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Categorizing Different Approaches to the Cosmological Constant Problem

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Abstract. We have found that proposals addressing the old cosmological constant problem come in various categories. The aim of this paper is to identify as many different, credible mechanisms as possible and to provide them with a code for future reference. We find that they all can be classified into five different schemes of which we indicate the advantages and drawbacks.

Besides, we add a new approach based on a symmetry principle mapping real to imaginary spacetime.

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Contents

1. Statement of the Problem

It is clear that the cosmological constant problem is one of the major obstacles for both particle physics and cosmology to further progress. Actually, after the remarkable discoveries and subsequent confirmations starting in 1997 that the universe really is accelerating its expansion, there are at present at least three cosmological constant problems! In a nutshell these are: Why is it so small, why is it then not exactly equal to zero and why is its energy density today of the same order of magnitude as the matter energy density? Although the recent observations concerning the accelerated expansion are usually attributed to a small, non-vanishing Λ, alternatives have been suggested, some of which we briefly discuss.

In this overview we will be mainly concerned with the first of these questions, the so-called "old cosmological constant problem". To phrase it more precisely, the question is why is the effective cosmological constant, Λ_{eff}, defined as $\Lambda_{eff} = \Lambda + 8\pi G\langle\rho\rangle$‡ so close to zero. Or, in other words, why is the vacuum state of our universe (at present) so close to the classical vacuum state of zero energy, or perhaps better, why is the resulting four-dimensional curvature so small, or why does Nature prefer a flat spacetime?

The different contributions to the vacuum energy density coming from ordinary particle physics would naively give a value for $\langle\rho\rangle$ of order M_P^4, which then would have to be (nearly) cancelled by the unknown 'bare' value of Λ. Note at this point that only the Λ_{eff} is observable, not Λ, so the latter quantity may be referred to as a 'bare', or perhaps classical quantity that has to be 'dressed' by quantum corrections, analogous to all other physical parameters in ordinary quantum field theory.

This cancellation has to be better than about 120 decimal places if we compare the zero-point energy of a scalar field, using the Planck scale as a cut-off, and the experimental value of $\rho_{vac} = \langle\rho\rangle + \Lambda/8\pi G$, being 10^{-47}GeV^4. As is well known, even if we take a TeV scale cut-off the difference between experimental and theoretical results still requires a fine-tuning of about 50 orders of magnitude. This magnificent fine-tuning seems to suggest that we miss an important point here. In this paper we give an overview of the main ideas that have appeared in trying to figure out what this point might be.

We have found that proposals addressing this problem come in various categories. The aim of this paper is to identify as many different, credible mechanisms as possible and to provide them with a code for future reference. Our identification code will look as follows, see table (1):

‡ Note that using this definition we use units in which the cosmological constant has dimension GeV^2 throughout.

Table 1. Classification of different approaches. Each of them can also thought of as occurring 1) Beyond 4D, or 2) Beyond Quantum Mechanics, or both.

Type 0: Just Finetuning	
Type I: Symmetry; A: Continuous	a) Supersymmetry
	b) Scale invariance
	c) Conformal Symmetry
B: Discrete	d) Imaginary Space
	e) Holography
	f) Sub-super-Planckian
	g) Antipodal Symmetry
	h) Duality Transformations
Type II: Back-reaction Mechanism	a) Scalar
	b) Gravitons
	c) Screening Caused by Trace Anomaly
	d) Brane and Black Hole Production
Type III: Violating Equiv. Principle	a) Massive Gravitons
	b) Non-local Gravity
	c) Fat Gravitons
	d) Composite graviton as Goldst. boson
Type IV: Statistical Approaches	a) Hawking Statistics
	b) Anthropic Principle, Cont.
	c) Anthropic Principle, Discrete

In other words, an approach examining 6-dimensional supersymmetry for a solution will be coded Type IAa1.

2. Type 0: Finetuning

One can set the cosmological constant to any value one likes, by simply adjusting the value of the bare cosmological constant. No further explanation then is needed. This fine-tuning has to be precise to better than at least 55 decimal places (assuming some TeV scale cut-off), but that is of course not a practical problem. Since we feel some important aspects of gravity are still lacking in our understanding and nothing can be learned from this 'mechanism', we do not consider this to be a physical solution. However, it is a possibility that we can not totally ignore and it is mentioned here just for sake of completeness.

3. Type I: Symmetry Principle

A natural way to understand the smallness of a physical parameter is in terms of a symmetry that altogether forbids any such term to appear. This is also often referred to as 'naturalness': a theory obeys naturalness only if all of its small parameters would lead to an enhancement of its exact symmetry group when replaced by zero. Nature has provided us with several examples of this. Often mentioned in this respect is the example of the mass of the photon. The upper bound on the mass (squared) of the photon from terrestrial measurements of the magnetic field yields:

$$m_\gamma^2 \lesssim \mathcal{O}(10^{-50})\text{GeV}^2. \tag{1}$$

The most stringent estimates on Λ_{eff} nowadays give:

$$\Lambda_{eff} \lesssim \mathcal{O}(10^{-84})\text{GeV}^2 \tag{2}$$

We 'know' the mass of the photon to be in principle exactly equal to 0, because due to the $U(1)$ gauge symmetry of QED, the photon has only two physical degrees of freedom (helicities). In combination with Lorentz invariance this sets the mass equal to zero. This suggests that there might also be a symmetry acting to keep the effective cosmological constant an extra 34 orders of magnitude smaller.

A perhaps better example to understand the smallness of a mass is chiral symmetry. If chiral symmetry were an exact invariance of Nature, quark masses and in particular masses for the pseudoscalar mesons (π, K, η) would be zero. The spontaneous breakdown of chiral symmetry would give pseudoscalar Goldstone bosons, which would be massless in the limit of zero quark mass. The octet (π, K, η) would be the obvious candidate and indeed the pion is by far the lightest of the mesons. Making this identification of the pion being the pseudo-Goldstone boson associated with spontaneous breaking of chiral symmetry, we can understand why the pion-mass is so much smaller than for example the proton mass.

3.1. Supersymmetry

One symmetry with this desirable feature is supersymmetry. The quantum corrections to the vacuum coming from bosons are of the same magnitude, but opposite sign compared to fermionic corrections, and therefore cancel each other. The vacuum state in an exactly supersymmetric theory has zero energy. However, supersymmetric partners of the Standard Model particles have not been found, so standard lore dictates that SUSY is broken at least at the TeV scale, which induces a large vacuum energy.

One often encounters some numerology in these scenario's, e.g. [1], linking the scale of supersymmetry breaking M_{susy} and the Planck mass M_P, to the cosmological constant. Experiment indicates:

$$M_{susy} \sim M_P \left(\frac{\Lambda}{M_P^2}\right)^\alpha, \qquad \text{with} \quad \alpha = \frac{1}{8} \tag{3}$$

The standard theoretical result however indicates $M_{susy} \sim \Lambda^{1/2}$.

However, to discuss the cosmological constant problem, we need to bring gravity into the picture. This implies making the supersymmetry transformations local, leading to the theory of supergravity or SUGRA for short, where the situation is quite different. In exact SUGRA the lowest energy state of the theory, generically has negative energy density: the vacuum of supergravity is AdS§. This has inspired many to consider so-

§ This negative energy density can also be forbidden by postulating an unbroken R-symmetry.

called no-scale supergravity models. See [2] or supersymmetry textbooks such as [3] for excellent reviews.

The important point is that there is an elegant way of guaranteeing a flat potential, with $V = 0$ after susy-breaking, by using a nontrivial form of the Kähler potential G. For a single scalar field z we have:

$$V = e^G \left[\frac{\partial_z G \partial_{z^*} G}{\partial_z \partial_{z^*} G} - 3 \right] \\ = \frac{9e^{4G/3}}{\partial_z \partial_{z^*} G} (\partial_z \partial_{z^*} e^{-G/3}), \tag{4}$$

where κ, the gravitational constant, has been set to zero. A flat potential with $V = 0$ is obtained if the expression in brackets vanishes for all z, which happens if:

$$G = -3 \log(z + z^*), \tag{5}$$

and one obtains a gravitino mass:

$$m_{3/2} = \langle e^{G/2} \rangle = \langle (z + z^*)^{-3/2} \rangle, \tag{6}$$

which as required is not fixed by the minimization of V. Thus *provided we are prepared to choose a suitable, nontrivial form for the Kähler potential G, it is possible to obtain a zero CC and to leave the gravitino mass undetermined*, just fixed dynamically through non-gravitational radiative corrections. The minimum of the effective potential occurs at:

$$V_{eff} \approx -(m_{3/2})^4, \tag{7}$$

where in this case after including the observable sector and soft symmetry-breaking terms we will have $m_{3/2} \approx M_W$. Such a mass seems to be ruled out cosmologically [4] and so other models with the same ideas have been constructed that allow a very small mass for the gravitino, also by choosing a specific Kähler potential.

That these constructions are possible is quite interesting and in the past there has been some excitement when superstring theory seemed to implicate precisely the kinds of Kähler potential as needed here, see for example [5]. However, that is not enough, these simple structures are not expected to hold beyond zeroth order in perturbation theory.

3.1.1. Unbroken SUSY To paraphrase Witten [6]:"Within the known structure of physics, supergravity in four dimensions leads to a dichotomy: either the symmetry is unbroken and bosons and fermions are degenerate, or the symmetry is broken and the vanishing of the CC is difficult to understand". However, as he also argues in the same article, in $2 + 1$ dimensions, this unsatisfactory dichotomy does not arise: SUSY can explain the vanishing of the CC without leading to equality of boson and fermion masses, see also [7].

The argument here is that in order to have equal masses for the bosons and fermions in the same supermultiplet one has to have unbroken global supercharges. These are determined by spinor fields which are covariantly constant at infinity. The supercurrents J^μ from which the supercharges are derived are generically not conserved in the usual sense, but covariantly conserved: $D_\mu J^\mu = 0$. However, in the presence of a covariantly constant spinor ($D_\mu \epsilon = 0$), the conserved current $\bar{\epsilon} J^\mu$ can be constructed and therefore, a globally conserved supercharge:

$$Q = \int d^3 x \bar{\epsilon} J^0. \tag{8}$$

But in a $2+1$ dimensional spacetime any state of non-zero energy produces a geometry that is asymptotically conical at infinity (see also [8]). The spinor fields are then no longer covariantly constant at infinity and so even when supersymmetry applies to the vacuum and ensures the vanishing of the vacuum energy, it does not apply to the excited states. Examples have been constructed in [9, 10]. Two further ideas in this direction, one in $D < 4$ and one in $D > 4$ are [11, 12], however the latter later turned out to be internally inconsistent [13].

In any case, what is very important is to make the statement of "breaking of supersymmetry" more precise. As is clear, we do not observe mass degeneracies between fermions and bosons, therefore supersymmetry, even if it were a good symmetry at high energies between excited states, is broken at lower energies. However, and this is the point, as the example of Witten shows, the issue of whether we do or do not live in a supersymmetric vacuum state is another question. In some scenarios it is possible to have a supersymmetric vacuum state, without supersymmetric excited states. This really seems to be what we are looking for. The observations of a small or even zero CC could point in the direction of a (nearly) supersymmetric vacuum state.

Obviously the question remains how this scenario and the absence nevertheless of a supersymmetric spectrum can be incorporated in 4 dimensions.

3.2. Imaginary Space

So far, the most obvious candidate-symmetry to enforce zero vacuum energy density, supersymmetry, does not seem to work; we need something else. What other symmetry could forbid a cosmological constant term? Einstein's equations are:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{9}$$

As was first observed by 't Hooft (unpublished), we can forbid the cosmological constant term by postulating that the transformations:

$$\vec{x} \to i\vec{x}, \qquad t \to it, \qquad g_{\mu\nu} \to g_{\mu\nu} \tag{10}$$

are symmetry operations. The different objects in Einstein's equations transform under this as follows:

$$\begin{aligned}
\Gamma^\lambda_{\mu\nu} &= \frac{1}{2} g^{\lambda\rho} \left[\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\sigma} - \partial_\rho g_{\mu\sigma}\right] \to -i\Gamma^\lambda_{\mu\nu} \\
R_{\mu\nu} &= \partial_\nu \Gamma^\lambda_{\mu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} - \Gamma^\rho_{\mu\nu}\Gamma^\sigma_{\rho\sigma} + \Gamma^\rho_{\mu\sigma}\Gamma^\sigma_{\nu\rho} \to -R_{\mu\nu} \\
R &= g^{\mu\nu} R_{\mu\nu} \to -R
\end{aligned}$$

Furthermore we have:

$$T_{\mu\nu} \rightarrow -T_{\mu\nu} \tag{11}$$

as long as there are no vacuum terms in the expression for $T_{\mu\nu}$. So Einstein's equations transforms as:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \rightarrow -G_{\mu\nu} + \Lambda g_{\mu\nu} = +8\pi G T_{\mu\nu} \tag{12}$$

Therefore, if we postulate (10) as a symmetry of nature, a CC term is forbidden!

However, at first sight, this symmetry does not seem to ameliorate the situation much, since this transformation is not a symmetry of the Standard Model. In particular, we have:

$$p^2 = m^2, \qquad \text{with} \quad p^\mu = i\partial^\mu \rightarrow -ip^\mu \tag{13}$$

Therefore, imposing (10) as a symmetry of nature, seems to imply that all particles should be massless. In other words, if we take this symmetry seriously, we should conclude that the smallness of the cosmological constant and the smallness of particle masses (relative to the Planck-scale) although of quite a different order of magnitude, have a common origin. What does it mean to state that the laws of physics should also hold in imaginary space?

One approach is to view this symmetry in combination with boundary conditions. Only the vacuum state satisfies boundary conditions in both real and imaginary space, excited states, obeying different boundary conditions, would violate the symmetry. Generally in quantum field theory we Fourier transform our field and impose (often periodic) boundary conditions only on its components in real space. It could be that the vacuum state has to obey boundary conditions both in real and imaginary space.

3.3. Scale Invariance, e.g. Conformal Symmetry

The above symmetry might be viewed as a specific example of the more general framework of conformal symmetry, $g_{\mu\nu} \rightarrow f(x^\mu) g_{\mu\nu}$. Massless particles are symmetric under a bigger group than just the Lorentz group, namely, the conformal group. This group does not act as symmetries of Minkowski spacetime, but under a (mathematically useful) completion, the "conformal compactification of Minkowski space". This group is 15-dimensional and corresponds to $SO(2,4)$, or if fermions are present, the covering group $SU(2,2)$. Conformal symmetry forbids any term that sets a length scale, so a cosmological constant is not allowed, and indeed also particle masses necessarily have to vanish.

General coordinate transformations and scale invariance, i.e. $g_{\mu\nu} \rightarrow f g_{\mu\nu}$, are incompatible in general relativity. The $R\sqrt{-g}$ term in the Einstein-Hilbert action is the only quantity that can be constructed from the metric tensor and its first and second derivatives only, that is invariant under general coordinate transformations. But this term is not even invariant under a global scale transformation $g_{\mu\nu} \rightarrow f g_{\mu\nu}$ for which f is constant. R transforms with Weyl weight -1 and $\sqrt{-g}$ with weight $+2$.

There are two ways to proceed to construct a scale invariant action: along the lines of Dirac, introducing a new scalar field, that transforms with weight -1, giving rise to so-called scalar-tensor theories, or consider Lagrangians that are quadratic in the curvature scalar. We consider the second. See for example [14, 15] for some resent studies and many references.

Gravity can be formulated under this bigger group, leading to "Conformal gravity", defined in terms of the Weyl tensor, which corresponds to the traceless part of the Riemann tensor:

$$\begin{aligned} S_G = & -\alpha \int d^4x\sqrt{-g}C_{\lambda\mu\nu\kappa}C^{\lambda\mu\nu\kappa} \\ = & -2\alpha \int d^4x\sqrt{-g}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) + (\text{boundary terms}), \end{aligned} \tag{14}$$

where $C^{\mu\nu\lambda\kappa}$ is the conformal Weyl tensor, and α is a dimensionless gravitational coupling constant. Thus the Lagrangian is quadratic in the curvature scalar and generates field equations that are fourth-order differential equations. Based on the successes of gauge theories with spontaneously broken symmetries and the generation of the Fermi-constant, one may suggest to also dynamically induce the Einstein action with its Newtonian constant as a macroscopic limit of a microscopical conformal theory. This approach has been studied especially by Mannheim and Kazanas, see [16, 17, 18, 19, 20, 21] to solve the CC problem.

These fourth-order equations reduce to a fourth-order Poisson equation:

$$\nabla^4 B(r) = f(r), \tag{15}$$

where $B(r) = -g_{00}(r)$ and the source is given by:

$$f(r) = 3(T^0_{\ 0} - T^r_{\ r})/4\alpha B(r), \tag{16}$$

For a static, spherically symmetric source, conformal symmetry allows one to put $g_{rr} = -1/g_{00}$ and the exterior solution to (14) can be written [21]:

$$g_{rr} = -1/g_{00} = 1 - \beta(2 - 3\beta\gamma)/r - 3\beta\gamma + \gamma r - kr^2. \tag{17}$$

The non-relativistic potential reads:

$$V(r) = -\beta/r + \gamma r/2 \tag{18}$$

which for a spherical source can be completely integrated to yield:

$$B(r > R) = -\frac{r}{2}\int_0^R dr' f(r')r'^2 - \frac{1}{6r}\int_0^R dr' f(r')r'^4. \tag{19}$$

Compared to the standard second-order equations:

$$\nabla^2\phi(r) = g(r) \qquad \rightarrow \qquad \phi(r > R) = -\frac{1}{r}\int_0^R dr' g(r')r'^2 \tag{20}$$

we see that the fourth-order equations contain the Newtonian potential in its solution, but in addition also a linear potential term that could dominate over Newtonian gravity at large distances. The factors β and γ in for example (18) are therefore given by:

$$\beta(2 - 3\beta\gamma) = \frac{1}{6}\int_0^R dr' f(r')r'^4 \qquad , \qquad \gamma = -\frac{1}{2}\int_0^R dr' f(r')r'^2 \tag{21}$$

Note in passing that in the non-relativistic limit of GR the $(0,0)$-component, where $R_{ij} \simeq (1/2R - \Lambda)g_{ij}$ and therefore $R = g^{\mu\nu}R_{\mu\nu} \simeq R_{00} + 3(1/2R - \Lambda)$, or $R \simeq -2R_{00} + 6\Lambda$ using also that $R_{00} \simeq (-1/2)\nabla^2 g_{00}$ becomes:

$$\nabla^2\phi = 4\pi G\left(\rho - \frac{\Lambda}{4\pi G}\right), \tag{22}$$

the Poisson equation for the normal Newtonian potential modified with a cosmological constant. This can easily be solved to give:

$$\phi = -\frac{GM}{r} - \frac{1}{6}\Lambda r^2. \tag{23}$$

However, modifying gravity only at large distances cannot solve the cosmological constant problem. The (nearly) vanishing of the vacuum energy and consequently flat and relatively slowly expanding spacetime is a puzzle already at distance scales of say a meter. We could expect deviations of GR at galactic scales, avoiding the need for dark matter, but at solar system scales GR in principle works perfectly fine. It seems hard to improve on this, since the world simply is not scale invariant.

There is also a more serious problem with the scenario of Mannheim and Kazanas described above. In order for the linear term not to dominate already at say solar system distances, the coefficient γ has to be chosen very small. Not only does this introduce a new kind of fine-tuning, it is simply not allowed to chose these coefficients at will. The linear term will always dominate over the Newtonian $1/r$-term, in contradiction with the perfect agreement of GR at these scales. See also [22] who raised the same objection.

This scenario therefore does not work.

3.3.1. Λ as Integration Constant Another option is to reformulate the action principle in such a way that a scale dependent quantity like the scalar curvature, remains undetermined by the field equations themselves. These are the so-called 'unimodular' theories of gravity, see e.g. [23, 24]. Note that although the action is not globally scale invariant, Einstein's equations in the absence of matter and with vanishing cosmological constant is. The dynamical equations of pure gravity in other words, are invariant with respect to global scale transformations, and since we have that $R = 0$, they are scale-free, i.e. they contain no intrinsic length scale.

There is a way to keep the scale dependence undetermined also after including matter which also generates a cosmological constant term. This well-known procedure assumes:

$$\sqrt{-g} = \sigma(x) \qquad \rightarrow \qquad \delta\sqrt{-g} = 0, \tag{24}$$

where $\sigma(x)$ is a scalar density of weight $+1$. The resulting field equations are:

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R = \kappa\left(T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T\right). \tag{25}$$

Using that the covariant derivative of Einstein's equation vanishes one obtains:

$$R + \kappa T = 4\Lambda, \tag{26}$$

where Λ now appears as an integration constant and the factor of 4 has been chosen for convenience since substituting this back we recover the normal Einstein equations with cosmological constant.

Obviously this does not solve anything, nor does it provide a better understanding of the cosmological constant. In a sense it brings us back to a 'Type 0' explanation, using just fine-tuning to arrive at the right value.

3.4. Holography

Gravitational holography [25] limits the number of states accessible to a system. The entropy of a region generally grows with its covering area (in Planck units) rather than with its volume, implying that the dimension of the Hilbert space, i.e. the number of degrees of freedom describing a region, is finite and much smaller than expected from quantum field theory. Considering an infinite contribution to the vacuum energy is not correct because states are counted that do not exist in a holographic theory of gravity.

It is a symmetry principle since there is a projection from states in the bulk-volume, to states on the covering surface.

In [26, 27] it is noted that in effective field theory in a box of size L with UV cutoff M the entropy S scales extensively, as $S \sim L^3 M^3$. A free Weyl fermion on a lattice of size L and spacing $1/M$ has $4^{(LM)^3}$ states and entropy $S \sim (LM)^3$. For a lattice theory of bosons represented by a compact field the scaling is likewise. The corresponding entropy density $s = S/V$ then is $s = M^3$. In $d = 4$ dimensions quantum corrections to the vacuum energy are therefore of order:

$$\rho_{vac} = \frac{\Lambda}{8\pi G} + \langle\rho\rangle = \frac{\Lambda}{8\pi G} + \mathcal{O}(s^{4/3}), \tag{27}$$

since both $\langle\rho\rangle$ and s are dominated by ultraviolet modes, (see also [28]). Thus finite s implies finite corrections to $\langle\rho\rangle$.

Using a cutoff M, $E \sim M^4 L^3$ is the maximum energy for a system of size L. States with $L < R_s \sim E$, or $L > M^{-2}$ (in Planckian units) have collapsed into a black-hole. If one simply requires that no state in the Hilbert space exists with $R_s \sim E > L$, than a relation between the size L of the region, providing an IR cutoff, and the UV cutoff M can be derived. Under these conditions entropy grows no faster than $A^{3/4} \sim L^{3/2}$, with A the area. If these black hole states give no contribution to $\langle\rho\rangle$, we obtain:

$$\langle\rho\rangle \sim s^{4/3} \sim \left(\frac{L^{3/2}}{L^3}\right)^{4/3} \sim L^{-2}. \tag{28}$$

In [26] this same scaling was obtained by assuming that $S < A$ as usual, but that the delocalized states have typical Heisenberg energy $1/L$:

$$\langle\rho\rangle \sim \frac{s}{L} \sim \frac{L^2}{L^3 L} \sim L^{-2}. \tag{29}$$

Plugging in for L the observed size of the universe today the quantum corrections are only of order 10^{-10} eV^4.

However, this does not yield the correct equation of state, [28]. During matter dominated epochs, to which WMAP and supernova measurements are sensitive, the horizon size grows as the RW-scale factor, $a(t)^{3/2}$, so the above arguments imply:

$$\Lambda_{eff}(L) \sim a(t)^{-3}, \tag{30}$$

or, $w \equiv p/\rho = 0$ at largest scales, since $\rho(t) \sim a(t)^{-3(1+w)}$. The data on the other hand give $w < -0.78$ (95% CL). In for example [26, 27] $\Lambda(L)$ is at all times comparable to the radiation + matter energy density, which is also argued to give problems for structure formation [29].

Holography-based scenarios thus naively lead to a cosmological constant that is far less constant than what the data require. This makes a connection between holography and dark energy a lot harder to understand‖.

More recently however, another proposal was made [31] where instead L is taken to be proportional to the size of the future event horizon:

$$L(t) \sim a(t) \int_t^\infty \frac{dt'}{a(t')} \tag{31}$$

This L describes the size of the largest portion of the universe that any observer will see. This could be a reasonable IR cutoff. It is argued that in this case the equation of state parameter w can be close enough to -1 to agree with the data. This relation is rather ad hoc chosen, and its deeper meaning, if any, still has to be discovered.

Another reason to discuss holography in the context of the cosmological constant problem lies in trying to reconcile string theory with the apparent observation of living in a de Sitter spacetime. The discussion centers around the semi-classical result that de Sitter space has a finite entropy, inversely related to the cosmological constant, see for example [32]. Thus one may reason that de Sitter space should be described by a theory with a finite number of independent quantum states and that a theory of quantum gravity should be constructed with a finite dimensional Hilbert space. In this reasoning a cosmological constant should be understood as a direct consequence of the finite number of states in the Hilbert space describing the world. Ergo, the larger the cosmological constant, the smaller the Hilbert space. However, in [33] it is argued that this relation between the number of degrees of freedom and the CC is not so straightforward.

3.5. Symmetry between Sub- and Super-Planckian Degrees of Freedom

This rather speculative reasoning originates from a comparison with condensed matter physics and is due to Volovik, see for example [34, 35, 36, 37, 38]. The vacuum energy of superfluid ^{4}Helium, calculated from an effective theory containing phonons as elementary bosonic particles and no fermions is:

$$\rho_\Lambda = \sqrt{-g} E^4_{Debye} \tag{32}$$

‖ In [30] in a different context somehow a similar relation between the CC and the volume of the universe is derived, thus suffering from the same drawbacks.

with g the determinant of the acoustic metric, since c is now the speed of sound, and $E_{Debye} = \hbar c/a$, with a the interatomic distance, which plays the role of the Planck length. However, in the condensed matter case, the full theory exists: a second quantized Hamiltonian describing a collection of a macroscopic number of structureless ^{4}Helium bosons or ^{3}Helium fermions, in which the chemical potential μ acts as a Lagrange multiplier to ensure conservation of the number of atoms.

Using this Hamiltonian to calculate the energy density of the ground state we get:

$$\rho_\Lambda = \frac{1}{V}\langle \mathrm{vac}|H - \mu N|\mathrm{vac}\rangle \tag{33}$$

An overall shift of the energy in H is cancelled in a shift of the chemical potential. Exact calculation shows that not only the low energy degrees of freedom from the effective theory, the phonons, but also the higher energy, "trans-Planckian" degrees of freedom have to be taken into account. Using just thermodynamic arguments, it is argued that in the infinite volume, zero temperature limit, this gives exactly zero vacuum energy density as long as there are no external forces acting on the quantum liquid, there is no matter, no curvature and no boundaries which could give rise to a Casimir effect [34].

A somewhat similar result is obtained from different arguments by [39]. In their formulation the world is like a crystal. The atoms of the crystal are in thermal equilibrium and exhibit therefore zero pressure, making the cosmological constant equal to zero.

3.6. Interacting Universes, Antipodal Symmetry

This is an approach developed by Linde [40, 41] arguing that the vacuum energy in our universe is so small because there is a global interaction with another universe where energy densities are negative. Consider the following action of two universes with coordinates x_μ and y_α respectively, $(x_\mu, y_\alpha = 0, 1, \ldots, 3)$ and metrics $g_{\mu\nu}(x)$ and $\bar{g}_{\alpha\beta}(y)$, containing fields $\phi(x)$ and $\bar{\phi}(y)$:

$$S = N \int d^4x d^4y \sqrt{g(x)}\sqrt{\bar{g}(y)} \left[\frac{M_P^2}{16\pi}R(x) + L(\phi(x)) - \frac{M_P^2}{16\pi}R(y) - L(\bar{\phi}(y))\right], \tag{34}$$

and where N is some normalization constant. This action is invariant under general coordinate transformations in each of the universes separately. The important symmetry of the action is $\phi(x) \to \bar{\phi}(x)$, $g_{\mu\nu}(x) \to \bar{g}_{\alpha\beta}(x)$ and under the subsequent change of the overall sign: $S \to -S$. He calls this an antipodal symmetry, since it relates states with positive and negative energies. As a consequence we have invariance under the change of values of the effective potentials $V(\phi) \to V(\phi) + c$ and $V(\bar{\phi}) \to V(\bar{\phi}) + c$ where c is some constant. Therefore nothing in this theory depends on the value of the effective potentials in their absolute minima ϕ_0 and $\bar{\phi}_0$. Note that because of the antipodal symmetry $\phi_0 = \bar{\phi}_0$ and $V(\phi_0) = V(\bar{\phi}_0)$.

In order to avoid the troublesome issues of theories with negative energy states, there can be no interactions between the fields $\phi(x)$ and $\bar{\phi}(y)$. Therefore also the equations of motion for both fields are the same and similarly, also gravitons from both universes do not interact.

However some interaction does occur. The Einstein equations are:

$$R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}R(x) = -8\pi G T_{\mu\nu}(x) - g_{\mu\nu}\langle\frac{1}{2}R(y) + 8\pi G L(\bar{\phi}(y))\rangle \quad (35)$$

$$R_{\alpha\beta}(y) - \frac{1}{2}\bar{g}_{\alpha\beta}R(y) = -8\pi G T_{\alpha\beta}(y) - \bar{g}_{\alpha\beta}\langle\frac{1}{2}R(x) + 8\pi G L(\phi(x))\rangle. \quad (36)$$

Here $T_{\mu\nu}$ is the energy-momentum tensor of the fields $\phi(x)$ and $T_{\alpha\beta}$ the energy-momentum tensor for the fields $\bar{\phi}(y)$ and the averaging means:

$$\langle R(x)\rangle = \frac{\int d^4x\sqrt{g(x)}R(x)}{\int d^4x\sqrt{g(x)}} \quad (37)$$

$$\langle R(y)\rangle = \frac{\int d^4y\sqrt{\bar{g}(y)}R(y)}{\int d^4y\sqrt{\bar{g}(y)}} \quad (38)$$

and similarly for $\langle L(x)\rangle$ and $\langle L(y)\rangle$.

Thus there is a global interaction between the universes X and Y: The integral over the whole history of the Y-universe changes the vacuum energy density of the X-universe. Assuming then that at late times the fields settle near the absolute minimum of their potential we have:

$$R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}R(x) = -8\pi G g_{\mu\nu}\left[V(\bar{\phi}_0) - V(\phi_0)\right] - \frac{1}{2}g_{\mu\nu}R(y) \quad (39)$$

$$R_{\alpha\beta}(y) - \frac{1}{2}\bar{g}_{\alpha\beta}R(y) = -8\pi G g_{\alpha\beta}\left[V(\phi_0) - V(\bar{\phi}_0)\right] - \frac{1}{2}g_{\alpha\beta}R(x). \quad (40)$$

Thus at late stages the effective cosmological constant vanishes:

$$R(x) = -R(y) = \frac{32}{3}\pi G\left[V(\phi_0) - V(\bar{\phi}_0)\right] = 0, \quad (41)$$

since because of the antipodal symmetry $\phi_0 = \bar{\phi}_0$ and $V(\phi_0) = V(\bar{\phi}_0)$.

This could also be seen as a back-reaction mechanism, from one universe at the other.

3.7. Duality Transformations

3.7.1. Hodge Duality This duality between a r-form and a $(D-r)$-form in D dimensions is studied [42], where the cosmological constant is taken to be represented by a 0-form field strength, which is just a constant. This is somewhat related to the unimodular approach of section (3.3.1) in the sense that they try to introduce the cosmological constant in a different way in the Einstein-Hilbert action.

3.7.2. S-Duality A different proposal was considered in [43], where S-duality acting on the gravitational field can mix gravitational and matter degrees of freedom. If theories A and B are S-dual then $f_A(\alpha) = f_B(1/\alpha)$. It relates type I stringtheory to the $SO(32)$ heterotic theory, and type IIB theory to itself.

The argument now is that after duality transformations a flat spacetime with zero cosmological constant can be obtained, although the original cosmological constant can have any value.

3.8. Summary

A symmetry principle as explanation for the smallness of the cosmological constant in itself is very attractive. A viable mechanism that sets the cosmological constant to zero would be great progress, even is Λ would turn out to be nonzero. Since supersymmetry does not really seem to help, especially some form of scale invariance stands out as a serious option. Needless to say, it is hard to imagine how scale invariance could be used, knowing that the world around us is not scale invariant. Particle masses are small, but many orders of magnitude larger than the observed cosmological constant.

4. Type II: Back-Reaction Mechanisms

In this approach it is argued that any cosmological constant will be automatically cancelled, or screened, to a very small value by back-reaction effects on an expanding space. Often these effects are studied in an inflationary background, where a cosmological constant is most dominant. The physical idea of this mechanism can be understood in the context of the energy-time uncertainty principle. For a particle with mass m and co-moving wavevector $\vec{k}$ in a spacetime with scalefactor $a(t)$ we have:

$$E(\vec{k},t) = \sqrt{m^2 + \|\vec{k}\|^2/a^2(t)}. \tag{42}$$

Thus growth of $a(t)$ increases the time a virtual particle of fixed m and $\vec{k}$ can exist and, during inflation, virtual particles with zero mass and long enough wavelength can exist forever. The rate (Γ) at which they emerge from the inflationary vacuum depends upon the type of particle. Most massless particles are conformally invariant. In that case, Γ gives the number of particles emerging from the vacuum per unit conformal time η, so the number per unit physical time is:

$$\frac{dn}{dt} = \frac{d\eta}{dt}\frac{dn}{d\eta} = \frac{\Gamma}{a}. \tag{43}$$

Their emergence rate thus falls like $1/a(t)$. This means that although those that are produced can exist forever, only very few are created, and their total effect during inflation is negligible, see e.g. [44].

However, two familiar massless particles are not conformally invariant, massless minimally coupled scalars and gravitons. Therefore in these two sections we consider their effects in more detail.

It should be noted that there exists a no-go theorem, derived by Weinberg, see [2] for details. The theorem states that the vacuum energy density cannot be cancelled dynamically, using a scalar field, without fine-tuning in any effective four-dimensional theory with constant fields at late times, that satisfies the following conditions:

(i) General Covariance;

(ii) Conventional four-dimensional gravity is mediated by a *massless* graviton;

(iii) Theory contains a finite number of fields below the cutoff scale;

(iv) Theory contains no negative norm states.

Under these rather general assumptions the theorem states that the potential for the compensator field, which should adjust the vacuum energy to zero, has a runaway behavior. This means that there is no stationary point for the potential of the scalar field that should realize the adjustment, and thus the mechanism cannot work.

4.1. Scalar Field, Instabilities in dS-Space

The first attempts to dynamically cancel a 'bare' cosmological constant were made by referring to instabilities in the case of a scalar field in de Sitter space. A massless minimally coupled scalar field ϕ has no de Sitter-invariant vacuum state and the expectation value of ϕ^2 is time-dependent. However, this breaking of de Sitter invariance is not reflected by the energy-momentum tensor, since $T_{\mu\nu}$ only contains derivatives and hence is not sensitive to long-wavelength modes. This changes if one includes interactions. Consider for example a $\lambda\phi^4$. Then:

$$\langle T_{\mu\nu}\rangle \sim \lambda\langle\phi^2\rangle^2 g_{\mu\nu} \propto t^2. \tag{44}$$

So in this case it is possible for $\langle T_{\mu\nu}\rangle$ to grow for some time, until higher order contributions become important. The infrared divergence results in a mass for the field which in turn stops the growth of $\langle T_{\mu\nu}\rangle$, see for example [45, 46].

Another illustrative, but unsuccessful attempt has been given by Dolgov [47]. He used a rather simple classical model for back-reaction:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\alpha\phi\partial^\alpha\phi - \xi R\phi^2\right), \tag{45}$$

where R is the scalar curvature and ξ a *negative* constant. The scalar field energy-momentum tensor at late times approaches the form of a cosmological constant term:

$$8\pi G\langle T_{\mu\nu}\rangle \sim \Lambda_0 g_{\mu\nu} + \mathcal{O}(t^{-2}). \tag{46}$$

$\Lambda_0 = 3H^2$ stands for the effective value of the cosmological constant during a de Sitter phase so the leading back-reaction term cancels this effect. The kinetic energy of the growing ϕ-field acts to cancel the cosmological constant. Unfortunately, not only the cosmological constant term is driven to zero, Newton's constant is also screened:

$$G_{eff} = \frac{G_0}{1 + 8\pi G|\xi|\phi^2} \sim \frac{1}{t^2}, \tag{47}$$

where G_0 is the "bare" value of G at times where $\phi = 0$.

Other models of this kind were also studied by Dolgov, see [48, 49] but these proved to be unstable, leading quickly to a catastrophic cosmic singularity.

In summary, the dynamical cancellation of a cosmological constant term by back-reaction effects of scalar fields does not seem to work. Let's focus therefore on a purely gravitational back-reaction mechanism.

4.2. Gravitons, Instabilities of dS-Space

Gravitational waves propagating in some background spacetime affect the dynamics of this background. This back-reaction can be described by an effective energy-momentum tensor $\tau_{\mu\nu}$.

4.2.1. Scalar-type Perturbations In [50, 51] the back-reaction for scalar gravitational perturbations is studied. They argue this might give a solution to the CC problem.

The idea is to first expand Einstein equations to second order in the perturbations, then to assume that the first order terms satisfy e.o.m. for linearized cosmological perturbations (and hence cancel). Next the spatial average is taken of the remaining terms and the resulting equations are regarded as equations for a new homogeneous metric $g_{\mu\nu}^{(0,br)}$, where the superscript $(0, br)$ denotes first the order in perturbation theory and the fact that back-reaction is taken into account:

$$G_{\mu\nu}\left(g_{\alpha\beta}^{(0,br)}\right) = 8\pi G\left[T_{\mu\nu}^{(0)} + \tau_{\mu\nu}\right] \tag{48}$$

and $\tau_{\mu\nu}$ contains terms resulting from averaging of the second order metric and matter perturbations:

$$\tau_{\mu\nu} = \langle T_{\mu\nu}^{(2)} - \frac{1}{8\pi G} G_{\mu\nu}^{(2)} \rangle. \tag{49}$$

In other words, they regard the first-order perturbations as contributing an extra energy-momentum tensor to the zeroth-order equations of motion; the effective energy-momentum tensor of the first-order equations renormalizes the zeroth-order energy-momentum tensor. This is a somewhat tricky approach and it is not clear whether one can consistently derive the equations of motion in this way, see for example [52, 53, 54, 55, 56].

Now work in longitudinal gauge and take the matter to be described by a single scalar field for simplicity. Then there is only one independent metric perturbation variable denoted $\phi(x, t)$. The perturbed metric is:

$$ds^2 = (1 + 2\phi)dt^2 - a(t)^2(1 - 2\phi)\delta_{ij}dx^i dx^j. \tag{50}$$

Calculating the τ_{00} and τ_{ij} elements and using relations valid for the period of inflation, Brandenberger's main result is that the equation of state of the dominant infrared contribution to the energy-momentum tensor $\tau_{\mu\nu}$ which describes back-reaction, takes the form of a negative CC:

$$p_{br} = -\rho_{br}, \qquad \rho_{br} < 0. \tag{51}$$

This leads to the speculation that gravitational back-reaction may lead to a dynamical cancellation mechanism for a bare CC and yield a scaling fixed point in the asymptotic future in which the remnant CC satisfies $\Omega_\Lambda \sim 1$.

However, as pointed out in [56], the spatially averaged metric is not a local physical observable: averaging over a fixed time slice, the averaged value of the expansion will not be the same as the expansion rate at the averaged value of time, because of the non-linear nature of the expansion with time. In other words, locally this 'achieved renormalization', i.e. the effect of the perturbations, is identical to a coordinate transformation of the background equations and not a physical effect. A similar conclusion was obtained in [57].

Brandenberger and co-workers have subsequently tried to improve their analysis by identifying a local physical variable which describes the expansion rate [58, 59]. This amounts to adding another scalar field that acts as an independent physical clock. Within this procedure they argue that back-reaction effects are still significant in renormalizing the cosmological constant.

It is however far from clear whether this scenario is consistent and whether the effects indeed are physical effects. Besides, this build-up of infrared scalar metric perturbations (vacuum fluctuations, stretched beyond the Hubble-radius) is set in an inflationary background and since the individual effects are extremely weak a large phase-space of IR-modes, i.e. a long period of inflation, is needed. The influence on today's cosmological constant is unclear.

4.2.2. Long-Wavelength Back-Reaction in Pure Gravity Closely related are studies by Tsamis and Woodard, see [60, 61, 62, 63, 64, 65, 66] concerning the back-reaction of long-wavelength gravitational waves in pure gravity with a bare cosmological constant. Leading infrared effects in quantum gravity are, contrary to what is often assumed, similar to those of QED, see [67].

When $\Lambda \neq 0$, the lowest dimensional self-interaction term is of dimension three, a three-point vertex with no derivatives (corresponding to the $\Lambda\sqrt{-g}$-term). The IR behavior of the theory with cosmological constant is therefore very different from that without. Tsamis and Woodard christen it Quantum Cosmological Gravity, or QCG for short.

They study QCG on an inflationary background, which powers the infrared divergences: since the spatial coordinates are exponentially expanded with increasing time, their Fourier conjugates, the spatial momenta, are redshifted to zero. The IR effects originate from the low end of the momentum spectrum, so they are strengthened when this sector is more densely populated.

Since other particles are either massive, in which case they decouple from the infrared, or conformally invariant, and therefore do not feel the de Sitter redshift, gravitons must completely dominate the far IR.

The classical background in conformal coordinates is:

$$-dt^2 + e^{2Ht} d\vec{x} \cdot d\vec{x} = \Omega^2 \left(-du^2 + d\vec{x} \cdot d\vec{x}\right) \tag{52}$$

$$\Omega \equiv \frac{1}{Hu} = \exp(Ht) \tag{53}$$

and $H^2 \equiv \frac{1}{3}\Lambda$. For convenience, perturbation theory is formulated in terms of a pseudo-graviton field $\psi_{\mu\nu}$:

$$g_{\mu\nu} \equiv \Omega^2 \tilde{g}_{\mu\nu} \equiv \Omega^2(\eta_{\mu\nu} + \kappa\psi_{\mu\nu}) \tag{54}$$

where $\kappa^2 \equiv 16\pi G$. What they then actually computed is the amputated expectation value of $\kappa\psi_{\mu\nu}(u,\vec{x})$ which takes the form:

$$D^{\rho\sigma}_{\mu\nu}\langle 0|\kappa\psi_{\mu\nu}(x)|0\rangle = a(u)\bar{\eta}_{\mu\nu} + c(u)\delta^0_\mu\delta^0_\nu, \tag{55}$$

where $D^{\rho\sigma}_{\mu\nu}$ is the gauge fixed kinetic operator, the coefficients a and c are some functions of conformal time u and a bar indicates that temporal components of the tensor are deleted.

After attaching the external legs they obtain the invariant element:

$$\hat{g}_{\mu\nu}(t,\vec{x})dx^\mu dx^\nu = -\Omega^2\left[1 - C(u)\right]du^2 + \Omega^2\left[1 + A(u)\right]d\vec{x}\cdot d\vec{x}. \tag{56}$$

The important result is that the backreaction of these IR gravitons acts to screen the bare cosmological constant, originally present. Their improved estimates¶ in terms of:

$$\epsilon \equiv \left(\frac{\kappa H}{4\pi}\right)^2 = \frac{G\Lambda}{3\pi} = \frac{8}{3}\left(\frac{M}{M_P}\right)^4 \tag{57}$$

turn out to be:

$$A(u) = \epsilon^2\left\{\frac{172}{9}\ln^3(Hu) + (\text{subleading})\right\} + \mathcal{O}(\epsilon^3), \tag{58}$$

$$C(u) = \epsilon^2\left\{57\ln^2(Hu) + (\text{subleading})\right\} + \mathcal{O}(\epsilon^3) \tag{59}$$

Therefore:

$$Ht = -\left\{1 - \frac{19}{2}\epsilon^2\ln^2(Hu) + \ldots\right\}\ln(Hu) \tag{60}$$

Since $\ln(Hu) \approx -Ht$ to very good approximation, we arrive at:

$$\begin{aligned} H_{eff}(t) &\approx H + \frac{1}{2}\frac{d}{dt}\ln(1+A), \\ &\approx H\left\{1 - \frac{\frac{86}{3}\epsilon^2\left(Ht\right)^2}{1 - \frac{172}{9}\epsilon^2\left(Ht\right)^3}\right\} \end{aligned} \tag{61}$$

The induced energy density, which acts to screen the original cosmological constant present gives:

$$\rho(t) \approx \frac{\Lambda}{8\pi G}\left\{-\frac{\frac{172}{3}\epsilon^2\left(Ht\right)^2}{1 - \frac{172}{9}\epsilon^2\left(Ht\right)^3} + \left(\frac{\frac{86}{3}\epsilon^2\left(Ht\right)^2}{1 - \frac{172}{9}\epsilon^2\left(Ht\right)^3}\right)^2\right\} \tag{62}$$

The number of e-foldings needed to make the backreaction effect large enough to even end inflation is:

$$N \sim \left(\frac{9}{172}\right)^{\frac{1}{3}}\left(\frac{3\pi}{G\Lambda}\right)^{\frac{2}{3}} = \left(\frac{81}{11008}\right)^{\frac{1}{3}}\left(\frac{M_P}{M}\right)^{\frac{8}{3}} \tag{63}$$

where M is the mass scale at inflation and M_P the Planck mass. For inflation at the GUT scale this gives $N \sim 10^7$ e-foldings. This enormously long period of inflation, much

¶ Papers before 1997 yield different results.

longer than in typical inflation models, is a direct consequence of the fact that gravity is such a weak interaction.

In other words, the effect might be strong enough to effectively kill any cosmological constant present, as long as such a long period of inflation is acceptable. There do exist arguments that the number of e-folds is limited to some 85, see [68] for details, but these are far from established. Another issue is that these results have been obtained for a very large cosmological constant during inflation. It is unclear what this means for the present day vacuum energy of the universe. Perturbative techniques break down when the effect becomes too strong, making this difficult to answer.

4.3. Screening as a Consequence of the Trace Anomaly

In [69] it is argued that the quantum trace anomaly of massless conformal fields in 4 dimensions leads to a screening of the cosmological constant. The effective action of 4D gravity yields an extra new spin-0 degree of freedom in the conformal sector, or trace of the metric. At very large distance scales this trace anomaly induced action dominates the standard Einstein action and gives an IR fixed point where scale invariance is restored.

Their result is that the effective cosmological constant in units of Planck mass decreases at large distances and that $G_N\Lambda \to 0$ at the IR fixed point in the infinite volume limit.

4.4. Brane and Black Hole Production

The idea here is that vacuum energy results in the production of membranes that subsequently collapse into black holes. Thus the net effect is to 'transform non-localized dark energy into localized dark matter' [70, 71, 72].

4.5. Summary

Finding a viable mechanism that screens the original possibly large cosmological constant to its small value today, is a very difficult task. Weinberg's no-go theorem puts severe limits on this approach. Back-reaction effects moreover, are generally either very weak, or lead to other troublesome features like a screened Newton's constant.

The underlying idea however that the effective cosmological constant is small simply because the universe is old, is attractive and deserves full attention.

5. Type III: Violating the Equivalence Principle

An intriguing way to try to shed light on the cosmological constant problem is to look for violations of the equivalence principle of general relativity. The near zero cosmological constant could be an indication that vacuum energy contrary to ordinary matter-energy sources does not gravitate.

The approach is based not on trying to eliminate any vacuum energy, but to make gravity numb for it. This requires a modification of some of the building blocks of general relativity. General covariance (and the absence of ghosts and tachyons) requires that gravitons couple universally to all kinds of energy. Moreover, this also fixes uniquely the low energy effective action to be the Einstein-Hilbert action. If gravity were not mediated by an exactly massless state, this universality would be avoided. One might hope that vacuum energy would then decouple from gravity, thereby eliminating the gravitational relevance of it and thus eliminate the cosmological constant problem.

5.1. Massive Gravitons

If gravitons are massive, they become unstable and will gradually decay. Gravity then no longer obeys the standard inverse-square law, but becomes weaker at large scales, leading to accelerated cosmic expansion. Of course, the extra degrees of freedom, extra polarizations of a massive graviton, would also become noticeable at much shorter distances, putting severe constraints on such scenarios.

5.1.1. Ghost Condensation or Gravitational Higgs Mechanism In this framework gravity is modified in the infrared as a result of interactions with a 'ghost condensate', leading among other things to a mass for the graviton, see [73].

Assume that for a scalar field ϕ we have:

$$\langle\dot{\phi}\rangle = M^2, \quad \rightarrow \quad \phi = M^2 t + \pi \tag{64}$$

and that it has a shift symmetry $\phi \rightarrow \phi + a$ so that it is derivatively coupled, and that its kinetic term enters with the wrong sign in the Lagrangian:

$$\mathcal{L}_\phi = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \dots \tag{65}$$

The consequence of this wrong sign is that the usual background with $\langle\phi\rangle = 0$ is unstable and that after vacuum decay, the resulting background will break Lorentz invariance spontaneously.

The low energy effective action for the π has the form:

$$S \sim \int d^4x \left[\frac{1}{2}\dot{\pi}^2 - \frac{1}{2M^2}(\nabla^2\pi)^2 + \dots\right], \tag{66}$$

so that the π's have a low energy dispersion relation like:

$$\omega^2 \sim \frac{k^4}{M^2} \tag{67}$$

instead of the ordinary $\omega^2 \sim k^2$ relation for light excitations. Time-translational invariance is broken, because $\langle\phi\rangle = M^2 t$ and as a consequence there are two types of energy, a "particle physics" and a "gravitational" energy which are not the same. The particle physics energy takes the form:

$$\mathcal{E}_{pp} \sim \frac{1}{2}\dot{\pi}^2 + \frac{(\nabla^2\pi)^2}{2M^2} + \dots, \tag{68}$$

whereas the gravitational energy is:

$$\mathcal{E}_{grav} = T_{00} \sim M^2 \dot{\pi} + \ldots \tag{69}$$

Although time-translation- and shift-symmetry are broken in the background, a diagonal combination is left unbroken and generates new "time" translations. The Noether charge associated with this unbroken symmetry is the conserved particle physics energy. The energy that couples to gravity is associated with the broken time translation symmetry. Since this energy begins at linear order in $\dot{\pi}$, lumps of π can either gravitate or anti-gravitate, depending on the sign of $\dot{\pi}$! The π thus maximally violate the equivalence principle.

Interestingly, IR modifications of general relativity could be seen at relatively short distances, but only after a certain (long) period of time! Depending on the mass M and the expectation value of ϕ, deviations of Newtonian gravity could be seen at distances 1000 km, but only after a time $t_c \sim H_0^{-1}$ where H_0 is the Hubble constant.

This generates the analog of the Higgs mechanism for gravity. But since the background is not Lorentz invariant, one does not get a massive graviton with 5 polarizations. Only one extra degree of freedom is added, in the form of the scalar excitation π. Moreover, not a Yukawa-like exponential suppression of the potential at large distances is obtained, but an oscillatory behavior.

This highly speculative scenario opens up a new way of looking at the cosmological constant problem, especially because of the distinction between particle physics energy, $\mathcal{E}_{pp}$ and gravitational energy, $\mathcal{E}_{grav}$. It has to be developed further to better judge it.

5.2. Non-Local Gravity, Infinite Volume Extra Dimensions

In [74, 75, 76, 77, 78, 79], a model based on infinite volume extra dimensions is presented. Embedding our spacetime in infinite volume extra dimensions has several advantages. If they are compactified, one would get a theory alike GR in the IR, facing Weinberg's no-go theorem again. Details of how these large dimension models circumvent the no-go theorem can be found in [77]. Moreover, often the assumption is made that the higher-dimensional theory is supersymmetric and that susy is spontaneously broken on the brane. These breaking effects can be localized on the brane only, without affecting the bulk, because the infinite volume gives a large enough suppression factor. Apart from that an unbroken R-parity might be assumed to forbid any negative vacuum energy density in the bulk.

They start with the following low-energy effective action:

$$S = M_*^{2+N} \int d^4x d^N y \sqrt{G}\mathcal{R} + \int d^4x \sqrt{g} \left(\mathcal{E} + M_P^2 R + \mathcal{L}_{SM}\right), \tag{70}$$

where M_*^{2+N} is the $(4+N)$-dimensional Planck mass, the scale of the higher dimensional theory, G_{AB} the $(4+N)$-dimensional metric, y are the 'perpendicular' coordinates and $\mathcal{E} = M_{Pl}^2 \Lambda$, the brane tension, or 4D cosmological constant. Thus the first term is the bulk Einstein-Hilbert action for $(4+N)$-dimensional gravity and the $M_P^2 R$ term is the induced 4D-Einstein-Hilbert action. So there are three free parameters: M_*, M_P and $\mathcal{E}$. M_* is assumed to be very small, making gravity in the extra dimensions much stronger than in our 4D world.

The higher dimensional graviton can be expanded in 4D Kaluza-Klein modes as follows:

$$h_{\mu\nu}(x, y_n) = \int d^N m \epsilon^m_{\mu\nu}(x)\sigma_m(y_n), \tag{71}$$

where $\epsilon^m_{\mu\nu}(x)$ are 4D spin-2 fields with mass m and $\sigma_m(y_n)$ are their wavefunction profiles in the extra dimensions. Each of these modes gives rise to Yukawa-type gravitational potentials, the coupling-strength to brane sources of which are determined by the value of σ_m at the position of the brane, say $y = 0$:

$$V(r) \propto \frac{1}{M_*^{2+N}} \int_0^\infty dm m^{N-1} |\sigma_m(0)|^2 \frac{e^{-rm}}{r}. \tag{72}$$

However, in this scenario there is a cut-off of this integral; modes with $m > 1/r_c$ have suppressed wavefunctions, where r_c is some cross-over scale, given by $r_c = M_{Pl}^2/M_*^3 \sim H_0^{-1}$. For $r \ll r_c$ the gravitational potential is $1/r$, dominated by the induced 4D kinetic term, and for $r \gg r_c$ it turns to $1/r^2$, in case of one extra dimension. In ordinary extra dimensional gravity, all $|\sigma_m(0)| = 1$, here however:

$$|\sigma_m(0)| = \frac{4}{4 + m^2 r_c^2}, \tag{73}$$

which decreases for $m \gg r_c$. Therefore, the gravitational potential interpolates between the 4D and 5D regimes at r_c. Below r_c almost normal 4D gravity is recovered, while at larger scales it is effectively 5-dimensional and thus weaker. This could cause the universe's acceleration.

For $N > 2$, solutions of the theory can be parameterized as:

$$ds^2 = A^2(y)g_{\mu\nu}(x)dx^\mu dx^\nu - B^2(y)dy^2 - C^2(y)y^2 d\Omega^2_{N-1}, \tag{74}$$

where $y \equiv \sqrt{y_1^2 + \ldots + y_n^2}$ and the functions A, B, C depend on $\mathcal{E}_4$ and M_*. Most importantly, one explicitly known solution, with $N = 2$, generates a flat 4D Minkowski metric and $R(g) = 0$ [80]. This is however not the only solution, and, besides, its interpretation is rather complicated because of the appearance of a naked singularity. Spacetime in N extra dimensions looks like $\Re_4 \times S_{N-1} \times R_+$, where $\Re_4$ denotes flat spacetime on the brane, and $S_{N-1} \times R_+$ are Schwarzschild solutions in the extra dimensions.

They argue that the final physical result is:

$$H \sim M_* \left(\frac{M_*^4}{\mathcal{E}^4}\right)^{\frac{1}{N-2}}. \tag{75}$$

According to the 4D result, $N = 0$, the expansion rate grows as $\mathcal{E}^4$ increases, but for $N > 2$ the acceleration rate H decreases as $\mathcal{E}^4$ increases. In this sense, vacuum energy can still be very large, it just gravitates very little; 4D vacuum energy is supposed to curve mostly the extra dimensions.

Furthermore, long wavelength sources, i.e. any source which has characteristic wavelength larger than H_0^{-1}, "such as vacuum energy" [78], would feel gravity only due to the graviton zero mode. This zero mode is very weakly coupled to brane sources since it is suppressed by the volume of the extra dimensions. However, the most severe quantum corrections to the cosmological constant are in the UV and it is unclear how suppression of the graviton zero-mode ameliorates that.

From a 4d-perspective, this can also be viewed as to make the effective Newton's constant frequency and wavelength dependent, in such a way that for sources that are uniform in space and time it is tiny [81]:

$$M_{Pl}^2\left(1+\mathcal{F}(L^2\nabla^2)\right)G_{\mu\nu}=T_{\mu\nu}. \tag{76}$$

Here $\mathcal{F}(L^2\nabla^2)$ is a filter function:

$$\begin{aligned}&\mathcal{F}(\alpha)\rightarrow \quad \text{for } \alpha\gg 1\\ &\mathcal{F}(\alpha)\gg 1 \text{ for } \alpha\ll 1\end{aligned} \tag{77}$$

L is a distance scale at which deviations from general relativity are to be expected and $\nabla^2\equiv\nabla_\mu\nabla^\mu$ denotes the covariant d'Alembertian. Thus (76) can be viewed as Einstein equation with $(8\pi G_N^{eff})^{-1}=M_{Pl}^2(1+\mathcal{F})$.

In the limit $L\rightarrow\infty$ they arrive at:

$$M_{Pl}^2G_{\mu\nu}-\frac{1}{4}\bar{M}^2g_{\mu\nu}\bar{R}=T_{\mu\nu}, \tag{78}$$

where

$$\bar{M}=\mathcal{F}(0)M_{Pl}^2\gg M_{Pl}^2, \qquad \bar{R}\equiv\frac{\int d^4x\sqrt{g}R}{\int d^4x\sqrt{g}} \tag{79}$$

$\bar{R}$ thus is the spacetime averaged Ricci curvature.

At the price of losing 4D-locality and causality, the new averaged term is both non-local and acausal, a model is constructed in which a huge vacuum energy does not lead to an unacceptably large curvature. The Planck scale is made enormous for Fourier modes with a wavelength larger than a size L. It is however argued that the acausality has no observable effect.

Other approaches arguing that a 4D cosmological constant curves mostly extra dimensions are proposed by [82] and also studied in [83, 84, 85].

Another idea based on a model of non-local quantum gravity and field theory due to Moffat [86, 87], also suppresses the coupling of gravity to the vacuum energy density and also leads to a violation of the Weak Equivalence Principle.

5.3. Fat Gravitons

A proposal involving a sub-millimeter breakdown of the point-particle approximation for gravitons has been put forward by Sundrum [88]. Diagrams with external gravitons and SM-particles in loops (see figure 1) give the dominant contribution to the effective CC with $\Lambda_{UV}/16\pi^2$ where Λ_{UV} is some ultraviolet cutoff and this leads to the enormous

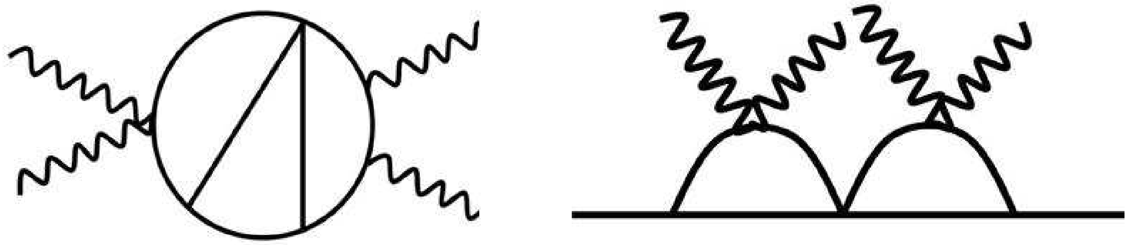

Figure 1. On the left-hand-side, a typical Standard Model contribution to $\Gamma_{eff}[g_{\mu\nu}]$. On the right, soft gravitons coupled to loop-correction to SM self-energy. Wiggly lines are gravitons and smooth lines are SM particles.

discrepancy with experimental results for any reasonable value of Λ_{UV}. However one might wonder what the risks are when throwing away these diagrams from the effective theory $\Gamma_{eff}[g_{\mu\nu}]$. Things at stake are: Unitarity, General Coordinate Invariance (GCI) and locality. In standard effective theory one also has diagrams where soft gravitons give corrections to the SM self energy diagrams, (figure 1). These cannot be thrown away, since they are crucial in maintaining the equivalence principle between inertial and gravitational masses. However, locally these diagrams are indistinguishable in spacetime, only globally can we discern their topological difference. Thus given locality of the couplings of the point particles in the diagrams, we cannot throw the first diagram away and keep the other. Therefore, it seems progress can be made by considering a graviton as an extended object. Define the graviton size:

$$l_{grav} \equiv \frac{1}{\Lambda_{grav}}. \tag{80}$$

Such a "fat graviton" does not have to couple with point-like locality to SM loops, but with locality up to l_{grav}. Thus a fat graviton can distinguish between the two types of diagrams, possibly suppressing the first while retaining the second.

The value of the CC based on usual power counting would then be:

$$\Lambda_{eff} \sim \mathcal{O}(\Lambda_{grav}^4/16\pi^2). \tag{81}$$

Comparing with the observational value this gives a bound on the graviton size of:

$$l_{grav} > 20 \text{ microns} \tag{82}$$

which would indicate a short-distance modification of Newton's law below 20 microns.

5.4. Composite Graviton as Goldstone boson

Another approach is to consider the possibility that the graviton appears as a composite Goldstone boson. There exists a theorem by Weinberg and Witten, [89], stating that a Lorentz invariant theory, with a Lorentz covariant energy-momentum tensor does not admit a composite graviton. It is therefore natural to try a mechanism where the graviton appears as a Goldstone boson associated with the spontaneous breaking of Lorentz invariance. Being a Goldstone boson, the graviton would not develop a potential, and hence the normal cosmological constant problem is absent, see for example [90, 91].

However, besides difficulties erasing the traces of broken Lorentz invariance to make the model agree with observations, also new fine-tunings are introduced.

5.5. Summary

Since General Relativity has only been thoroughly tested on solar system distance scales it is a very legitimate idea to consider corrections to GR at galactic and/or cosmological distance scales. However, often these models are not so harmless as supposed to be: changing the laws of gravity also at shorter scales, or leading to violations of locality. The scenario's described in this section do not directly solve the cosmological constant problem, but offer new ways of looking at it.

On the more positive side, many theories that predict modifications of GR in the IR, reproduce Einstein gravity at smaller distances, but up to some small corrections. These corrections are discussed in [92] and could be potentially observable at solar system distance scales. At the linearized level gravity is of the scalar-tensor type, because the graviton has an extra polarization that also couples to conserved energy-momentum sources. If these models are correct, an anomalous perihelion precession of the planets is expected to be observed in the near future.

6. Type IV: Statistical Approaches

6.1. Hawking Statistics

If the cosmological constant would appear as a constant of integration, then in a kind of quantum cosmology the state vector of the universe would be a superposition of states with different values of Λ. Eleven dimensional supergravity contains a three-form gauge field, with a four-form field strength. When reduced to four dimensions, this acts as a cosmological constant. Thus Hawking used [93] a three-form gauge field $A_{\mu\nu\lambda}$ with gauge transformations like:

$$A_{\mu\nu\rho} \rightarrow A_{\mu\nu\rho} + \nabla_{[\mu} C_{\nu\rho]} \tag{83}$$

This field would be an addition to the effective action:

$$S_A \propto -\int d^4x \sqrt{-g} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}, \quad \text{with} \quad F_{\mu\nu\rho\sigma} = \nabla_{[\mu} A_{\nu\rho\sigma]} = c\epsilon_{\mu\nu\rho\sigma}/\sqrt{-g} \tag{84}$$

Such a field F has no dynamics, but the F^2 term in the action behaves like an effective cosmological constant term, whose value is determined by the unknown scalar field c. The observable value of the CC in this framework is not a fundamental parameter. It is necessary to consider universes with different values of Λ distributed according to a certain distribution $\mathcal{P} \propto \exp(-S(\Lambda))$ in which Λ is promoted to be a quantum number. The action S is determined to be:

$$S(\Lambda) = -3\pi \frac{M_p}{\Lambda} \tag{85}$$

and thus the most probable configurations are those with $\Lambda = 0$. The idea also goes back to Baum [94].

It is the first appearance of the idea that the CC could be fixed by the shape of the wavefunction of the universe.

6.1.1. Wormholes In a somewhat similar approach Coleman [95] argued that one did not need to introduce a 3-form gauge field, if one includes the topological effects of wormholes. His final result is that the probability is $\mathcal{P} \propto \exp\left[\exp(-S(\Lambda))\right]$ and thus is even sharper peaked at $\Lambda = 0$ than in Hawking's case.

6.2. Anthropic Principle

Several inflationary scenario's [96, 97, 98], quantum cosmologies, [93, 99, 100, 101, 102] and string theory [103, 104, 105] predict different domains of the universe, or even different universes, with widely varying values for the different coupling constants.

Since the conditions for life to evolve as we know it are very constrained, one can use a form of the anthropic principle to select a certain state, with the right value for the cosmological constant, the fine structure constant, etc, from a huge ensemble.

A not very technical and almost foundational introduction to the anthropic principle is given by [41]. See [106] for a recent critique.

6.2.1. Discrete It might be worthwhile to make a distinction between a continuous anthropic principle and a discrete version. Imagine we have a theory at our hands describing an ensemble of universes (different possible vacuum solutions) with different discrete values for the fine structure constant: $\alpha = \ldots, 1/134, 1/135, 1/136, 1/137, \ldots$ etc. An anthropic argument could then be used to explain why we are in the universe with $\alpha = 1/137$. Best would be if this theory would also give us a prescription how to calculate the higher order corrections for each of these values, i.e. for each of these universes. Such use of an anthropic principle might be easier to accept than one where all digits are supposed to be anthropically determined.

6.3. Summary

This very much discussed approach offers a new line of thought, but so far, unfortunately, predictions for different constants of Nature, like the cosmological constant and the fine-structure constant, are not interrelated. We try to look for a more satisfying approach.

7. Beyond 4 Dimensions

Since the Casimir effect troubles our notion of a vacuum state, the cosmological constant problem starts to appear when considering distances larger than a millimeter or so. Therefore, extra dimensions with millimeter sizes might provide a mechanism to understand almost zero 4D vacuum energy. This size really is a sort of turn-over scale. Somehow all fluctuations with sizes between a Planck length and a millimeter are cancelled or sum up to zero.

Often the extra dimensions are compactified, but they can also be kept very large as in section (5.2). Alternatively, the space can be kept uncompactified, but warped, as in the Randall-Sundrum type-II models [107]. In this case the size of the extra dimensions can be infinite, but their volume is still finite. This warping causes the graviton wavefunction to be peaked near the brane, in other words, gravity is localized:

$$\bar{M}_{Pl}^2 = 2M_*^3 \int_0^{\pi r_c} dy e^{-2k|y|} = \frac{M_*^3}{k}\left(1 - e^{-2\pi k r_c}\right), \qquad (86)$$

where $\bar{M}_{Pl}^2$ is the 4D reduced Planck scale at 10^{18} GeV. The warp factor $e^{-2\pi k r_c}$ is a very small quantity, which implies that $\bar{M}_{Pl}, M_*$ and k all have essentially comparable magnitudes. The same warping would occur for a 4D-Planck scale Higgs vev, thereby providing a solution to the hierarchy problem.

Gravity in the $4D$ subspace reduces to GR up to some very small Yukawa-type corrections. Unfortunately however, the model is no candidate for a solution to the cosmological constant problem, since in order to get very small or zero tension on the SM brane, finetuning of bulk cosmological constant and brane tension at the other Planck brane is still necessary. All fundamental energy scales are at TeV level, but the vacuum energy density in our 4D-world is much smaller.

7.1. Self-Tuning Solutions

Transmitting any contribution to the CC to the bulk parameters, in such a way that a 4D-observer does not realize any change in the 4D geometry seems quite spurious. It would become more interesting if this transmission would occur automatically, without the necessity of re-tuning the bulk quantities by hand every time the 4D vacuum energy changes. Models that realize this are called *self-tuning* models (see for example [108]). A severe drawback that supposedly all these models face is that this scenario does not exclude '*nearby curved solutions*'. This means that in principle there could exist solutions for neighboring values of some bulk parameters, which result in a curved 4D space. Besides, there are additional problems such as a varying effective Planck mass, or varying masses for fields on the brane. So far no mechanism without these drawbacks has been found.

7.2. Extra Time-like Dimensions

Since the symmetry $x \to ix$ discussed in section (3.2) can be seen as transforming our (3+1)-dimensional world into a (1+3)-dimensional world, let us for completeness also briefly mention approaches using extra time-like dimensions. See e.g. in $D = 11$ supergravity [109]. Classical vacuum solutions can be obtained with zero cosmological constant and without ghosts or tachyons in the low energy limit. There are however many different solutions with different characteristics so the predictive power seems to be minimal.

8. Conclusions

In this paper we categorized the different approaches to the cosmological constant problem. The many different ways in which it can be phrased often blurs the road to a possible solution and the wide variety in approaches makes it difficult to distinguish real progress.

So far we can only conclude that in fact none of the approaches described above is a real outstanding candidate for a solution of the 'old' cosmological constant problem. Most effort nowadays is in finding a physical mechanism that drives the Universe's acceleration, but as we have seen these approaches, be it by modifying general relativity in the far infrared, or by studying higher dimensional braneworlds, generally do not convincingly attack the old and most basic problem.

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