Geometric Algorithm for Received Signal Strength Based Mobile Positioning

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Abstract. *Mobile positioning is one of the fastest growing areas for the development of new technologies, services and applications. This paper describes a simple and efficient geometric algorithm using received signal strength measurements extracted from at least three base stations. This method is compared with standard Least Squares method. The simulation results show, that geometric algorithm gives more accurately location estimation than LS algorithm in multipath propagation.*

Keywords

Received signal strength (RSS), Least squares (LS), Non-line-of-sight (NLoS), Line-of-sight (LoS), radio channel.

1. Introduction

Recently, Mobile Station (MS) location information has received significant interest. Different location-based applications include location sensitive billing, fleet tracking, package and personnel tracking, mobile yellow pages, location based messaging, route guidance and providing traffic information. The frequency and accuracy of location requests vary with the required application. Location based services is one of the most important services for the next generation mobile systems.

Wireless location systems usually require two or more base stations (BSs) to intercept a mobile station (MS) signal. Common location approaches are based on Cell Identification (Cell ID), time-of-arrival (TOA), received signal strength (RSS), time-difference-of-arrival (TDOA), angle-of-arrival (AOA) measurements determined from the MS signals received at the BSs [7, 11]. In this correspondence, we focus on mobile positioning using the RSS information.

The advantage of the RSS technique is in fairly low costs since power measurements are already made by handsets (part of the handoff algorithm). If a propagation model is known, power measurement can be mapped to a distance measurement. If the distance from the mobile terminal to three or more base stations is known, the mobile terminal position can be calculated. Unfortunately, RSS estimation error is higher than the other localization methods. Also, the mobile terminal usually moves and fast fading caused by multipath propagation causes significant errors. This is showed in the next section. The results acquired by geometric approach are compared with standard LS results.

2. Channel Propagation Model

The mobile radio channel defines fundamental limitations on the performance of wireless communications systems and will play an important role in these positioning systems. Radiolocation using signal strength is a well known location method that uses a mathematical model describing a signal path loss with a distance [1]. Therefore it is suitable to define basic parameters necessary for RSS location method.

Radio channel is generally hostile in nature. It is very difficult to predict its behavior. Therefore, radio channel is modeled in a statistical way using real propagation measurement data. In general, the signal fading in a radio environment between a transmitter and a receiver can be decomposed into a large-scale path loss component together with a medium-scale slow-varying component (having a lognormal distribution) and a small-scale fast varying component (with Rician or Rayleigh distribution).

2.1 Large-Scale Propagation Model

A path loss predicts signal power attenuation with the distance from a BS. Well known models for path loss are Hata's, Lee's, COST231 model etc. Hata [3-5, 10] developed a useful model for a path loss in macrocells on experimental results of Okumura [1]. The model expresses path loss as a function of BS height, MS height, carrier frequency and the type of environment (urban, suburban or rural). In Hata's model, the path loss is expressed as [in dB]

$$\begin{aligned} L_{PL}(Urban) &= 69{,}55 + 26{,}16\log(f) - 13{,}82\log(h_b) \\ &\quad - a(h_m) + (44{,}9 - 6{,}55\log(h_b))\log(d) \end{aligned}, \tag{1}$$

$$a(h_m) = (1{,}1\log(f) - 0{,}7)h_m - 1{,}56\log(f) + 0{,}8\,,$$

$$\begin{aligned} L_{PL}(Rural) &= L_{PL}(Urban) - 4{,}78(\log(f))^2 \\ &\quad + 18{,}33\log(f) - 40{,}94 \end{aligned} \tag{2}$$

where d is the distance between BS and MS in km, h_b is the height of BS (transmitter), and h_m is the height of MS (receiver), both in m.

The range of parameters over Hata model is valid under the following conditions

$$150 \le f[MHz] \le 1500, \quad 30 \le h_b[m] \le 200, \\ 1 \le h_m[m] \le 10. \tag{3}$$

2.2 Medium and Small Scale Propagation Model

The mobile radio envelope $r(t)$, illustrated in Fig. 1, is composed of $m(t)$ and $r(t)$

$$s(t) = m(t).r(t). \tag{4}$$

The component $m(t)$ is called a long-term fading or a log-normal fading and its variation is due to a terrain contour between a base station and a mobile station. It represents a slow variation in the mean envelope over a distance. Medium-scale variations take on Gaussian statistics when the signal is expressed in dB. The Gaussian or normal density function takes the form

$$p(a) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(a-\mu)^2}{2\sigma^2}\right] \tag{5}$$

where a is the signal level [dB], σ^2 is the variance of the signal distribution [dB], and μ is the mean signal level stated [dB].

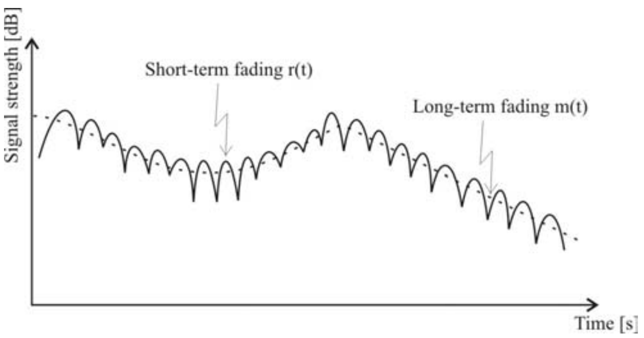

Fig. 1. A mobile radio signal fading representation.

Small-scale propagation models are characterized by the fast variation of the received signal strength over a short distance on the order of a few wavelengths or over short time durations on the order of seconds. The second component represents a short-term fading or a multipath fading. Variation of this fading is due to a wave reflection from surrounding buildings and other structures [2, 4, 5].

A channel with a large number of paths can be modeled as a Rayleigh channel. The signal amplitude is Rayleigh distribution in a case of large number of obstacles and there are dominant reflected waves (NLoS environment – Non Line of Sight). NLoS propagation always exists in cities or built-up environments. Rayleigh distribution is defined

$$p(a) = \frac{a}{\sigma^2} \exp\left\{-\frac{a^2}{2.\sigma^2}\right\}, \tag{6}$$

where a is the signal amplitude and σ^2 is the variance of the signal distribution [dB^2].

In a situation where the distance between the transmitter and receiver is small and the environment is static, there is a fixed spatial pattern of maxima and minima. Mostly there is a dominant stationary (non-fading) signal component. This is a case of LoS (Line of Sight) propagation path. The LoS propagation model is given by the Rician distribution [4, 5, and 10]

$$p(a) = \frac{a}{\sigma^2} \exp\left\{-\frac{(a^2+s^2)}{2.\sigma^2}\right\} I_0\left(\frac{a.s}{\sigma^2}\right), \tag{7}$$

where $I_0(.)$ is modified Bessel function of the first kind and zeroth order, s is the peak value of the specular radio signal, variable a is the signal amplitude and σ^2 is the variance of Ricean distribution.

3. System Model

We assume the system model as following:

- The number of BS is N.
- Signals from N - BSs are measured at MS.
- Received Signal Strength from each BS is independent to each other.

The service area consists of 7 cells as shown in Fig. 2. All cells in the system are deployed with omni directional antennas. The cell radius is 1000 m.

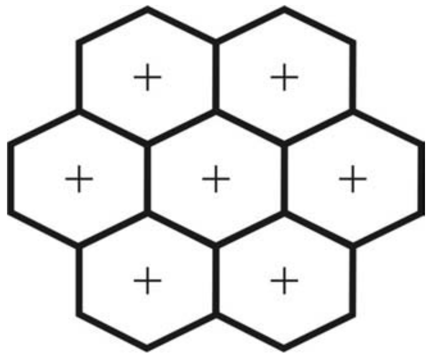

Fig. 2. Configuration of cells arrangement.

Simulations are done in two different environments, urban and rural. For each environment proper Hata's model is used, for urban environment (1) and for rural environment (2). In urban environment we consider NLoS propagation and in rural environment we consider LoS propagation.

4. RSS Measurement Model

In this section, we introduce the proposed algorithm for RSS based mobile positioning:

$$RSS = T_x - \left(L_{LS} + L_{MS} + L_{SS}\right), \tag{8}$$

where RSS is the received signal strength, T_x is the transmitted signal strength. Following parameters influence signal attenuation: L_{LS} is the signal degradation, caused by a large-scale propagation, L_{MS} is the signal degradation, caused by a medium-scale propagation, and finally L_{SS} is the signal degradation, caused by a small-scale propagation. All of the parameters are in [dB].

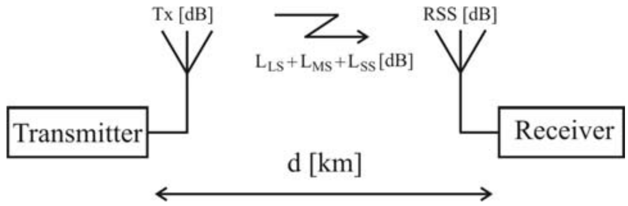

Fig. 3. RSS measurement model.

It is easy to define the distance between MS and BS from RSS measurement at the MS from the i^{th} BS. For simplification we consider the location in a 2-D plane. Let the true location of MS is $[x_s, y_s]^T$ (receiver) and the coordinates of the i^{th} BS is $[x_i, y_i]^T$ (source), i=1, 2, …, N. The distance between MS and the i^{th} BS, denoted by d_i, is given by

$$d_i = \sqrt{\left(x_s - x_i\right)^2 + \left(y_s - y_i\right)^2}, \quad i = 1, 2, \ldots, N\ . \tag{9}$$

In presence of disturbance a measured distance is r_i

$$r_i = d_i + n_i = \sqrt{\left(x_s - x_i\right)^2 + \left(y_s - y_i\right)^2} + n_i, \quad i = 1, 2, \ldots, N \tag{10}$$

where n_i is the noise or range error at the i^{th} BS. For the simplification we assume that measurement errors $\{n_i\}$ are zero mean Gaussian variables with known variance σ^2.

The distance r_i determines the radius of a circle. If we use at least three BSs to resolve ambiguities, the position of a MS is given by the intersection of circles. The circles are given by equations

$$\left(x_s - x_i\right)^2 + \left(y_s - y_i\right)^2 = r_i^2, \quad i = 1, 2, \ldots, N\ . \tag{11}$$

In general, two approaches can be taken for solving MS location on the basis of signal strength measurements [1]. The straightforward approach is to use a geometric interpretation of the measurements and compute the intersection of the lines of position (in this case it means circles). The second approach is a statistical approach.

5. Mobile Location Algorithms

5.1 Geometric Algorithm - GA

The function of the suggested algorithm is an estimation of the MS location point (with the highest probability).

In the ideal case the circles are intersected in one point, the MS location is defined unequivocally. In a real environment we can't assume the previous case. In this case there are 3 relevant intersections and the algorithm has to calculate 1 point (MS location). These 3 points and circles circumscribe the Region of Possible Positions (Fig. 4).

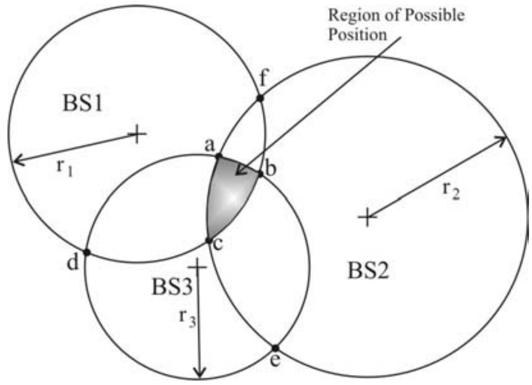

Fig. 4. The Region of Possible Positions formed by the Intersection of Circles.

The best calculation of the position would be to evaluate the center of gravity of the Region of Possible Positions [8]. A simple and fairly straightforward estimate of this is to determine the mean position of the triangle corner points (forming the Region of Possible Positions). This is done in a simple two-stage process. The set of all the intersections of circle pairs is calculated at first. Methods for finding the two intersections (two sets of values for x and y) from these equations requires considerable amount of algebraic manipulation.

The process for finding the two intersections (between two circles) is described at the following. At first the distance between centers of circles is calculated

$$ix = x_{i+1} - x_i, \quad i = 1, 2 \text{ - the horizontal distance,} \tag{12}$$

$$iy = y_{i+1} - y_i \text{ - the vertical distance,} \tag{13}$$

$$c = \sqrt{ix_i^{\ 2} + iy_i^{\ 2}} \text{ - the center distance.} \tag{14}$$

We adjust the angle β for the proper quadrant with respect to the coordinate system

$$\beta = \arctan\left(\frac{iy}{ix}\right). \tag{15}$$

For further description we have to define the variable α:

$$\alpha = \arccos\left(\frac{c^2 + r_i^2 + r_{i+1}^2}{2cr_i}\right). \tag{16}$$

$ix_i = r_i \cos(\beta - \alpha)$ the center i^{th} circle to 1st intersection, horizontal, (17)

$iy_i = r_i \sin(\beta - \alpha)$ the center i^{th} circle to 1st intersection, vertical, (18)

$ix_{i+1} = r_i \cos(\beta + \alpha)$ the center i^{th} circle to 2nd intersection, horizontal, (19)

$\mathrm{iy}_{i+1} = \mathrm{r}_i\ sin(\beta - \alpha)$ the center i^{th} circle to 2nd intersection, vertical. (20)

Finally, the coordinates of circles intersections are:

$$\begin{aligned} x_1 &= x_i + ix_i \\ y_1 &= y_i + iy_i \end{aligned} \quad \text{the 1}^{\text{st}} \text{ intersection coordinates,} \tag{21}$$

$$\begin{aligned} x_2 &= x_{i+1} + ix_i \\ y_2 &= y_{i+1} + iy_i \end{aligned} \quad \text{the 2}^{\text{nd}} \text{ intersection coordinates.} \tag{22}$$

After the first step we know intersections' coordinates

$$I = \lfloor x_j ; y_j \rfloor,\ j = 1, 2, \ldots, 2N\ . \tag{23}$$

Secondly a subset of interior intersections is created in the following way. If an intersection point of a pair of circles is inside of all the other circles, then it is defined as point from a subset of interior intersections. Otherwise, it is not included in the subset of interior intersections

$$I_{INTER} = \lfloor x_j ; y_j \rfloor,\ j = 1, 2, \ldots, N\ . \tag{24}$$

Our algorithm uses three circles for the location calculation, it means that N=3. There are known coordinates of circles centers and circles radiuses. From these parameters it is possible to determine intersections of circles. After the first step, six intersections *a*, *b, c, d, e* and *f* are known, but only three of them belong to interior intersections (as shown in Figure 3). The algorithm has to choose these 3 intersections. After the second step, three points (*b, c* and *a*) are known and consequently we have to calculate the mean value of the correct points. The mean of these points is identical with the location estimation of this algorithm.

5.2 LS Algorithm

$$\begin{aligned} r_i^2 &= (x_s - x_i)^2 + (y_s - y_i)^2 \\ &= R_s - 2x_s x_i - 2y_s y_i + \left(x_i^2 + y_i^2\right), \\ &\qquad i = 1, 2, \ldots, N \end{aligned} \tag{25}$$

where $R_s^2 = x_s^2 + y_s^2$; r_i is the measured distance between the source and the i^{th} receiver. By means of the new variable definition $K_i = x_i^2 + y_i^2$, we rewrite (25) through a set of linear expressions

$$2x_i x_s + 2y_i y_s - R_s = K_i - r_i^2,\ i = 1, 2, \ldots, N\ . \tag{26}$$

Equation (26) can be expressed in a matrix form

$$\mathbf{GZ} = \mathbf{h} \tag{27}$$

where

$$\mathbf{G} = \begin{bmatrix} 2x_1 & 2y_1 & -1 \\ 2x_2 & 2y_2 & -1 \\ \vdots & \vdots & \vdots \\ 2x_N & 2y_N & -1 \end{bmatrix},$$

$$\mathbf{Z} = \begin{bmatrix} x_s & y_s & R_s^2 \end{bmatrix}^T, \tag{28}$$

$$\mathbf{h} = \begin{bmatrix} K_1 - r_1^2 & K_2 - r_2^2 & \cdots & K_N - r_N^2 \end{bmatrix}^T.$$

The position can be estimated using standard least squares

$$\mathbf{Z} = \mathbf{G}^{-1}\mathbf{h}\ . \tag{29}$$

6. Simulation Results and Comparison

Presented numerical simulations compare the performance of the proposed geometric algorithm and the standard LS method. Simulations are done in two cases. In the first case we assume urban environment. In the second case we assume rural environment. These conditions are important for correct choosing of environment properties.

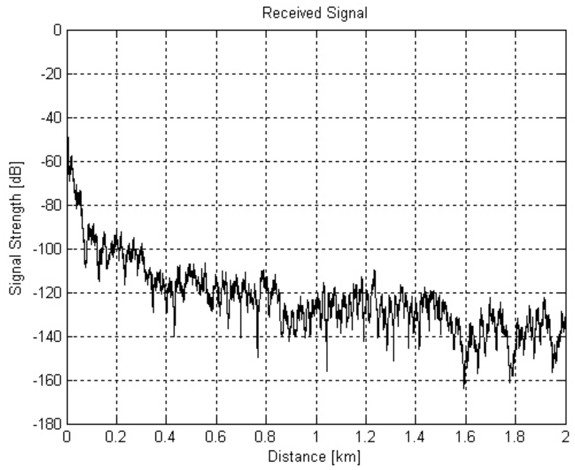

Fig. 4. Signal strength of mobile radio signal in urban environment.

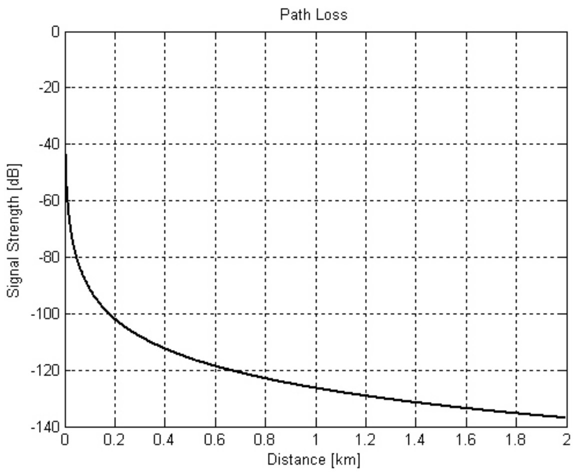

Fig. 5. Path Loss in urban environment according to (1).

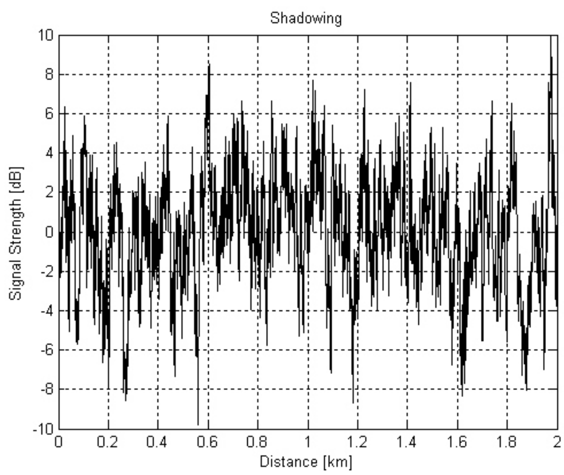

Fig. 6. Shadowing in urban environment.

Considering urban environment, Fig. 4 to 7 show the signal strength received at the MS in urban environment. The signal can be decomposed to particular parts – long-term fading (path loss and shadowing) and short-term fading.

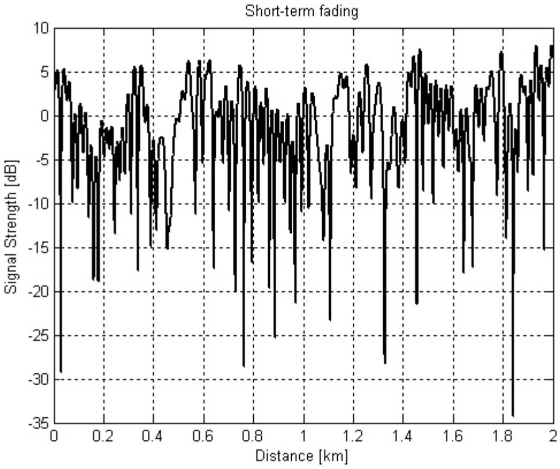

Fig. 7. Short-term (Rayleigh) fading in urban environment.

The situation for the second case (rural environment) is different (see Fig. 8.).

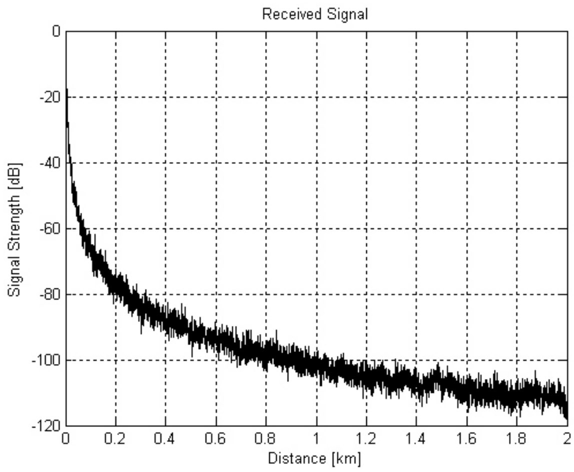

Fig. 8. Mobile radio signal in rural environment.

Environment	Urban		Rural	
Fading	Shadowing	Short - term	Shadowing	Short - term
Distribution	Log - normal	Rayleigh	Log - normal	Ricean
σ [dB]	4	6	1	2
μ [dB]	0	-1,5	0	-1

Tab. 1. Table of particular environments, their parameters and distributions used for the description.

We assume that there are seven transmitters (BSs), and the receivers are close to each other. Their positions are [0, 0] m, [2000, 0] m, [1000,-2000] m, [-1000, -2000] m, [-2000, 0] m, [-1000, 2000] m, and [1000, 2000] m. The receiver (MS) moves randomly in the square

$$-2000\ m \le x_s, y_s \le 2000\ m\ . \qquad (30)$$

Simulations are performed using three transmitters (with the shortest distance between the transmitter and the receiver, depending on the signal strength). All results were based on 5000 independent runs.

The accuracy of both methods is compared by Root Mean Square Error (RMSE)

$$RMSE = \sqrt{(X-x)^2 + (Y-y)^2}\ \text{[m]}, \qquad (31)$$

where $[X,Y]$ are the coordinates of the true location and $[x,y]$ are the coordinates of the estimated location.

At first it is necessary to determine the influence of fading to the location accuracy. The problem of fading is partially eliminated by estimating Local Average Power. It is calculated by equation (32)

$$\overline{RSS} = \frac{1}{N_s}\sum_{i=0}^{N_s-1} RSS_i\ , \qquad (32)$$

where N_s is the number of samples.

Comparing of the results is shown in the table 2.

Environment	Urban		Rural	
RSS	***RSS*** N_S=10	***RSS*** N_S=1	***RSS*** N_S=10	***RSS*** N_S=1
RMSE [km] GA	0,3298	0,8661	0,2205	0,395
RMSE [km] LS	0,4681	1,5946	0,2937	0,521

Tab. 2. Comparing of results for both algorithms with conditions defined in the table 1.

For the next experiments we consider using of the algorithm with Local Average Power (N_s = 10 samples). Fig. 9 and 10 show the distribution of the MS position estimates obtained by the GA and LS, for different environments.

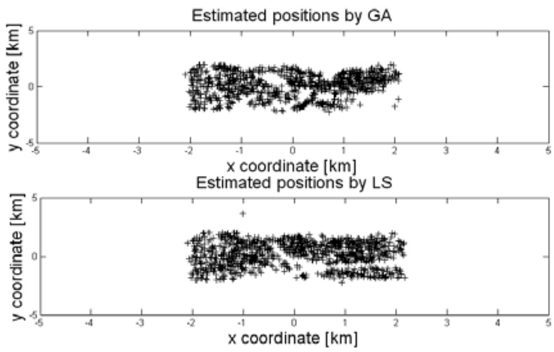

Fig. 9. Estimated positions of MS for rural environment.

In Fig. 9 a 10 we illustrate an environment impact on the positioning accuracy. Both methods work just reliably in the rural environment (Fig. 9). Conditions for positioning in the urban environment are much more complicated and therefore the distribution of MS positions is larger. These results are numerically expressed in Tab. 2. From Tab. 2 (and Fig. 9, 10) we can see that the geometric algorithm is more suitable (in comparison with standard LS method) for the estimation of a position in the presence of NLoS propagation environment.

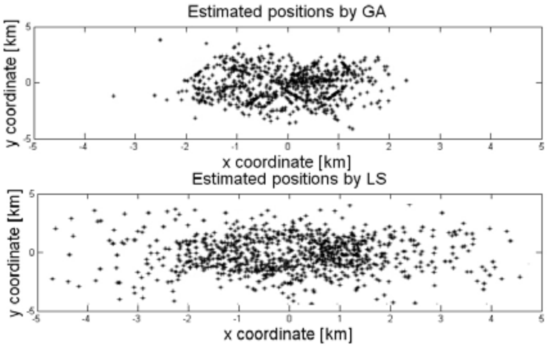

Fig. 10. Estimated positions of MS for urban environment.

7. Conclusions

The proposed location algorithm reduces the induced errors in the presence of NLoS propagation. We compare the proposed geometric algorithm with the standard LS method. According to these results, the performance of the proposed algorithm is better in comparison to the conventional method.

Acknowledgements

The research described in the paper was supported by the state programme of the Slovak Republic, No. 2003 SP 51/028 09 00/028 09 10 “Communication Networks and Services of New Generations”.

This paper was supported by the grant VEGA - 1/0140/03 “Effective Radio Resources Management Methods in Next Generations of Mobile Communication Networks”.

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