate system. Otherwise the space-time is assumed to be flat, i.e.

$$g_{\mu\nu} = \begin{pmatrix} c^2 & -B_g y/2 & B_g x/2 & 0 \\ -B_g y/2 & -1 & 0 & 0 \\ B_g x/2 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (2)$$

Strictly speaking, this would be the situation inside a steadily rotating massive shell. This field has been studied by Lense and Thirring[6] soon after the publication of the General Theory of Relativity, by Brill and Cohen[7] in the 1960's, in connection with the effect of rotating masses on inertial frames, and also, more recently, by Ciufolini and Ricci[8], who studied the effect of rotation on the time delay.

It is interesting to compare the metric tensor (2) with the line element of Minkowski space time as seen from a steadily rotating reference frame (rotation about the z axis):

$$ds^2 = (c^2 - \omega^2 x^2 - \omega^2 y^2) dt^2 - dx^2 - dy^2 - dz^2 - 2\omega y dx dt + 2\omega x dy dt \quad (3)$$

As far as the peripheral rotation speed $\omega\sqrt{x^2 + y^2}$ is negligible with respect to c (2) and (3) coincide with $B_g/2$ playing the role of ω .

Explicitly writing the Christoffels corresponding to (2) produces:

$$\begin{aligned}\Gamma_{tx}^t &= \frac{B_g^2 x}{4c^2 + B_g^2(x^2 + y^2)} & \Gamma_{ty}^t &= \frac{B_g^2 y}{4c^2 + B_g^2(x^2 + y^2)} \\ \Gamma_{tx}^x &= -\frac{B_g^3 xy}{8c^2 + 2B_g^2(x^2 + y^2)} & \Gamma_{ty}^x &= \frac{B_g(B_g^2 x^2 + 4c^2)}{8c^2 + 2B_g^2(x^2 + y^2)} \\ \Gamma_{tx}^y &= -\frac{B_g(B_g^2 y^2 + 4c^2)}{8c^2 + 2B_g^2(x^2 + y^2)} & \Gamma_{ty}^y &= -\frac{B_g^3 xy}{8c^2 + 2B_g^2(x^2 + y^2)}\end{aligned}$$

Assuming, as said in the Introduction, that all velocities are small when compared to the speed of light, we can write, at the lowest approximation order, $ds = cdt$. Furthermore, under the same assumption the analogy between B_g and an angular speed suggests that $B_g\sqrt{(x^2 + y^2)} \ll c$. The significant Christoffels are then reduced to

$$\begin{aligned}\Gamma_{ty}^x &= \frac{B_g}{2} \\ \Gamma_{tx}^y &= -\frac{B_g}{2}\end{aligned}$$

The components of the covariant four-acceleration a (same approximation as above) are:

$$\begin{aligned}a^0 &\simeq 0 \\ a^x &= \frac{d^2x}{c^2 dt^2} + B_g \frac{dy}{c^2 dt} \\ a^y &= \frac{d^2y}{c^2 dt^2} - B_g \frac{dx}{c^2 dt} \\ a^z &= \frac{d^2z}{c^2 dt^2}\end{aligned}\tag{4}$$

Suppose now that a material point is constrained to move so that:

$$\begin{aligned}x &= R \cos \theta \\ y &= R \sin \theta \cos(\Omega t) \\ z &= R \sin \theta \sin(\Omega t)\end{aligned}\tag{5}$$

In practice we are assuming that, whatever the probe we are using is, it is free to move along a rigidly steadily rotating circular ring; otherwise stated, we are considering a fluid inside a rotating annular tube. R is the radius of the ring, Ω is its angular velocity; rotation takes place about the x axis; $\theta(t)$ is an angular parameter showing the position along the ring. This is the same configuration as for a typical electromagnetic induction experiment in an alternator.

Let us now introduce (5) into (4). Since the motion is not geodesic the result will in general be different from zero; it will correspond to the components of the four-force per unit mass constraining the probe to stay in the ring.

Explicitly, and considering the only space components:

It will be

$$\begin{aligned}
c^2 a^x &= a^x = -R \frac{d^2 \theta}{dt^2} \sin \theta - R \left(\frac{d\theta}{dt} \right)^2 \cos \theta + B_g R \left(\frac{d\theta}{dt} \cos \theta \cos(\Omega t) - \Omega \sin \theta \sin(\Omega t) \right) \\
c^2 a^y &= a^y = R \frac{d^2 \theta}{dt^2} \cos \theta \cos(\Omega t) - R \left(\frac{d\theta}{dt} \right)^2 \sin \theta \cos(\Omega t) - 2R\Omega \frac{d\theta}{dt} \cos \theta \sin(\Omega t) \\
&\quad - R\Omega^2 \sin \theta \cos(\Omega t) + B_g R \frac{d\theta}{dt} \sin \theta \\
c^2 a^z &= a^z = R \frac{d^2 \theta}{dt^2} \cos \theta \sin(\Omega t) - R \left(\frac{d\theta}{dt} \right)^2 \sin \theta \sin(\Omega t) + 2R\Omega \frac{d\theta}{dt} \cos \theta \cos(\Omega t) \\
&\quad - R\Omega^2 \sin \theta \sin(\Omega t)
\end{aligned}$$

The constraints represented by the walls of the circular ring of course react to the forces orthogonal to the wall. The only unconstrained direction will be the one tangent to the ring. For that direction we can write:

$$a^x \sin \theta - a^y \cos \theta \cos(\Omega t) - a^z \cos \theta \sin(\Omega t) = 0 \quad (6)$$

Finally, introducing (6) in (6), the approximated expression for the angular acceleration along a rotating ring in presence of a gravito-magnetic field will be:

$$\frac{d^2 \theta}{dt^2} \simeq -B_g \Omega \sin(\Omega t) \sin^2 \theta + \Omega^2 \sin \theta \cos \theta$$

The second term corresponds to the transverse contribution of the centrifugal acceleration. The other one is the true gravito-magnetic induction contribution, if we like to call it so.

The linear acceleration will of course be

$$R \frac{d^2 \theta}{dt^2} \simeq -B_g R \Omega \sin(\Omega t) \sin^2 \theta + R \Omega^2 \sin \theta \cos \theta$$

Integrating along the length of the ring one has a work per unit mass (gravito-electromotive force)

$$\begin{aligned}
\mathcal{F} &= \int_0^{2\pi} R^2 \left(-B_g \Omega \sin(\Omega t) \sin^2 \theta + \Omega^2 \sin \theta \cos \theta \right) d\theta \\
&= -\pi R^2 \Omega B_g \sin \Omega t
\end{aligned} \quad (7)$$

This in practice is the equivalent of Faraday-Henry law for classical electrodynamics, in integral form; the usual application of Stoke's theorem brings about the differential form (1).

In fact if we consider the g_{ti} 's of our metric tensor as the components of a gravito-magnetic vector potential, the corresponding gravito-magnetic field will be B_g and (7) is the time derivative of the flux of \vec{B}_g across the area of the ring. Furthermore, we have also practically checked the validity of a Lorentz-like force law $\vec{a} = \vec{v} \wedge \vec{B}_g$. Indeed, considering (4) and recalling that the constrained motion has only v_y and v_z components of the velocity, and the gravito-magnetic field is along the z axis we see that the B_g dependent acceleration has the form of the typical vector product in the Lorentz force formula.

Up to this moment no troubles arise, provided the approximation conditions are satisfied.

If one considers the ponderomotive force corresponding to (7) we can think of a principle means to reveal the existence of a gravito-magnetic field. This could happen in a superconductor ring equipped with a Josephson junction, though in that case there would be a serious problem with the pure magnetic effects. Non inertial effects in superconductors, according to the scheme we mentioned, have been studied by Fisher et al. [9]. Another possibility would be to consider a superfluid flow in a ring shaped tube; in this case one would avoid troubles with magnetic interactions.

3 Gravitational Meissner effect

Considering superconducting devices implies considering the typical Meissner effect too. Is there a gravito-magnetic analog of the Meissner effect? The answer we can find in the literature is partly yes, as in DeWitt[10], Li-Torr[11],[12]¹; there however a superconductor was analyzed so that there was an interplay between electromagnetic and gravito-electromagnetic interactions. The conclusion was that the gravito-magnetic, as well as the magnetic field, weaken when penetrating into the bulk of a superconductor, though the typical penetration length for the gravito-magnetic part is so big that the effect is practically irrelevant. If however one wants to consider a real analog of the pure Meissner effect, one should treat the pure gravito-

¹However, the papers by Li-Torr have been criticized by Kowitt[13], who pointed out that they *grossly overestimated* the magnitude of the effects; we may add that the paper [11] contains a little mistake: the typical 1/2 factor expressing the difference between the gravito-electric and gravito-magnetic mass is missing.

magnetic case. To be more definite, we should envisage a situation where matter can flow without friction in response to a gravito-electromagnetic field, i.e. matter in a pure superfluid state.

Let us start with the Maxwell-like equations for the gravito-electromagnetic field:

$$\left\{ \begin{array}{l} \vec{\nabla} \cdot \vec{E}_g = -4\pi G\rho \\ \vec{\nabla} \cdot \vec{B}_g = 0 \\ \vec{\nabla} \wedge \vec{E}_g = -\frac{1}{2} \frac{\partial \vec{B}_g}{\partial t} \\ \vec{\nabla} \wedge \vec{B}_g = -\frac{8\pi G}{c^2} \vec{j}_g + \frac{2}{c^2} \frac{\partial \vec{E}_g}{\partial t} \end{array} \right. \quad (8)$$

So far we have seen that the third equation is O.K. deducing it, in its integral form, directly from the equations of motion. As for the other ones, they are obtained from the linearized Einstein equations[14]. Let us concentrate on the last pair of equations and try and solve them in general.

In this case we must also consider that \vec{E}_g is the three-acceleration effective in changing the absolute value of the three-velocity of matter along a loop, the one entering the very definition of the matter current density. In weak field conditions, it is

$$\vec{j}_g = \frac{\rho \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (9)$$

As far as the velocity of the matter flow is small enough and we can assume that the self-perturbation of the matter density ρ is negligible, we have

$$\frac{\partial \vec{j}_g}{\partial t} = \rho \vec{E}_g$$

Let us now differentiate the last Maxwell-like equation with respect to time, then introduce into it the third one:

$$\vec{\nabla} \wedge \vec{\nabla} \wedge \vec{E}_g = \frac{4\pi G}{c^2} \rho \vec{E}_g - \frac{1}{c^2} \frac{\partial^2 \vec{E}_g}{\partial t^2} \quad (10)$$

This corresponds to

$$-4\pi G \vec{\nabla}(\rho) - \nabla^2 \vec{E}_g = \frac{4\pi G}{c^2} \rho \vec{E}_g - \frac{1}{c^2} \frac{\partial^2 \vec{E}_g}{\partial t^2}$$

Suppose that inside matter it is $\vec{\nabla}(\rho) = 0$; the equation reads

$$\nabla^2 \vec{E}_g = \frac{1}{c^2} \frac{\partial^2 \vec{E}_g}{\partial t^2} - \frac{4\pi G}{c^2} \rho \vec{E}_g$$

Introducing the time Fourier transform $\vec{\mathcal{E}}(r, \omega) = \int_{-\infty}^{\infty} \vec{E}_g e^{i\omega t} dt$ we have

$$\nabla^2 \vec{\mathcal{E}} = -\frac{\omega^2}{c^2} \vec{\mathcal{E}} - \frac{4\pi G}{c^2} \rho \vec{\mathcal{E}}$$

The solution is

$$\begin{aligned} \vec{\mathcal{E}} &= \vec{\mathcal{E}}_0(\omega) e^{ikr} \\ k &= \frac{1}{c} \sqrt{\omega^2 + 4\pi G\rho} \end{aligned}$$

Here r stands for a space coordinate orthogonal to the surface of the material. Introducing this result into the third equation of (8) we see of course that the space distribution of the gravito-magnetic \vec{B}_g field is of the same type as for the gravito-electric field. It oscillates in space, rather than being damped. There is no equivalent of the electromagnetic plasma frequency: the exponential e^{ikr} never becomes real. The wavelength of the space oscillation in static conditions ($\omega = 0$) is

$$\lambda = c \sqrt{\frac{\pi}{G\rho}}$$

In practice the numeric value of λ for ordinary situations would be $\sim 10^{12}$ m. The difference with respect to a damped trend is substantially irrelevant, however we can state that in principle in superfluids there is no analog of the Meissner effect in superconductors. Differently phrased, recalling a remark by Pascual-Sanchez [15], superfluids in gravito-magnetic fields display a paramagnetic-like behavior rather than a diamagnetic-like one.

4 Inconsistencies of the gravito-electromagnetic analogy

The result regarding the Meissner effect may induce the suspect that something is wrong with the last Maxwell-like equation. In fact the behavior we have seen depends on the '-' sign in front of the matter current density in the last equation of (8). That little '-' produces indeed other apparently anomalous, not to say incongruous, consequences. Essentially we can say that the lines of force of \vec{B}_g circulate around the matter flux in clockwise sense, opposite to what happens with a magnetic field with respect to an electric current.

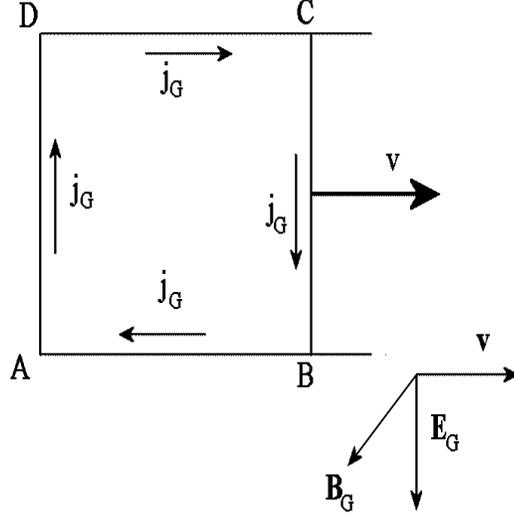


Figure 1: Scheme of a fluid circuit with a moving arm, immersed in a gravito-magnetic field oriented orthogonally with respect to the figure, toward the observer.

Let us begin with a simple example. The gravito-magnetic Faraday-Henry law (7) is formally equivalent to the induction law for the electromagnetic fields. So, we can reasonably expect that a gravito-magnetic time-varying flux Φ_g induces a gravito-electric field \vec{E}_g . We born ourselves to the simplest situation one can imagine.

Suppose that a massive fluid is constrained to move in a circuit $ABCD$, where the CB side is moving with constant velocity V ; everything is immersed in a gravito-magnetic field \vec{B}_G , perpendicular to the plane and directed toward the reader (see figure 1). The massive particles in the segment CB are acted upon by a Lorentz-like force

$$\vec{F} = m\vec{V} \wedge \vec{B}_g$$

so they start moving under the influence of the induced gravito-electric field $\vec{E}_g = \frac{\vec{F}}{m}$ and a mass current density \vec{j}_g appears, as depicted.

According to eq. $\vec{\nabla} \wedge \vec{B}_g = -\frac{8\pi G}{c^2} \vec{j}_g + \frac{2}{c^2} \frac{\partial \vec{E}_g}{\partial t}$ this current is the source for a gravito-magnetic field \vec{b}_g , which satisfies the gravito-magnetic Ampere

law

$$\oint \vec{b}_g \cdot \vec{dl} = -\frac{8\pi G}{c^2} i_g$$

Of course i_g is the total mass current in the circuit.

Because of the "minus" sign, this field is directed in the same direction as the initial \vec{B}_g field, whose varying flux induces the current! In practice the system would diverge. Of course an increasing current leads soon to a violation of the linearization conditions, but the relevant fact is that, in a sense, the approximation is unstable.

5 Discussion

The fact that the linearization of the Einstein equations, which produces the Maxwell-like equations (8) has strong limitations is well known[16],[5], since it leads to a non self consistent theory. The point which has been considered in the literature concerns the issue of energy, since gravitational forces "*do no significant work*" and the energy-stress tensor $T^{\mu\nu}$ is conserved independently of the action of the gravitational fields.

Gravitational energy, however, is in general an open problem even in the exact theory.

Here we can see that the energy balance, in the style of special relativity, is indeed satisfied.

We have shown, with the simple example represented in figure 1, that the gravito-electromagnetic induction would produce an indefinitely increasing matter flux in a fluid. This result is of course paradoxical in a non-relativistic approach. However, as far as the speed of matter particles (the flux, indeed) increases, purely special relativistic effects cannot be neglected. In practice to keep the translation speed constant requires a force. Using special relativistic formulas we expect the force (in the direction of \vec{V}) to be

$$\begin{aligned} F &= m \frac{d}{dt} \left(\frac{V}{\sqrt{1 - \frac{V^2+v^2}{c^2}}} \right) \\ &= \frac{mV}{\left(1 - \frac{V^2+v^2}{c^2}\right)^{3/2}} \frac{v}{c^2} \frac{dv}{dt} \\ &= \frac{mV}{1 - \frac{V^2+v^2}{c^2}} \frac{j}{\rho c^2} E \end{aligned} \tag{11}$$

In the formula, m is the rest mass of the flowing matter in the moving arm of the circuit; the mass of the container (that is also moving) has been

neglected, because, under the hypothesis $V = \text{constant}$ its time derivative would be zero. The small v stands for the velocity of the flowing matter, which, in our example, is orthogonal to the translational motion of the CB arm. E is the acceleration induced in the flow by the gravito-magnetic interaction.

A consequence of what we have seen is that no static fluid flow orthogonal to \vec{B}_g is possible since it would require an asymptotically infinite force and would absorb an infinite energy. Nothing can be kept in motion in a gravito-magnetic field if not along the lines of the field.

The same conclusion can be attained in the case of a closed fluid circuit set to rotation about an axis not aligned with the field. In other words a fluid gyroscope cannot maintain its angular momentum constant if not aligned with the field.

In drawing this conclusion we have released the condition of small velocities but not the one of having a weak external field. Of course if the system actually reaches relativistic conditions we can no longer neglect the back reaction on the structure of the global field, however we do not expect the non-linearities to modify qualitatively the result concerning the untenability of stationary motions in a gravito-magnetic field.

Actually all we have said is referred to uncharged flowing matter. The situation changes when electric currents or supercurrents are considered. As already said, Li and Torr[11] showed that in a superconductor magnetic and gravito-magnetic fields are indeed coupled and the final result is that, in the material, both fields decay exponentially with increasing depth.

In our case we can describe the situation as follows. In a pure superconductor a varying magnetic field \vec{B} induces a supercurrent whose back reaction is a field that entirely compensates the changes in \vec{B} . If a varying external gravito-magnetic field is also present, its effect is to contribute to a mass flow in the superconductor whose effect would lead to a continuous increase of the flow itself. However, since the matter flow is also an electric current, the magnetic field of the current will be strong enough to stabilize the system. Summing up, a superconductor loop in a varying gravito-magnetic field will indeed reach a state of dynamic equilibrium with a supercurrent (then the corresponding magnetic field) a bit stronger than it would be the case without the gravito-magnetic interaction. Unfortunately, if the gravito-magnetic field is the one of the Earth its effect is extremely small [17].

We think that our description of the gravito-magnetic induction, compared and contrasted with the electro-magnetic one, can help in shedding light on the properties of the gravitational field of rotating bodies, to the

benefit of graduate students who are interested in the subject, but seldom have the opportunity to adequately approach it. Even for undergraduate students, the comparison of electro-magnetism and gravitation, if presented together with a historical sketch of the evolution of both theories, should lead to a better understanding of the phenomena they describe.

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