Fertility, Income Distribution, and Growth*

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Abstract

In this paper I develop a unified theory of fertility, inequality, and growth. The model is consistent with a phase of stagnation during which the economy exhibits Malthusian features, followed by a transition to a balanced-growth regime. A special emphasis is placed on the role of education and child labor policies. I use the model to explain the different transition experiences of Brazil and Korea. While Korea had a fast demographic transition and consistently low inequality, the demographic transition was slow in Brazil, and the income distribution was unequal. Numerical experiments show that policy differences can explain these observations. In a further empirical application I show that the model can reproduce observed patterns in inequality and fertility in nineteenth-century England, once policy changes towards the end of the century are accounted for.

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1 Introduction

The last 200 years have been a period of unprecedented economic and social change. Starting with the onset of the industrial revolution in Britain, the inhabitants of an increasing number of countries have experienced sustained increases in their living standards. The transition from preindustrial stagnation to modern growth has been accompanied by equally sweeping changes in other areas of economic and social life. In this paper I am focusing on two aspects of this change: The fall in mortality and fertility rates known as the "demographic transition" and the pattern of initially increasing and then decreasing inequality known as the "Kuznets curve."

In countries that have not entered the demographic transition both mortality and fertility rates are high by the standards of industrialized countries. The demographic transition typically begins with a fall in mortality. Since fertility stays initially high, the fall in mortality leads to high population growth during the early phases of the transition. With a lag, fertility begins to fall as well, so that population growth slows down. Most industrial countries today have fertility rates at or below the replacement level, so that without immigration population growth comes to a halt. Every industrialized country has experienced a demographic transition. Yet while the overall pattern is repeating itself throughout the world, there are striking differences in the experiences of individual countries. In Britain, there was a lag of almost a century between the start of mortality decline and the phase of most rapid fertility decline. In contrast, in this century countries like Japan or Korea have completed the demographic transition in a period of about 30 years. Other countries are just beginning to enter the demographic transition. In many developing countries, fertility and population growth are still high.

Simon Kuznets' (1955) hypothesis on the connection between development and the income distribution states that inequality first increases and later decreases during industrialization. His conjecture of an inverted-U-shape relationship between inequality and growth was based on observations from Britain, Germany, and the United States. However, unlike the demographic transition the concept of a Kuznets curve proved to be an elusive one. In this century a number of countries, notably the Asian "Tiger economies," have industrialized and achieved spectacular growth rates without any pronounced increases in inequality. Other countries, many of them in Latin America, have maintained high inequality without a clear trend in either direction.

The goal of this research is to develop a unified theory that can account for observed differences in demographic change and the evolution of the income distribution during industrialization. There are a number of reasons to suspect that demographic change and the income distribution are related. One determinant of inequality is the distribution of education and skill in the economy. From the perspective of the economics of the family, to the degree that parents make educational decisions for their children, education can be understood as a decision on the quality of children. If decisions on the quality of children are made jointly with decisions on the quantity of children, there is a direct link between fertility and population growth and the income distribution. This link is amplified by the fact that fertility differs across different groups within a population, with high fertility typically occurring in groups with low income and low education.

I develop a model in which the demand for skill and the relative cost of education are the driving forces behind the demographic transition and changes in the income distribution. I use this model to investigate how much of the differences in demographic change and the evolution of the income distribution across countries can be explained by differing policies and regulations in the areas of education and child labor. In the model, there are people of two skill levels, skilled and unskilled. Output can be produced with an agricultural technology that uses land and the two types of labor, and with an industrial technology that uses skilled and unskilled labor only. Productivity in both sectors grows at exogenous and possibly different rates. People live for two periods, and as adults they decide on the number and on the education level of their children. If parents send their children to school, the children will be skilled as adults, otherwise they remain unskilled. If children do not go to school, they engage in child labor.

Initially, as long as productivity in the industrial sector is low, the model exhibits Malthusian features. Wages are constant over time, and population growth is just fast enough to offset the improvements in agricultural technology. This part of the model is consistent with the economic history of the world before the industrial revolution.

At some point, productivity in the industrial sector reaches a sufficiently high level for the industrial technology to be used. Since the industrial technology has constant returns, population growth no longer depresses wages, so that for the first time wages start to rise. During the transition period, the proportion of the population working in the industrial sector increases. Since the industrial technology is assumed to be more skill-intensive, the relative number of skilled workers increases, as more unskilled parents send their children to school. Ultimately, the economy reaches a balanced growth path. Wages increase at the rate of technological progress, the ratio of skilled to unskilled people is constant, and population grows at a constant rate.

The timing of the demographic transition and the evolution of the income distribution depends crucially on government policies. If parents have to pay for schooling and child labor is unrestricted, industrialization leads to increasing inequality, and demographic change is slow. In contrast, with public education and restrictions on child labor, inequality stays low, and the demographic transition is completed in a very short time.

I use the model to examine differences in the transition experience of South Korea and Brazil. Korea has a good education system, low child labor, low inequality, and experienced a fast demographic transition. In contrast, inequality is high in Brazil, the education system is weak, there is a high incidence of child labor, and the demographic transition proceeded much slower. In order to explain these differences, I simulate the model under two different assumptions on education policies and child labor restrictions. The numerical results show that the theory is consistent with the observed patterns in these countries. The model also produces additional implications pertaining to per capita incomes and fertility differentials across income groups, which match the data as well.

As a further test of the theory, I turn to the country in which the industrial revolution once started, England. In England the demographic transition was spread out over about 100 years. Inequality increased during early industrialization, and started falling towards the end of the 19th century, about at the same time when fertility started to fall more quickly as well. I conjecture that this pattern is partly caused by changes in education and child labor policies in the second half of the nineteenth century. To test this conjecture, I simulate the model under the assumption that education and child labor policies change 100 years after the beginning of industrialization. The simulated paths for fertility and inequality follow the same pattern that is observed in the data.

This paper is related to previous research in a number of fields. My approach is similar to Becker, Murphy and Tamura (1990) in that I emphasize a quantity-quality tradeoff in the decision on children. Their model also exhibits two steady states, one in which wages stagnate and fertility is high, and another in which there is sustained growth in per capita income and fertility is low. However, there is no transition between the two steady states, and the authors do not consider the distribution of income. Galor and Weil (1999), Tamura (1996, 1998), and Jones (1999) are all models that are consistent with a long phase of stagnation, followed by a transition to a continued growth regime. The key emphasis of these models is the actual cause of the transition, the typical explanation being an external effect of population on technological progress. Goodfriend and McDermott (1995) and Kremer (1993) work along similar lines without considering fertility decisions. In contrast to those papers, I concentrate on the dynamics of the income distribution and fertility, conditional on technological change. None of the mentioned papers considers the role of child labor and education policies. The technologies used in this paper are related to Hansen and Prescott (1998) and Laitner (1998), who also consider economies with an agricultural and an industrial sector, subject to exogenous productivity growth. However, they do not endogenize fertility or consider the income distribution. Lam (1986) and Chu and Koo (1990) examine the relationship between the income distribution, fertility, and intergenerational mobility in a Markov context with exogenously given income-specific fertility rates. Raut (1991), Kremer and Chen (1999), and Dahan and Tsiddon (1998) are three papers that examine the relationship of endogenous fertility and the income distribution. Unlike those papers, I emphasize the quantity-quality tradeoff in the decision on children, and I generate a transition from a phase of stagnation to modern growth.

In the next section, I will introduce the model and define a recursive competitive equilibrium. Section 3 analyzes the two production sectors in the model economy, and in Section 4 I derive some important properties of the decision problem of adults. Section 5 discusses the behavior of the model in the Malthusian regime, the modern growth regime, and the transition between the two. In Section 6, I use the model to explain the transition experiences of Brazil and Korea. Section 7 uses the model to understand the evolution of fertility and the income distribution in England in the last 200 years. Section 8 concludes and provides directions for further research.

2 The Model

The economy is populated by overlapping generations of people who live for two periods, childhood and adulthood. Children receive education, do not enjoy any utility, and do not get to decide anything. Adults can be either skilled or unskilled, depending on their education. In each period there is a continuum of adults of each type; N_S is the measure of skilled adults, and N_U is the measure of unskilled adults. Adults decide on their consumption, labor supply, and on the number and the education of their children.

The single consumption good in this economy can be produced with two different methods. There is agricultural technology that uses skilled labor, unskilled labor, and land as inputs, and an industrial technology that only uses the two types of labor. Production in each sector is carried out by competitive firms. I will now describe the two technologies in more detail, and then turn to the decision problem of an adult.

Technology

The agricultural technology uses the two types of labor and land. Since I want to abstract from land ownership and bequests, I assume that land is a public good. From the perspective of a small individual firm, there are constant returns to labor. However, since there is a limited amount of land, labor input by one firm imposes a negative externality on all other firms. The situation resembles a hunter-and-gatherer society, or a world of fishermen who all fish in the same sea. Output y_F (F stands for "Farm") for a firm that uses the agricultural technology and employs l_S units of skilled labor and l_U units of unskilled labor is given by:

$$y_F = \tilde{A}_F \, (l_S)^{\frac{\theta_S}{\theta_S+\theta_U}} \, (l_U)^{\frac{\theta_U}{\theta_S+\theta_U}}, \tag{1}$$

where:

$$\tilde{A}_F = A_F \, [(L_{FS})^{\theta_S}(L_{FU})^{\theta_U}]^{-\frac{1-\theta_S-\theta_U}{\theta_S+\theta_U}} \, (Z)^{1-\theta_S-\theta_U}.$$

Here A_F is a productivity parameter, L_{FS} and L_{FU} are the aggregate amounts of skilled and unskilled labor employed in the agricultural sector, and Z is the total amount of land. Thus the total amount of labor employed has a negative effect on the productivity of an individual firm. The specific form of the external effect was chosen such that the aggregate agricultural production function is given by (2) below.

Assumption 1 *The parameters* θ_S *and* θ_U *satisfy* $\theta_S, \theta_U > 0$ *and* $\theta_S + \theta_U < 1$.

Profit maximization implies that all firms choose the same ratio of skilled to unskilled labor. Aggregating (1) yields the following aggregate agricultural production function:

$$Y_F = A_F \,(L_{FS})^{\theta_S} \,(L_{FU})^{\theta_U} \,(Z)^{1-\theta_S-\theta_U}. \tag{2}$$

As far as the analysis in this paper is concerned, the main feature of the agricultural production function are decreasing returns to labor. The assumption of decreasing returns is essential for generating the Malthusian regime.[1]

The industrial production function, on the other hand, exhibits constant returns to scale even in the aggregate. From the perspective of an individual firm, the production function is given by:

$$y_F = A_I \,(l_S)^{1-\alpha} \,(l_U)^{\alpha}, \tag{3}$$

and since there are no externalities, aggregate industrial output is:

$$Y_I = A_I \,(L_{IS})^{1-\alpha} \,(L_{IU})^{\alpha},$$

where L_{IS} and L_{IU} are aggregate amounts of skilled and unskilled labor employed in the industrial sector.

[1]The assumption of an external effect from labor, on the other hand, is not essential, and is used only to abstract from land ownership. Alternatives to this formulation include a socialistic society in which everyone owns an equal share of land, or an economy with a separate land-owning class. Each of these formulations would lead to the same qualitative results as the model described here.

Assumption 2 *The parameter α satisfies* $0 < \alpha < 1$.

The productivities of both technologies grow at constant, though possibly different rates:

$$A'_F = \gamma_F A_F, \tag{4}$$

$$A'_I = \gamma_I A_I, \tag{5}$$

where $\gamma_F, \gamma_I > 1$. The state vector x in this economy consists of the productivity levels A_F and A_I in the agricultural and industrial sectors, and the measures N_S and N_U of skilled and unskilled people:

$$x \equiv \{A_F, A_I, N_S, N_U\}.$$

The only restriction on the state vector is that it has to consist of nonnegative numbers. Therefore the state space X for this economy is given by $\mathbf{R}^4_+$:

$$X \equiv \mathbf{R}^4_+.$$

In equilibrium wages are a function of the state. The problem of a firm in sector j, where $j \in \{F, I\}$, is to maximize profits subject to the production function, taking wages as given:

$$\max_{l_S, l_U} \{y_j - w_S(x)l_S - w_U(x)l_U\}, \tag{6}$$

subject to (1) or (3) above. It will be shown in Proposition 1 below that firms will always be operating in the agricultural sector, while the industrial sector is only operated if the wages satisfy the following condition:

$$w_S(x)^{1-\alpha}\, w_U(x)^{\alpha} \le A_I\, (1-\alpha)^{1-\alpha}\, \alpha^{\alpha}.$$

Profit maximization implies that wages equal marginal products in each sector. Writing labor demand as a function of the state, for the agricultural sector we get the following conditions:

$$w_S(x) = A_F\, \frac{\theta_S}{\theta_S+\theta_U}\, \frac{L_{FU}(x)^{\theta_U}}{L_{FS}(x)^{1-\theta_S}}\, Z^{1-\theta_S-\theta_U}, \tag{7}$$

$$w_U(x) = A_F\, \frac{\theta_U}{\theta_S+\theta_U}\, \frac{L_{FS}(x)^{\theta_S}}{L_{FU}(x)^{1-\theta_U}}\, Z^{1-\theta_S-\theta_U}. \tag{8}$$

If the industrial sector is operating, wages have to equal marginal products as well:

$$w_S(x) = A_I\, (1-\alpha)\, \left(\frac{L_{FU}(x)}{L_{FS}(x)}\right)^{\alpha} \quad \text{if} \quad L_{FS}(x), L_{FU}(x) > 0, \tag{9}$$

$$w_U(x) = A_I\, \alpha\, \left(\frac{L_{FS}(x)}{L_{FU}(x)}\right)^{1-\alpha} \quad \text{if} \quad L_{FS}(x), L_{FU}(x) > 0. \tag{10}$$

Instead of writing out the firms' problem in the definition of an equilibrium below, I will impose (7)–(10) as equilibrium conditions.

Preferences

I will now turn to the decision problem of the adults. Adults care about consumption and the number and utility of their children. In this model, there are no gender differences; every adult is able to produce children without outside help. The preference structure is an extension of Becker and Barro (1989) to the case of different types of children. Adults discount the utility of their children, and the discount factor is decreasing in the number of children. In other words, the more children an adult already has, the smaller is the additional utility from another child. I specialize the utility function to the constant-elasticity case. The utility of an adult who consumes c units of the consumption good and has n_S skilled children and n_U unskilled children is given by:

$$U(c, n_S, n_U) = c^\sigma + \beta(n_S + n_U)^{-\epsilon}[n_S V_S' + n_U V_U'].$$

Here V_S' is the utility skilled children will enjoy as adults, and V_U' is the utility of unskilled children, both foreseen perfectly by the parent. The parameter σ determines the elasticity of utility with respect to consumption, β is the general level of altruism, and ϵ is the elasticity of altruism with respect to the number of children. The utilities V_S' and V_U' are outside of the control of parents and are therefore taken as given. The utility of children depends on the aggregate state vector in the next period, and since there is a continuum of people, aggregates cannot be influenced by any finite number of people.

Assumption 3 *The utility parameters satisfy* $0 < \beta < 1$, $0 < \sigma < 1$, *and* $0 < \epsilon < 1$.

Adults are endowed with one unit of time, and they allocate their time between working and child-raising. Children are costly, both in terms of goods and in terms of time. Raising each child takes $\rho > 0$ units of the consumption good and fraction $\phi > 0$ of the total time available to an adult. Adults also have to decide on the education of their children. Children need a skilled teacher to become skilled. It takes fraction ϕ_S of a skilled adult's time to teach one child. Therefore, if parents want skilled children, they have to send their children to school and pay the skilled teacher. Children who do not go to school stay unskilled and work already during childhood. Children can perform only the unskilled task, and one working child is equivalent to fraction ϕ_U of an unskilled adult who works full time. The parameter ϕ_U is smaller than one since children do not work from birth on, and since they are not as productive as adults. I also assume $\phi_U < \phi$, so that even after accounting for child labor there is still a net time cost associated with having unskilled children.

The budget constraint of an adult of type i, where $i \in \{U, S\}$, is given by:

$$c + (\phi w_i(x) + \rho)(n_S + n_U) + \phi_S w_S(x) n_S \leq w_i(x) + \phi_U w_U(x) n_U \quad (11)$$

The right hand side is the full income of the adult plus the income from working unskilled children. On the left hand side are consumption, the cost that accrues for every child (goods cost and time cost), and the cost for the education of children who go to school. For simplicity, adults are not restricted to choose integer numbers of children. Also notice that there is no uncertainty in this model. Whether a child becomes skilled does not depend on chance or unobserved abilities, but is under full control of the parent.

In equilibrium, wages and the utilities of skilled and unskilled people are functions of the state vector. The maximization problem of an adult of type i, where $i \in \{S, U\}$, is described by the following Bellman equation:

$$V_i(x) = \max_{c, n_U, n_S \geq 0} \left\{ c^\sigma + \beta(n_S + n_U)^{-\epsilon}[n_S V_S(x') + n_U V_U(x')] \right\}$$

subject to the budget constraint (11) and the equilibrium law of motion $x' = g(x)$.

The fact that only parents, not children, make educational decisions leads to a market imperfection. With perfect markets, children would be able to borrow funds to finance their own education. In equilibrium, children would have to be indifferent between going to school or not, so that net income of skilled and unskilled adults would be equalized. Since there are no differences in ability or stochastic income shocks, the market imperfection is necessary to create inequality in this model. I also rule out the possibility that parents write contracts that bind their children. Otherwise, parents could borrow funds from richer adults, and have their children pay back the loan to the children of the lender. Both assumptions are in line with reality: In the real world, children are usually not responsible for the debts of their parents, and we do not observe many children who receive loans to pay for their primary or secondary education.

Equilibrium

It will be shown in section 4 below that the adults' problem has only corner solutions. Adults either send all their children to school, or none of them; there are never both skilled and unskilled children within the same family. It is possible, however, that adults of a specific type are just indifferent between sending all their children to school or not. In that case, some parents of a given type might decide to have skilled children, while others go for the unskilled variety. In equilibrium, the typical situation will be that all skilled parents have skilled children, while there are both unskilled parents with unskilled children and unskilled parents who send their children to school. In other words, there is upward intergenerational mobility.

In the definition of an equilibrium I have to keep track of the fractions of adults of each type who have skilled and unskilled children. The function $\lambda_{i \to j}(\cdot)$ gives the fraction of adults of type i who have children of type j, as a function of the state x. Of course, for each type of parent and for all $x \in X$ these fractions have to sum to one:

$$\lambda_{S \to S}(x) + \lambda_{S \to U}(x) = \lambda_{U \to S}(x) + \lambda_{U \to U}(x) = 1. \tag{12}$$

The policy function $n_j(i, \cdot)$ gives the number of children for i-type parents who have j-type children, as a function of the state. For example, $n_S(U, x)$ is the number of children born to an unskilled adult who decides to send the children to school. Notice that in equilibrium adults have only one type of children.

I will now introduce the remaining equilibrium conditions, starting with the determination of labor supply. Skilled adults distribute their time between working, raising and teaching their own children, and teaching children of unskilled parents. Therefore the total supply of skilled labor L_S is given by:

$$\begin{aligned} L_S(x) =& [1 - (\phi + \phi_S)\ \lambda_{S\to S}(x)\ n_S(S,x) - \phi\ \lambda_{S\to U}(x)\ n_U(S,x)]\ N_S \\ & - \phi_S\ \lambda_{U\to S}(x)\ n_S(U,x)\ N_U. \end{aligned} \tag{13}$$

Notice that L_S only refers to skilled labor used for producing the consumption good; the time skilled adults spend as teachers is not counted. This is merely a matter of notational convenience, since it simplifies the market-clearing constraints for the labor market. Unskilled labor L_U is supplied by unskilled adults and by children who do not go to school:

$$\begin{aligned} L_U(x) =& [1 - \phi\ \lambda_{U\to S}(x)\ n_S(U,x) - \phi\ \lambda_{U\to U}(x)\ n_U(U,x)]\ N_U \\ & + \phi_U\ [\lambda_{S\to U}(x)\ n_U(S,x)\ N_S + \lambda_{U\to U}(x)\ n_U(U,x)\ N_U]. \end{aligned} \tag{14}$$

In equilibrium, labor supply has to equal labor demand for each type of labor. I assume that skilled adults can perform both the skilled and the unskilled work, while unskilled adults can do unskilled work only. Going to school does not lead to a loss of the ability to do unskilled work. Under this assumption, the skilled wage cannot fall below the unskilled wage, because then all skilled adults would decide to do unskilled work:

$$w_S(x) \geq w_U(x). \tag{15}$$

Unless the economy starts out with a very high number of skilled adults, even without this assumption the skilled wage never falls below the unskilled wage. Still, it will be analytically convenient to impose (15). The market-clearing conditions for the labor market are:

$$L_{FS}(x) + L_{IS}(x) \leq L_S(x), \qquad = \text{ if } w_S(x) > w_U(x), \tag{16}$$

$$L_{FU}(x) + L_{IU}(x) = L_U(x) + [L_S(x) - L_{FS}(x) - L_{IS}(x)]\,. \tag{17}$$

The final equilibrium condition is the law of motion for population. Since I abstract from child mortality, the number of adults of a given type tomorrow is given by the number of children of that type today:

$$N'_S = \lambda_{S\to S}(x)\; n_S(S,x)\; N_S + \lambda_{U\to S}(x)\; n_S(U,x)\; N_U, \tag{18}$$

$$N'_U = \lambda_{S\to U}(x)\; n_U(S,x)\; N_S + \lambda_{U\to U}(x)\; n_U(U,x)\; N_U. \tag{19}$$

We now have all the ingredients at hand that are needed to define an equilibrium.

Definition 1 (Recursive Competitive Equilibrium) *A recursive competitive equilibrium consists of value functions V_S and V_U, labor supply functions L_S and L_U, labor demand functions L_{FS}, L_{FU}, L_{IS}, and L_{IU}, wage functions w_S and w_U, mobility functions $\lambda_{S\to S}$, $\lambda_{S\to U}$, $\lambda_{U\to S}$, and $\lambda_{U\to U}$, all mapping X into $\mathbf{R}_+$, policy functions n_S and n_U mapping $\{S,U\} \times X$ into $\mathbf{R}_+$, and a law of motion g mapping X into itself, such that:*

(i) The value functions satisfy the following functional equation for $i \in \{S,U\}$:

$$V_i(x) = \max_{c,n_S,n_U \geq 0} \left\{ c^{\sigma} + \beta(n_S + n_U)^{-\epsilon} \left[n_S V_S(x') + n_U V_U(x') \right] \right\} \tag{20}$$

subject to the budget constraint (11) *and the law of motion $x' = g(x)$.*

(ii) For $i,j \in \{S,U\}$, if $\lambda_{i\to j}(x) > 0$, $n_j(i,x)$ attains the maximum in (20).

(iii) The wages w_S and w_U and labor demand L_{FS}, L_{FU}, L_{IS}, and L_{IU} satisfy (7)–(10) *and* (15).

(iv) Labor supply L_S and L_U satisfies (13) *and* (14).

(v) Labor supply L_S and L_U and labor demand L_{FS}, L_{FU}, L_{IS}, and L_{IU} satisfy (16) *and* (17).

(vi) The mobility functions $\lambda_{S \to S}$, $\lambda_{S \to U}$, $\lambda_{U \to S}$, and $\lambda_{U \to U}$ satisfy (12).

(vii) The law of motion g for the state variable x is given by (4), (5), (18), *and* (19).

Notice that the equilibrium conditions do not include a market-clearing constraint for the goods market, because it holds automatically by Walras' Law. In condition (ii) above, it is understood that parents choose only one type of children. In other words, saying that $n_S(S, x)$ attains the maximum in (20) means that $\{n_S = n_S(S, x), n_U = 0\}$ and the consumption c that results from the budget constraint maximize utility. Maximization is only required if a positive number of parents choose the type of children in question. Condition (iii) requires that wages equal marginal products and that the skilled wage does not fall below the unskilled wage, condition (iv) links labor supply to population and education time, condition (v) is the market-clearing condition for the labor market, condition (vi) requires that for each type of adult the fractions having skilled and unskilled children sum to one, and condition (vii) defines the law of motion.

Schooling Subsidies, Taxes, and Child Labor Restrictions

In the model described above, parents pay for the schooling of their children, and there are no restrictions on child labor. In this section I extend the model to allow for schooling subsidies by the government and child labor legislation. In the real world, most countries finance a large part of the education of their citizens, and child labor is usually subject to restrictions. In much of the analysis below I will be concerned with the effects of changes in these policies during the transition from agriculture to industry.

Incorporating child labor restrictions is straightforward. The government can limit the amount of time that children work, which in the model amounts to lowering the parameter ϕ_U. To stay in the recursive framework, I let the government choose a function $\phi_U(\cdot)$ which determines how much time children work, depending on the state. Since restrictions can only lower the legal amount of child labor, I require $0 \leq \phi_U(x) \leq \phi_U$ for all x. In the applications below, I will consider a one-time change in child labor policy. Such a policy can be represented by using a $\phi_U(\cdot)$ function that changes once the industrial technology reaches a certain threshold level. For example, if child labor is abolished completely in the period when A_I reaches $\bar{A}_I$, the function is given by:

$$\phi_U(x) = \begin{cases} \phi_U & \text{if } A_I < \bar{A}_I, \\ 0 & \text{if } A_I \geq \bar{A}_I. \end{cases}$$

Introducing education policies is more complicated. The government cannot decree the amount of time required to teach a child. Instead, I assume that the government subsidizes a fixed amount of the schooling cost for all children at school. The expenditure is financed with a flat income tax, and budget balance is observed in every period. The government chooses a function δ that determines the fraction of the schooling cost to be paid by the government, where $0 \leq \delta(x) \leq 1$ for all x. Contingent on this function, the flat tax τ is chosen to observe budget balance. The tax rate is given by dividing the total expenditure on schooling subsidies by total wage income:

$$\tau(x) = \frac{\delta(x)\ \phi_S\ N_S'(x)\ w_S(x)}{L_S(x)\ w_S(x) + L_U(x)\ w_U(x) + \phi_S\ N_S'(x)\ w_S(x)}. \tag{21}$$

Here $N_S'(x)$ is shorthand notation for the total number of skilled children:

$$N_S'(x) = \lambda_{S \to S}(x)\ n_S(S,x)\ N_S + \lambda_{U \to S}(x)\ n_S(U,x)\ N_U.$$

Notice that for the computation of total labor income we have to add the income of the teachers to the wage of the usual workers. Teachers receive wages for their work and are taxed like all other adults in the model economy.

With taxes and the subsidy, the budget constraint of an adult of type i becomes:

$$
\begin{aligned}
c + (\phi\ (1-\tau(x))\ w_i(x) + \rho)\ (n_S + n_U) + \delta(x)\ \phi_S w_S(x)\ n_S \\
\leq (1-\tau(x))\ (w_i(x) + \phi_U(x)\ w_U(x)\ n_U).
\end{aligned}
\tag{22}
$$

Finally, we also have to adjust the expression for unskilled labor supply (14) for the child labor policy:

$$
\begin{aligned}
L_U(x) =& [1 - \phi\ \lambda_{U\rightarrow S}(x)\ n_S(U,x) - \phi\ \lambda_{U\rightarrow U}(x)\ n_U(U,x)]\ N_U \\
&+ \phi_U(x)\ [\lambda_{S\rightarrow U}(x)\ n_U(S,x)\ N_S + \lambda_{U\rightarrow U}(x)\ n_U(U,x)\ N_U].
\end{aligned}
\tag{23}
$$

Apart from these changes, the definition of an equilibrium is parallel to the case without child labor and education policies.

Definition 2 (Equilibrium with Government Policy) *Given a government policy* $\{\phi_U, \delta\}$*, a recursive competitive equilibrium consists of a tax function* τ*, value functions* V_S *and* V_U*, labor supply functions* L_S *and* L_U*, labor demand functions* L_{FS}*,* L_{FU}*,* L_{IS}*, and* L_{IU}*, wage functions* w_S *and* w_U*, mobility functions* $\lambda_{S\rightarrow S}$*,* $\lambda_{S\rightarrow U}$*,* $\lambda_{U\rightarrow S}$*, and* $\lambda_{U\rightarrow U}$*, all mapping* X *into* $\mathbf{R}_+$*, policy functions* n_S *and* n_U *mapping* $\{S,U\} \times X$ *into* $\mathbf{R}_+$*, and a law of motion* g *mapping* X *into itself, such that:*

(i) The value functions satisfy the following functional equation for $i \in \{S,U\}$*:*

$$
V_i(x) = \max_{c,n_S,n_U\geq 0} \left\{ c^\sigma + \beta(n_S + n_U)^{-\epsilon} \left[n_S V_S(x') + n_U V_U(x') \right] \right\}.
\tag{24}
$$

subject to the budget constraint (22) *and the law of motion* $x' = g(x)$*.*

(ii) For $i, j \in \{S, U\}$, if $\lambda_{i\to j}(x) > 0$, $n_j(i, x)$ attains the maximum in (24).

(iii) The tax function τ satisfies the government budget constraint (21).

(iv) The wages w_S and w_U and labor demand L_{FS}, L_{FU}, L_{IS}, and L_{IU} satisfy (7)–(10) *and* (15).

(v) Labor supply L_S and L_U satisfies (13) *and* (23).

(vi) Labor supply L_S and L_U and labor demand L_{FS}, L_{FU}, L_{IS}, and L_{IU} satisfy (16) *and* (17).

(vii) The mobility functions $\lambda_{S\to S}$, $\lambda_{S\to U}$, $\lambda_{U\to S}$, and $\lambda_{U\to U}$ satisfy (12).

(viii) The law of motion g for the state variable x is given by (4), (5), (18), *and* (19).

3 Production in the Agricultural and Industrial Sectors

In this section, I will take a closer look at the two production sectors in the economy. The main result is that while the agricultural sector is always operating, industrial firms produce only if wages are sufficiently low relative to industrial productivity. The following proposition derives the condition that is necessary for production in industry.

Proposition 1 *Firms will be operating in the industrial sector only if the skilled and unskilled wages $w_S(x)$ and $w_U(x)$ satisfy the condition:*

$$w_S(x)^{1-\alpha}\; w_U(x)^{\alpha} \leq A_I\; (1-\alpha)^{1-\alpha}\; \alpha^{\alpha}. \tag{25}$$

Proof: The profit-maximization problem of a firm in the industrial sector is given by:

$$\max_{l_S, l_U} \left\{ A_I\; (l_S)^{1-\alpha}\; (l_U)^{\alpha} - w_S(x)\; l_S - w_U(x)\; l_U \right\}. \tag{26}$$

The first-order condition for a maximum with respect to l_U gives:

$$(l_S)^{1-\alpha} = \frac{w_U(x)(l_U)^{1-\alpha}}{A_I \alpha}.$$

Plugging this expression back into (26) yields a formulation of the profit maximization problem as a function of unskilled labor only:

$$\max_{l_U} \left\{ \frac{w_U(x)}{\alpha} \, l_U - w_S(x) \left(\frac{w_U(x)}{A_I \alpha} \right)^{\frac{1}{1-\alpha}} l_U - w_U(x) \, l_U \right\}. \tag{27}$$

Since this expression is linear in l_U, production in the industrial sector will be profitable only if we have:

$$\frac{w_U(x)}{\alpha} - w_S(x) \left(\frac{w_U(x)}{A_I \alpha} \right)^{\frac{1}{1-\alpha}} - w_U(x) \geq 0,$$

which can be rearranged to get:

$$w_S(x)^{1-\alpha} \, w_U(x)^{\alpha} \leq A_I \, (1-\alpha)^{1-\alpha} \, \alpha^{\alpha},$$

which is (25). □

The next proposition shows that in contrast to firms in the industrial sector, agricultural firms always operate.

Proposition 2 *For any skilled and unskilled wages $w_S(x)$ and $w_U(x)$ firms will be operating in the agricultural sector.*

Proof: The first-order necessary conditions for a maximum of the profit-maximization problem of a firm in agriculture are given by the wage conditions (7) and (8):

$$w_S(x) = A_F \frac{\theta_S}{\theta_S + \theta_U} \frac{L_{FU}^{\theta_U}}{L_{FS}^{1-\theta_S}} Z^{1-\theta_S-\theta_U}, \tag{28}$$

$$w_U(x) = A_F \frac{\theta_U}{\theta_S + \theta_U} \frac{L_{FS}^{\theta_S}}{L_{FU}^{1-\theta_U}} Z^{1-\theta_S-\theta_U}. \tag{29}$$

Since the problem is concave, the first-order conditions are also sufficient for a maximum. It is therefore sufficient to show that for any $w_S(x), w_U(x) > 0$ we can find values for skilled and unskilled labor supply L_{FS} and L_{FU} such that (28) and (29) are satisfied. The required values are given by:

$$L_{FS} = \left(\frac{A_F}{\theta_S + \theta_U}\right)^{\frac{1}{1-\theta_S-\theta_U}} \left(\frac{\theta_S}{w_S(x)}\right)^{\frac{1-\theta_S}{1-\theta_S-\theta_U}} \left(\frac{\theta_U}{w_U(x)}\right)^{\frac{\theta_U}{1-\theta_S-\theta_U}} Z$$

and:

$$L_{FU} = \left(\frac{A_F}{\theta_S + \theta_U}\right)^{\frac{1}{1-\theta_S-\theta_U}} \left(\frac{\theta_S}{w_S(x)}\right)^{\frac{\theta_S}{1-\theta_S-\theta_U}} \left(\frac{\theta_U}{w_U(x)}\right)^{\frac{1-\theta_U}{1-\theta_S-\theta_U}} Z,$$

which are positive for any positive wages $w_S(x)$ and $w_U(x)$. □

It is easy to check whether the industrial sector will be operated for a given supply of skilled and unskilled labor. We can use conditions (28) and (29) to compute wages in agriculture under the assumption that there is agricultural production only. If the resulting wages satisfy condition (25), the industrial technology is used. Skilled and unskilled labor is allocated so that the wage for each skill is equalized across the two sectors. If condition (25) is violated, production takes place in agriculture only.

In equilibrium, it will be the case that initially only the agricultural technology is used. At some point the industrial technology becomes sufficiently productive to be introduced alongside agriculture. An “industrial revolution” occurs, and ultimately the fraction of output produced in agriculture converges to zero. This behavior arises from an interaction between the properties of the two production sectors and the population dynamics in the model. Since population is determined by fertility decisions, I will now turn to the decision problem of an adult in the model economy.

4 The Decision Problem of an Adult

From the point of view of an adult, the utility of a potential skilled or unskilled child is given by a number that cannot be influenced. There are no individual state variables, and the utility of children is determined by fertility decisions in the aggregate, which adults take as given since there is a continuum of people. This allows us to analyze the decision problem of an adult in detail without solving for a complete equilibrium first. In this section, we will analyze the decision problem of an adult who receives wage $w > 0$ and who knows that skilled children will receive utility $V_S > 0$ in the next period, whereas unskilled children can expect $V_U > 0$. I restrict attention to positive utilities, because if children receive zero utility it is clearly optimal not to have any children. In order to keep notation simple, I will express the cost of children directly in terms of the consumption good. The cost for a skilled child is p_S, and the cost for an unskilled child is denoted as p_U. In this model, we have $p_S = \phi w + \phi_S w_S + \rho$ and $p_U = \phi w - \phi_U w_U + \rho$. Obviously, this implies that $p_S > p_U$; skilled children are always more expensive than unskilled children.

The analysis leads to two main results. The first one is that the problem of the adult has only corner solutions. That is, adults have either skilled or unskilled children, but there are no adults who have children of both kinds. The second result is that if an adult is just indifferent between skilled and unskilled children, the total expenditure on children is independent of the chosen type of children. If one type of children is more expensive, this will be made up exactly by a lower number of children.

Corner Solutions

I want to analyze the following maximization problem of an adult:

$$\max_{n_S, n_U \geq 0} \left\{ (w - p_S n_S - p_U n_U)^\sigma + \beta (n_S + n_U)^{-\epsilon} [n_S V_S + n_U V_U] \right\}. \tag{30}$$

An alternative way of formulating this problem is to imagine the adults as choosing the total education cost E they spend on raising children, and the fraction f of this cost that they spend on skilled children. The number of children is then given by $n_S = fE/p_S$ and $n_U = (1-f)E/p_U$. This formulation is more convenient to work with, and it is equivalent to the original one. In the new formulation, the maximization problem of the adult is:

$$\max_{0 \leq E \leq w, 0 \leq f \leq 1} \left\{ (w - E)^\sigma + \beta E^{1-\epsilon} (f/p_S + (1-f)/p_U)^{-\epsilon} [f V_S/p_S + (1-f) V_U/p_U] \right\}. \tag{31}$$

We are now in position to show the first main result.

Proposition 3 *Given Assumption 3, for any pair $\{E, f\}$ that attains the maximum in (31) we have either $f = 0$ or $f = 1$.*

Proof: See Appendix. □

Proposition 3 implies that adults have either skilled or unskilled children, but they never mix both types in one family. While the actual proof is a little tedious, the result is intuitive. If we had $\epsilon = 0$ (which is ruled out by Assumption 3), both the utility gained from having children and the cost of children would be linear in the numbers of the two types of children. If we have $V_S/p_S = V_U/p_U$, it has to be the case that the adult is indifferent between unskilled and skilled children, and any combination of the two.

However, if we now have $\epsilon > 0$, as assumed, the term $(f/p_S + (1-f)/p_U)^{-\epsilon}$ in (31) becomes a convex function of f, and the adult will choose a corner solution.

Given the fact that there are only corner solutions, we can determine the optimal number of children by separately computing the optimal choices assuming that there are only unskilled or only skilled children. We can then compare which yields higher utility. Parents who decide to have only children of type i solve:

$$\max_{0 \leq n_i \leq w/p_i} \left\{ [(w - p_i n_i)]^\sigma + \beta (n_i)^{1-\epsilon} V_i \right\}.$$

The first-order condition is:

$$-\sigma p_i (w - p_i n_i)^{\sigma - 1} + \beta (1 - \epsilon)(n_i)^{-\epsilon} V_i \leq 0,$$

and this equation holds with equality if $n_i > 0$. In fact, since the marginal utility of an additional child tends to infinity if the number of children goes to zero, and the marginal utility from consumption tends to infinity if consumption goes to zero, there is an interior solution, characterized by:

$$\beta (1 - \epsilon)(n_i)^{-\epsilon} V_i = \sigma p_i (w - p_i n_i)^{\sigma - 1},$$

or:

$$\beta (1 - \epsilon)(w - p_i n_i)^{1-\sigma} V_i = \sigma p_i (n_i)^\epsilon. \quad (32)$$

We cannot solve for n_i explicitly apart from certain parameter combinations, but we can be sure that there is a unique n_i solving (32): the right-hand side equals zero if $n_i = 0$ and is strictly increasing in n_i, and the left-hand side is strictly decreasing in n_i and equals zero if $n_i = w/p_i$. The second-order condition for a maximum is given by:

$$-\sigma(1-\sigma)p_i^2(w-p_in_i)^{\sigma-2} - \beta\epsilon(1-\epsilon)(n_i)^{-\epsilon-1}V_i < 0, \tag{33}$$

which is satisfied since we assume $0 < \sigma < 1$ and $0 < \epsilon < 1$. The first-order condition is therefore necessary and sufficient for a maximum.

The next question is how the optimal number of children varies with the wage w and the utility of children V_i.

Proposition 4 *The optimal number of children n_i is increasing in V_i and in the wage w.*

Proof: Totally differentiating (32) gives:

$$n_i\ dV_i + (1-\sigma)(w-p_in_i)n_iV_i\ dw = [\epsilon V_i + (1-\sigma)p_in_iV_i]\ dn_i.$$

We therefore have:

$$\frac{dn_i}{dV_i} = \frac{n_i}{\epsilon V_i + (1-\sigma)p_in_iV_i} > 0, \tag{34}$$

and:

$$\frac{dn_i}{dw} = \frac{(1-\sigma)(w-p_in_i)n_iV_i}{\epsilon V_i + (1-\sigma)p_in_iV_i} > 0. \tag{35}$$

□

Thus children are a normal good in this model. On the other hand, if the cost of children p_i is directly proportional to the wage w, as in the case of a pure time cost for children, the optimal number of children decreases with the wage. To see this, assume that the time cost of raising one child of type i is ϕ, so that the price of children is $p_i = \phi w$.

Plugging this into (32) and bringing w to the right-hand side we get:

$$\beta(1-\epsilon)(1-\phi n_i)^{1-\sigma} V_i = \sigma\phi w^{\sigma}(n_i)^{\epsilon}. \tag{36}$$

Totally differentiating yields:

$$\frac{dn_i}{dw} = -\frac{\sigma n_i}{\epsilon w} < 0. \tag{37}$$

Thus if the cost of children is a pure time cost, the substitution effect outweighs the income effect, and the optimal number of children decreases with income.

Another important property of the decision problem of an adult is that if the adult is indifferent between skilled and unskilled children, the total expenditure on children does not depend on the type of the children.

Proposition 5 *An adult is indifferent between skilled and unskilled children if and only if the costs and utilities of children satisfy:*

$$\frac{V_S}{(p_S)^{1-\epsilon}} = \frac{V_U}{(p_U)^{1-\epsilon}}. \tag{38}$$

If an adult is indifferent, the total expenditure on children does not depend on the type of children that is chosen.

Proof: It is helpful to consider the formulation of the problem in which adults choose the total education cost E, so that the number of children equals E/p_i for adults who choose to have children of type i. The maximization problem in this formulation is:

$$\max_{0 \leq E \leq w/p_i} \left\{ (w-E)^{\sigma} + \beta(E/p_i)^{1-\epsilon} V_i \right\}. \tag{39}$$

This can also be written as:

$$\max_{E\geq 0}\left\{(w-E)^{\sigma}+\beta(E)^{1-\epsilon}\frac{V_i}{(p_i)^{1-\epsilon}}\right\}. \tag{40}$$

Since the costs and utilities of children enter only in the last term, clearly an adult is indifferent between skilled and unskilled children if and only if:

$$\frac{V_S}{(p_S)^{1-\epsilon}}=\frac{V_U}{(p_U)^{1-\epsilon}}, \tag{41}$$

Notice that this condition does not depend on the wage of the adult. Also, if condition (41) is satisfied, adults face the same maximization problem regardless whether they decide for unskilled or skilled children. This implies that the optimal total education cost E does not depend on the type of the children. The higher cost of having skilled children will be exactly made up by a lower number of children. □

Implications for Equilibrium Behavior

Propositions 3 and 5 have important implications for intergenerational mobility in the model. Simply put, Propositions 5 states that for given utilities of skilled and unskilled children, the ratio of the prices of skilled and unskilled children determines whether parents send their children to school. As long as the wage for skilled labor is higher than the unskilled wage, skilled children are relatively cheaper for skilled parents, since $w_S>w_U$ implies:

$$\frac{\phi w_S+\phi_S w_S+\rho}{\phi w_S-\phi_U w_U+\rho}<\frac{\phi w_U+\phi_S w_S+\rho}{\phi w_U-\phi_U w_U+\rho}.$$

The term on the left-hand side is the ratio of the prices for skilled and unskilled children for skilled adults, and the right hand side is the ratio for unskilled adults. The opportunity cost of time is higher for skilled adults. Since the opportunity cost of child-rearing makes up a larger fraction of the cost of unskilled children, skilled children are relatively cheaper for skilled parents. The only case when this is not true is when the skilled and unskilled wage is the same. However, in equilibrium the skilled wage is always going to be higher, with the possible exception of the initial period. If both wages were equal in any given period, all adults in the preceding period would have decided to have unskilled children, since they are cheaper to educate. In equilibrium there always have to be some skilled children, so this situation never arises.

Since the relative price of skilled and unskilled children differs for skilled and unskilled parents, it can never be the case that both types of adults are indifferent between the two types of children at the same time. Since skilled children are relatively cheaper for skilled parents, in equilibrium there are always skilled parents who have skilled children. Otherwise, there would be no skilled children at all, which cannot happen in equilibrium. Likewise, there are always unskilled adults with unskilled children.

Taking these facts together, exactly three situations can arise in any given period. The first possibility is that skilled parents prefer skilled children, while unskilled parents prefer unskilled children. In that case, there is no intergenerational mobility. The second possibility is that skilled parents are indifferent between the two types of children, while all unskilled parents have unskilled children. The third option is that all skilled parents have skilled children, while the unskilled adults are indifferent between the two types: some unskilled adults have unskilled children, while others decide to send their children to school. This last case is the typical one along an equilibrium path, as will be explained in more detail later. In this situation, there is upward intergenerational mobility, because some unskilled adults have skilled children, but no downward mobility.

The following corollary sums up the implications of these results for an equilibrium.

Corollary 1 *In equilibrium, for any $x \in X$ such that $w_S(x) > w_U(x)$, the following must be true:*

- *A positive fraction of skilled adults has skilled children, and a positive fraction of unskilled adults has unskilled children:*

$$\lambda_{S\to S}(x), \lambda_{U\to U}(x) > 0.$$

- *Just one type of adult can be indifferent between the two types of children:*

$$\lambda_{S\to U}(x) > 0 \text{ implies } \lambda_{U\to S}(x) = 0,$$

$$\lambda_{U\to S}(x) > 0 \text{ implies } \lambda_{S\to U}(x) = 0.$$

- *Specifically, $\lambda_{S\to U}(x) > 0$ implies:*

$$\left(\frac{\phi w_S(x) + \phi_S w_S(x) + \rho}{\phi w_S(x) - \phi_U w_U(x) + \rho}\right)^{1-\epsilon} = \frac{V_S(g(x))}{V_U(g(x))},$$

and $\lambda_{U\to S}(x) > 0$ implies:

$$\left(\frac{\phi w_U(x) + \phi_S w_S(x) + \rho}{\phi w_U(x) - \phi_U w_U(x) + \rho}\right)^{1-\epsilon} = \frac{V_S(g(x))}{V_U(g(x))}.$$

Proof: Follows directly from Proposition 5. □

5 Outline of the Behavior of the Model

Assuming that the economy starts at a time when productivity in industry is low compared to agriculture, the economy evolves through three different regimes: The Malthusian regime, the transition regime, and the balanced-growth regime. In the Malthusian regime the industrial technology is too inefficient to be used for some time. Therefore the model behaves like one in which there is an agricultural sector only. The economy displays Malthusian features—wages stagnate, and population growth offsets any improvements in productivity. The economy reaches a stable steady state in which wages are constant and population growth just offsets productivity growth in agriculture. If there were sudden technological improvements in technology, per-capita incomes would rise only temporarily, until higher population growth makes up for the higher productivity.

The transition starts when productivity in industry becomes high enough for the industrial technology to be introduced. Since the industrial technology does not exhibit decreasing returns, population growth no longer offsets productivity growth, so that wages start to rise. If productivity growth in industry is sufficiently high, the fraction of output produced in industry will increase, until the agricultural sector ultimately becomes negligible. The economy will then reach a second steady state, the growth regime. Here the model behaves like one in which there is the industrial technology only. Whether population growth and fertility is higher in the growth regime than in the Malthusian regime is determined by the relative importance of skill in the two technologies. If the industrial technology is sufficiently skill-intensive, in the growth regime most children will go to school. Since schooling is costly, this will tend to lower fertility and population growth. On the other hand, as wages grow the physical cost ρ of children becomes less important. This effect makes children relatively cheaper in the growth regime, which will tend to increase fertility. Which effect dominates depends on the specific parameters chosen. During the transition, it is possible that fertility first increases in response to higher wages, but decreases later as the industrial technology starts to dominate and more children go to school.

The transition can also be influenced by public policy. Both an education subsidy and child labor restrictions lower the relative cost of skilled children. Therefore both policies have a positive effect on the number of children going to school. The effects on fertility, however, are different. Since a subsidy lowers the cost of children, an education subsidy tends to increase fertility, even though more children are going to school. Child labor restrictions, on the other hand, increase the cost of children, and therefore lead to lower fertility. Since in this model inequality is linked to the relative cost of skilled and unskilled children, both policies decrease inequality in subsequent generations. I will now analyze the three regimes in more detail.

The Malthusian Regime

It was shown in Section 3 that if productivity in the industrial sector is low, the industrial technology is not used. If productivity is low enough so that the industrial technology will not be used any time in the near future, the economy behaves approximately like one in which the industrial technology does not exist at all. In this regime, the model exhibits Malthusian features. That is, wages are constant, and population growth just offsets productivity growth. Sudden improvements in productivity or sudden decreases in population lead to temporarily higher wages, until higher population growth drives wages back to the steady-state values.

There are two key features of the model that generate the Malthusian steady state. First, it is important that children are a normal good, as shown in Proposition 4. This property ensures that population growth increases once improvements in technology lead to higher wages. Because the agricultural technology exhibits decreasing returns, higher population growth depresses wages and pushes the economy back to the steady state. If the income effect were negative (a common assumption in fertility models that do not consider a quantity-quality tradeoff), higher wages would lead to less population growth, which increases wages even further.

The second key assumption is that there is a goods cost ρ for each child. Without this cost, it would be possible that population growth stays ahead of productivity growth, and wages converge to zero. The goods cost rules out this possibility, because at some point wages will fall to a level were the current population level can just be maintained.

Apart from the wages, the ratio of the number of skilled and unskilled adults is also constant in the Malthusian steady state.[2] If the wage of skilled adults temporarily increases for an exogenous reason, more unskilled adults will decide to have skilled children, until the ratio of skilled to unskilled adults reaches its steady-state value again.

Without making specific assumptions on parameters, it is not clear whether fertility will be higher for skilled or for unskilled adults in the Malthusian steady state. On the one hand, children are a normal good. If the only costs for children were the goods cost and the schooling cost (i.e., $\phi = \phi_U = 0$, $\phi_S, \rho > 0$), the absolute price of skilled and unskilled children would be the same for both types of parents. In that case the adults with the higher wage (the skilled adults) would have more children. On the other hand, it was shown above that if the cost of children is a pure time cost (i.e., $\phi_S = \rho = \phi_U = 0$, $\phi > 0$), higher wages cause lower fertility. This effect favors lower fertility for skilled adults. Finally, in equilibrium a larger fraction of skilled adults has skilled children. Since skilled children are more expensive, this effect tends to lower the fertility of skilled adults as a group. Which of these effects dominates depends on assumptions on parameters. If the parameters are chosen in a way that is consistent with a demographic transition, the last effect tends to be the most important one. I will therefore concentrate on the case in which skilled adults have less children on average in the steady state.

In the steady state, the ratio of skilled to unskilled adults is constant. Since at the same time average fertility in steady state is higher for unskilled parents, it has to be the case that a fraction of unskilled adults has skilled children. Otherwise, the fraction of

[2]This is not true for all parameter combinations, however. If the schooling cost is very high, it is possible that the ratio of skilled to unskilled adults converges to zero, and the wage for skilled adults converges to infinity. In the numerical illustrations below I restrict attention to the case where a steady-state exists.

skilled adults would decrease over time. Therefore unskilled adults have to be indifferent between skilled and unskilled children in the steady state. Using Corollary 1, steady state wages $\bar{w}_S$ and $\bar{w}_U$ and utilities $\bar{V}_S$ and $\bar{V}_U$ then have to satisfy the following condition:

$$\left(\frac{\phi\bar{w}_U + \phi_S\bar{w}_S + \rho}{\phi\bar{w}_U - \phi_U\bar{w}_U + \rho}\right)^{1-\epsilon} = \frac{\bar{V}_S}{\bar{V}_U}. \tag{42}$$

This equation determines the wage differential, i.e., the degree of inequality in the Malthusian steady state. Of course, without other equilibrium conditions we cannot solve for the steady-state wages directly, but (42) will be useful for comparing the degree of inequality in the Malthusian steady state and in the growth steady state.

Within the Malthusian regime, it is straightforward to compute an equilibrium numerically via value function iteration (some details are given in the Appendix). Steady-state values can either be inferred from the computed solution, or derived directly from steady-state conditions. No closed-form solutions are available, however. Since my main interest is in the transition from the Malthusian regime to the growth regime, I defer a discussion of numerical results until the next section.

The Transition

In the Malthusian Regime, wages depend on preference parameters and the growth rate of agricultural productivity, but are independent of the level of productivity. At some point, productivity in industry will reach a level at which industrial production is profitable at the wages that prevail in the Malthusian steady state. From that time on the industrial technology will be used alongside the agricultural technology. Since population growth does not depress wages in industry, wages and per capita incomes start to grow.

The evolution of fertility, inequality, and intergenerational mobility depends on the specific properties of the industrial production function. I assume that production in industry is more skill intensive than production in agriculture:

Assumption 4 *The industrial sector is more skill-intensive, i.e., the production function parameters satisfy* $\alpha < \theta_U$*, which implies* $1 - \alpha > \theta_S$.

Under this assumption, the introduction of the industrial sector increases the wage premium for skilled labor. This will increase the returns to education, so that more unskilled adults will choose to have skilled children. Because there are more skilled children, more skilled adults work as teachers. This in turn leaves less skilled adults to work in industry and in agriculture, which causes an additional upward effect on the skilled wage. However, increases in the skilled wage also increase the cost of education. This will make up for the higher utility of skilled children, and unskilled parents will be indifferent between skilled and unskilled children once again. The overall effect is an increase in skilled wages, and increased upward mobility in the sense that more unskilled adults send their children to school.

Government policy can have large effects during the transition. Education subsidies induce more unskilled adults to have skilled children. The wage premium will still rise initially, because many skilled adults are needed as teachers. Child labor restrictions lower the relative cost of education and therefore also lead to increased mobility. The short-run effects on the wage premium are ambiguous. On the one hand, higher demand for skilled children requires more teachers, which increases the skill premium. On the other hand, with child labor restrictions unskilled labor supply falls. This has a positive effect on the unskilled wage, and therefore decreases the premium for skilled labor.

The Growth Regime

As the economy continues to grow, the fraction of the population working in the agricultural sector falls, and under a wide range of conditions ultimately approaches zero[3].

[3] If productivity growth is significantly higher in the agricultural sector than in industry or if population

In the limit, the economy behaves like one in which there is no agricultural sector at all. Also, as income grows, the goods cost ρ for children will become negligible.

The limit economy without agriculture can be described with just two state variables. Since the industrial production function exhibits constant returns, wages are determined by the ratio of skilled to unskilled labor supply. The only state variables are therefore the ratio of the number of skilled to unskilled adults, and the productivity level in industry. The setup can further be simplified by noting that the period utility function is of the constant-elasticity form, and that wages are linear in the productivity level. This results in equilibrium value functions that are homogeneous in the industrial productivity level:

$$V_i(A_I, N_S/N_U) = A_I^\sigma V_i(N_S/N_U).$$

This reduces the growth regime essentially to a one-dimensional system, with the ratio of skilled to unskilled adults as the state variable.

Before I turn to the properties of the growth regime, one further technical assumption is needed. The assumption is that the effective discount factor is not higher than one, i.e., adults cannot place higher weight on the utility of their children than on their own utility. Since the discount factor depends on the number of children, we have to consider the highest number of children possible, which is reached by an unskilled adult who spends all income in children. The resulting number of children is $1/(\phi - \phi_U)$.

Assumption 5 *The parameters β, ϵ, σ, ϕ, ϕ_U, and γ_I satisfy:*

$$\beta\gamma^\sigma \left(\frac{1}{\phi - \phi_U}\right)^{1-\epsilon} < 1.$$

growth turns negative, there can be situations where the fraction of output produced in the agricultural sector does not tend to zero. I concentrate on the case in which agriculture disappears. Judging from the experience of industrial countries, this seems to be the empirically interesting case.

Given this assumption, the problem of adults is always well defined.

In the growth regime the economy can exhibit two types of behavior. In the first case, the one that I am going to concentrate on, there is a globally stable steady state. That is, the ratio of skilled to unskilled adults reaches a fixed number, population growth and fertility are constant, and wages and consumption grow at the rate of technical progress. In the steady state, average fertility is lower for skilled than for unskilled adults. This would be true even if the schooling cost were zero and if there were no child labor, since then the only remaining cost of children would be a time cost. It was shown in Section 4 that if there is a pure time cost for having children, wages and fertility are negatively related. With positive schooling cost and child labor the relative cost of skilled children increases, and since relatively more skilled adults have skilled children, this will further increase the fertility differential between the two types of adults.

Given that fertility is higher for unskilled adults in the steady state, it has to be the case that some unskilled adults have skilled children. As in the Malthusian steady state, this is necessary because otherwise the fraction of unskilled adults would increase over time. Therefore unskilled adults are just indifferent between the two types of children. Steady state wages $\bar{w}_S$ and $\bar{w}_U$ and utilities $\bar{V}_S$ and $\bar{V}_U$ then have to satisfy the following condition:

$$\left(\frac{\phi\bar{w}_U + \phi_S\bar{w}_S}{\phi\bar{w}_U - \phi_U\bar{w}_U}\right)^{1-\epsilon} = \frac{\bar{V}_S}{\bar{V}_U}. \tag{43}$$

The difference to the parallel condition (42) for the Malthusian regime is that the goods cost ρ for children is negligible in the growth regime. Comparing (42) and (43), we can infer that skilled children are relatively more expensive in the growth steady state, thus inequality will be higher than in the Malthusian steady state.

The second possible behavior in the growth regime arises when the schooling cost, i.e., the parameter ϕ_S, is very high. In that case it is possible that the ratio of skilled to unskilled adults converges to zero, and the ratio of the skilled to the unskilled wage tends to infinity. The increasing wage premium makes education very expensive from the perspective of unskilled adults, and if ϕ_S is sufficiently high, this more than outweighs the high utility of potential skilled children. Since in real-world countries the number of skilled adults generally does not converge to zero, I consider this case degenerate, and concentrate on parameterizations for which a growth steady state exists.

Education and child labor policies influence the relative cost of the two types of children much in the same way as described in the discussion of the transition period. Both education subsidies and child labor restrictions lower the relative cost of educating children. This leads to a higher fraction of skilled adults in the growth steady state, lower inequality, and a lower fertility differential between skilled and unskilled adults.

6 Recent Transitions: Comparing Brazil and Korea

In this section I use the model to analyze differences in the demographic transition and the evolution of inequality in Brazil and Korea. Brazil and Korea make an interesting comparison because they are similar in aspects that the model takes as exogenous, but strikingly different along the dimensions the model is designed to explain. Both countries entered the demographic transition about at the same time, and the fertility levels were about the same before the decline started. Also, both countries achieved high growth rates in per capita income in the period following the fertility decline. The major differences are that in Korea the demographic transition was completed (i.e., fertility fell below replacement level) within 25 years. Over the entire transition period, inequality was low. In Brazil the fertility decline was slower, with fertility being still above the replacement level today. Also, Brazil started out with high inequality, and, if anything, inequality increased during the transition.

Facts on Fertility, Education, Inequality, and Child Labor

The main facts that I am trying to explain are displayed in Figures 2 and 3, which show the total fertility rate and the Gini coefficient for Brazil and Korea from 1950 until today (the fertility rates are also displayed in Table 2). Both countries had high fertility until about 1960, when fertility started to fall. In Korea, however, the fertility decline was fast, and replacement fertility was reached within 25 years. In Brazil fertility declined more slowly. If we look at subgroups of the population, it turns out that fertility differentials by education level are much higher in Brazil. In 1986, Brazilian women without formal education had a total fertility rate of 6.7, while for women with seven or more years of education the number is 2.4. Thus the fertility differential amounts to more than four children. In 1974, total fertility for Korean women without formal education was 5.7, while for women with seven or more years of education the rate is 3.4. This gives a fertility differential of 2.3, roughly half of the differential in Brazil.[4]

In both countries the Gini coefficient stays roughly constant. In Brazil the Gini is above .5 throughout, while in Korea the value is closer to .3. Table 3 displays income shares by quintile for Brazil and Korea in 1970 and 1987/88. These data allow a more detailed comparison of the income distribution in Brazil and Korea. I find it especially interesting that the share of income that goes to the poorest 40% of the Brazilians is only slightly higher than the share of the poorest 20% in Korea. This implies drastic differences in living standard between poor Brazilians and poor Koreans. There are also large differences at the top of the income distribution: In 1970, the income share of the top quintile was 61.7% in Brazil, and only 41.6% in Korea.

[4]Unfortunately, there is no data for Brazil and Korea for the same year. However, given the overall fall in fertility, it seems likely that the fertility differential by education in Korea was even lower in 1986 than in 1974.

The differences in fertility and inequality between Brazil and Korea after 1960 are especially interesting because both countries had a similar growth experience, at least initially. As Figure 1 shows, real GDP per capital grew rapidly in both countries in the 1960s and 1970s. Per capita income was initially higher in Brazil, and Brazil maintained the lead until the early 1980s. At that point, the similarity breaks down; growth slowed considerably in Brazil and remained high in Korea. Today, GDP per capita in Brazil is only about half of GDP per capita in Korea.

I will use the model to explore how much of the differences described above can be explained by different policies in the areas of education and child labor. In the following description of the policies in Korea and Brazil I will rely mainly on empirical data, as opposed to legal regulations. The laws pertaining to education and child labor are similar Korea and Brazil; the main differences lie in implementation, financing, and enforcement. For example, according to the law, primary education was both compulsory and free in Brazil since 1930. Yet for many years primary schooling was simply not available in many rural areas, and the available schools were often of poor quality.

Table 4 shows average years of schooling for the adult population in Brazil and Korea from 1960 to 1985. In both countries average years of schooling increase over time, but the increase is much faster in Korea. In 1960, the difference in average years of schooling between Korea and Brazil was less than a year. By 1985, the difference increased to more than four years, so that Koreans on average have more than twice as much schooling as Brazilians. Table 5 shows adjusted enrollment ratios for primary and secondary education from 1960 to 1990. While primary enrollment rates are similar, Korea has a large and growing advantage in secondary education. The differences in enrollment rates understate the differences in education between Korea and Brazil, because they do not account for the quality of education. As a crude measure of educational success, in Table 6 I display adult illiteracy rates. In 1960, 39% of adult Brazilians were illiterate, compared to

29% of adult Koreans. In 1995, in Brazil illiteracy is still at 16.7%, while Koreans are almost completely literate with an illiteracy rate of only 2%. In Brazil illiteracy is high even among groups who went to school only recently. In 1991, illiteracy was 12.1% for the age group from 15 to 19 years. Thus even though the primary enrollment ratio is 100% in Brazil and Korea, the quality of education is clearly lower in Brazil.

In my model, the alternative to education is child labor. The data for Brazil and Korea indicate that the same tradeoff is present in the real world. In Korea, there was already a low incidence of child labor at the beginning of the demographic transition. According to the ILO, in 1960 1.1% of the children from zero to fifteen years were economically active, which compares to 4.3% in Brazil. With the enforcement of compulsory education, child labor virtually disappeared in Korea. In 1985 .3% of the children between ages ten and fourteen participated in the labor market. In Brazil the number is 18.7% for 1987. Child labor is even more prevalent among male children, with 25.3% listed as economically active in 1987, and still 24.3% in 1990.

The facts paint a clear picture. Korea provided better access to education to its citizens, and the quality of education is superior to education in Brazil. While both countries have child labor regulations and compulsory schooling laws, the enforcement of these rules is evidently stronger in Korea. In the next section, I simulate my model under two different assumptions on government policies. I model the Korean policy as an education subsidy combined with the abolition of child labor. In the stylized Brazilian policy, parents pay for education, and child labor is unrestricted.

Simulations

I use a parameterized version of the model to investigate whether the observed differences in education policy and child labor restrictions can explain the demographic transition and the evolution of the income distribution in Brazil and Korea. The model parameters are chosen in order to be consistent with a demographic transition under both policy regimes. However, the parameters are not calibrated to match specific observations precisely. In the future, I plan to calibrate the model so that it matches steady state observations both before and after the transition.

The model parameters used for the simulations are listed in Table 1. The initial values for agricultural productivity and the number of skilled and unskilled adults are chosen to start the economy in the Malthusian steady state. The productivity in industry is chosen so that the transition to industry starts about four periods after the start of the simulation. Apart from different education and child labor policies, the simulations for Brazil and Korea are identical. In the Brazilian simulation, parents have to pay for the education of their children, and child labor is unrestricted. The Korean simulation starts with the same policy. In the third model period (1950), synchronized with the start of industrialization, the policy changes: The government pays for 50% of the schooling cost, and child labor is abolished.

Figures 4 and 5 show the simulated paths for the total fertility rate and the Gini coefficient. The time axis was labeled so that each model period corresponds to 25 years, and the start of the transition was placed in the year 1950. Figure 4 shows the simulation results for Brazil. The corresponding real-world data is contained in Figure 2. In the simulation, the fertility rate declines slowly after 1950 and is still above replacement at the end of the simulation (the year 2050). The decline in fertility occurs because the industrial technology is skill-intensive. As the economy grows, more parents decide to have skilled children, and since skilled children are expensive, fertility falls. The main difference between the simulation and the data is that fertility start out at a higher level in the data.

I will now turn to the income distribution. The Gini coefficient starts at about .15 in the model, and then increases to about .5 by the year 2000. In the model, inequality is determined by the relative cost of skilled and unskilled children. As the economy grows, the goods cost ρ becomes irrelevant. This increases the relative cost of education, and therefore leads to more inequality. In the real-world data, the Gini ends up in a similar area, but starts already at a high level and is roughly constant over time. Thus the model explains why inequality is high today, but it does not explain why inequality was already high in 1950. My interpretation is that Brazilian inequality in 1950 stemmed mainly from other sources than the premium for skilled workers, but that by today educational inequality is the major determinant of the income distribution in Brazil.

Figure 5 shows the simulation results for Korea, while the real-world data is contained in Figure 3. As in the data, fertility falls sharply at the beginning of industrialization, ending up around replacement level. Notice that the time scale is different on the two pictures; the real-world figure shows only a subset of the simulation period. Fertility falls sharply because the policy change occurs at the same time when the skill-intensive industrial technology is introduced. Many unskilled adults who otherwise would have had unskilled children now decide to send their children to school. Since even with the education subsidy skilled children are more expensive than unskilled children, fertility falls rapidly.

Inequality is low both in the data and in the simulation. In the simulation, the bump in the Gini graph is due to the fact that many skilled adults work as teachers early in the transition, which tends to increase the wage premium. Still, relative to Brazil, inequality stays low throughout.

Figures 6 and 7 show simulation results for some additional variables. The graphs in the upper left-hand corner show the fraction of skilled and unskilled adults working in the industrial sector. The pattern of industrialization looks very similar for the two countries. In the lower left hand corner the number of skilled and unskilled adults is displayed. In Korea, skilled adults outnumber the unskilled before the year 2000, while in Brazil the majority stays unskilled. The reason for this, of course, is that the relative price of skilled children is much lower in Korea. The graph in the lower right-hand corner shows fertility by skill level. In Brazil the fertility differential is much higher, as is observed in the data.

Especially given the fact that parameters were not specifically chosen to closely match observations, the model shows a remarkable ability to reproduce the stylized features of the demographic transition and the evolution of inequality in Brazil and Korea. In the next section I explore whether the model also succeeds in the country where industrialization once started.

7 A Nineteenth-Century Transition: England

England is an especially interesting case for two reasons. As the country that once started the industrial revolution, England is a natural testing ground for any theory of transition. In addition, England is one of the countries where inequality followed the inverted-U-shape pattern that was described by Simon Kuznets, whereas both in Brazil and Korea the income distribution changed little during transition. Therefore England is a good example to test whether the model can generate a Kuznets curve.

Facts on Fertility, Education, Inequality, and Child Labor

Table 7 and Figure 9 show total fertility rates for England from 1700 until 1992. Even though industrialization started in the late eighteenth century and per capita income was growing throughout the nineteenth century, fertility stays relatively high until 1880. Then fertility starts to fall rapidly around the turn of the century, and reaches replacement level before 1940. There is only sparse information on fertility differentials during the transition (Haines 1979, 1989), but it seems likely that fertility differentials by education were higher during the transition than today. In 1974, the difference in the total fertility rate between women with higher education and women with incomplete primary education or less was only about .4 (Jones 1982).

Even though there is a lack of high-quality data on the income distribution in the nineteenth century, the available evidence shows that inequality increased during the first half of the nineteenth century, and then decreased in the late nineteenth and early twentieth centuries. Williamson (1991) reports that income shares at the top of the income distribution rose from about 1760, the premium on skilled labor increased, and the income distribution widened. Inequality peaked around 1860. Afterwards, income shares at the top fell, as did the wage premium, the relative position of the unskilled workers improved, and the income distribution narrowed. Using tax assessment data, Williamson (1985) constructs a series of Gini coefficients for the income distribution in England. Figure 9 displays Gini coefficients from 1820. Even though there might be substantial measurement error, the initial increase and later decrease in inequality, the "Kuznets curve," is clearly visible.

At the beginning of the industrial revolution, education was not widespread in England. In 1780, only about 50% of brides and grooms were able to sign their name. Educational expansion proceeded slowly in the early nineteenth century. While overall enrollment rates increased, much of the increase was initially accounted for by Sunday schools, which were less effective than day schools in providing education. Mitch (1999) reports that in 1818 most students were enrolled in private, for-profit schools, many of which were of questionable educational value. Public education also started to expand in the nineteenth century, but there were large regional differences and no universal access to affordable education. The situation changed drastically with the Victorian education reforms, starting in the 1870s. The Forster Education Act of 1870 placed primary education under public control. School boards were established, and many new schools were built as a consequence of the Forster Act. In 1880 compulsory schooling was introduced, and starting in 1891 primary education was free. While schooling quality is hard to measure, literacy data suggests that the reforms were successful. While in 1880 about 15% of grooms and 20% of brides were unable to sign their name, these numbers decreased to less than 2% until 1910.

Child labor restrictions were expanded on a number of occasions during the nineteenth century. The first restrictions were put in place with the Factory Act of 1833. Only a small set of industries was affected, however, and Nardinelli (1980) concludes that the impact on child labor was small. The Factory Acts were amended in 1844 and 1874, when the minimum age for child laborers was raised to 10. At that time the restrictions became universal, instead of being limited to certain industries. Compulsory schooling laws also had an effect on child labor. As a result, the incidence of child labor was decreasing late in the nineteenth century. Activity rates for children aged from ten to fourteen reached a peak 1861, when 29% of all children in that age group were economically active. In 1871, the number was still at 26%, but then it fell to about 20% in 1881 and 1891, 17% in 1901, and 14% in 1911.

Simulation

To reproduce the stylized facts of the English transition, I use the same model parameters that were applied for Brazil and Korea. The policy change is the same that was used for the Korean simulation, but it now occurs four model periods, or 100 years, after the start of industrialization. In other words, in the simulation England follows the Brazilian model until late in the nineteenth century, and then switches policies to a Korean regime. Figures 8 and 10 show the results, while Figure 9 shows some corresponding real-world data. The time axis is labeled to place the start of industrialization in 1775 and the policy change in 1875. In the simulation, fertility falls slowly throughout the nineteenth century, and then drops in the year of the policy change. This matches the overall pattern in the data. The main discrepancy is that the drop in fertility is even more pronounced in the simulation than in the data. Also, in the real world fertility increased initially before it started to fall.

The Gini coefficient behaves according to the Kuznets hypothesis: Inequality increases from the beginning of industrialization, and then starts decreasing late in the nineteenth century. The similarity between the simulation and the data is striking. As in the Korean simulation, fertility differentials between skilled and unskilled adults are low after the policy change, which matches the contemporary observations from Britain.

The overall results show that the model is capable of reproducing the Kuznets curve in England and the associated changes in fertility behavior. Even though I concentrated on the transition, it should be pointed out that the model is also consistent with the behavior of the English economy before and after the nineteenth century. Both in England and in the model wages stagnate before industrialization, and grow at roughly constant rates after the industrialization.

8 Conclusions

In this paper I develop a unified theory of fertility, inequality, and growth. The model is consistent with a phase of stagnation during which the economy exhibits Malthusian features, followed by a transition to a balanced-growth regime. A special emphasis is placed on the role of education and child labor policies. Previous models that are capable of generating a demographic transition have generally not considered the role of policy variables. My results suggest that accounting for policy changes is important for understanding the experience of different countries during the demographic transition. Another novelty is that the model allows an analysis of fertility differentials by income and education within a country. The predictions of the model match empirical observations in this dimension as well.

I use the model to explain the different transition experiences of Brazil and Korea. While Korea had a fast demographic transition and consistently low inequality, the demographic transition was slow in Brazil, and the income distribution was unequal. I link these differences to policies in the areas of education and child labor. Brazil has a weak education system, and child labor restrictions are not strictly enforced. In contrast, Korea provides excellent education to its citizens, and child labor restrictions are enforced. Numerical experiments show that these policies can explain a major part of the observed differences between Brazil and Korea. In a further empirical application, I show that the model can reproduce the observed patterns in inequality and fertility in nineteenth-century England, once changes in educational and child labor policies are accounted for.

As a first step in future research, I plan to improve the empirical part by calibrating the model parameters to match steady state observations in actual economies before and after the demographic transition. This will allow a more precise comparison of data and model outcomes during the transition. On the theoretical side, I plan to work on a sharper characterization of the steady states. Also, at least for the Malthusian and the growth regimes it should be possible to prove some theorems concerning the existence and uniqueness of equilibria. In the current model, I abstract from capital accumulation. Even though I do not expect that the main conclusions will change, incorporating capital accumulation adds an important dimension of reality, and would allow to analyze the wealth distribution alongside the income distribution. On the other hand, adding capital accumulation poses some difficult technical problems, and it is not clear whether an analysis will be technically feasible.

Instead of adding complications, another possibility is to simplify the model. Specifically, the model is relatively easy to analyze when there is the industrial technology only.

I plan to use this simpler model to analyze a wider range of policies aimed at population growth and the income distribution.

A Mathematical Appendix

Nonexistence of Interior Solutions for the Adults' Problem

We consider the following maximization problem:

$$\max_{E\geq 0, 0\leq f\leq 0} \left\{(w-E)^\sigma + \beta E^{1-\epsilon}(f/p_S + (1-f)/p_U)^{-\epsilon}[fV_S/p_S + (1-f)V_U/p_U]\right\}.$$

By Assumption 3, the parameters β, σ and ϵ are all strictly bigger than zero and strictly smaller than one. It is also assumed that the inequality $0 < p_U < p_S$ holds, and that we have $V_S, V_U \geq 0$. These assumptions could be relaxed, at the price of complicating the proof.

Proposition 1 *There are no interior solutions in f, i.e., if there is a solution to the maximization problem above, the optimal f is either zero or one.*

Proof: To show that there are no interior solutions, assume that we have already determined the optimal E. Given this E and the fact that the function to be maximized is twice continuously differentiable in f, if there were an interior solution, the optimal f would have to satisfy first- and second-order conditions for a maximum. I solve for the unique f which solves the first-order condition, and show that this f does not satisfy the second-order condition. This proves that there are only corner solutions.

I will name the maximand $U(\cdot)$. The first and second derivatives of U with respect to f are:

$$\begin{aligned}\frac{\partial U}{\partial f} &= \beta E^{1-\epsilon}\left[-\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)\right.\\ &\quad\left.+\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\right],\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 U}{\partial f^2} &= \beta E^{1-\epsilon}\left[\epsilon(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)^2\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-2}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)\right] \\ &\quad -2\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-1}\left(\frac{V_U}{p_U}-\frac{V_S}{p_S}\right)\Bigg] \\ &= \beta E^{1-\epsilon}\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-1} \\ &\quad \left[(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)-2\left(\frac{V_U}{p_U}-\frac{V_S}{p_S}\right)\right].\end{aligned}$$

In the first derivative, for $0 \leq f \leq 1$, the first term within the outer brackets is positive. For an interior solution to be possible, it has to be the case that $V_S/p_S < V_U/p_U$, because otherwise the second term is also positive and the first-order condition cannot be satisfied. Therefore if $V_S/p_S \geq V_U/p_U$, we are done. For the case that $V_S/p_S < V_U/p_U$, I set the first derivative equal to zero, and solve for f.

$$\begin{aligned}0 &= \beta E^{1-\epsilon}\left[-\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)\right. \\ &\quad \left.+\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-\epsilon}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\right],\end{aligned}$$

$$0 = -\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)+\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right),$$

$$\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right) = \epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right),$$

$$\left(\frac{f}{p_S}-\frac{f}{p_U}+\frac{1}{p_U}\right)\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right) = \epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{fV_S}{p_S}-\frac{fV_U}{p_U}+\frac{V_U}{p_U}\right),$$

$$(1-\epsilon)\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)f=\epsilon\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\frac{V_U}{p_U}-\frac{1}{p_U}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right),$$

$$f=\frac{\epsilon\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)-\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)}{(1-\epsilon)p_U\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)}.$$

I will now plug this value for f into the second derivative to verify that the second derivative is positive, so that the critical point is not a maximum. The second derivative is positive if the following inequality holds:

$$(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right)^{-1}\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)-2\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)<0,$$

or:

$$(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{fV_S}{p_S}+\frac{(1-f)V_U}{p_U}\right)<2\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\left(\frac{f}{p_S}+\frac{1-f}{p_U}\right),$$

$$(1+\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left[\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)f+\frac{V_U}{p_U}\right]<2\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\left[\left(\frac{1}{p_S}-\frac{1}{p_U}\right)f+\frac{1}{p_U}\right],$$

$$(1+\epsilon)\frac{1}{p_U}\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)<(1-\epsilon)\left(\frac{1}{p_S}-\frac{1}{p_U}\right)\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)f+2\frac{1}{p_U}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right).$$

Plugging in our value for f yields:

$$(1+\epsilon)\frac{1}{p_U}\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)<\frac{1}{p_U}\left[\epsilon\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)-\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right)\right]+2\frac{1}{p_U}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right),$$

$$\frac{1}{p_U}\left(\frac{V_U}{p_S}-\frac{V_U}{p_U}\right)<\frac{1}{p_U}\left(\frac{V_S}{p_S}-\frac{V_U}{p_U}\right),$$

$$V_U < V_S,$$

Thus if $V_S > V_U$ we are done. But if V_S were smaller than V_U, there would be only unskilled children for sure, since they are cheaper to educate. Therefore there are only corner solutions, families have either unskilled or skilled children, but they do not mix. □

B Notes on Computation

The Malthusian Regime and the Growth Regime

Within the Malthusian regime and the growth regime, the model can be computed via standard value function iteration on a discretized state space. The initial guess for the value functions is computed by setting the number of children to zero, so that utility stems from consumption only. During the iterations, for a given state x, the algorithm finds a state x' in the next period such that the resulting fertility decisions of adults are consistent with state x'. After the iterations converge, the equilibrium law of motion can be used to compute steady state values.

The Transition

In principle it is possible to compute the entire model, encompassing the Malthusian regime, the transition, and the growth regime, with the same method described above. However, since the state vector is four-dimensional, computations would either take very long or would be imprecise. Therefore I use an algorithm that directly computes the equilibrium path over T periods from any starting value x_0 for the state vector.

In my computations I concentrate on transitions that start in the Malthusian steady state, where all skilled adults have skilled children. Since during the transition the returns to being skilled increase, it will be the case that during the entire transition skilled adults continue to prefer skilled children. We therefore have $\lambda_{S\rightarrow S\ t} = 1$ and $\lambda_{S\rightarrow U\ t} = 0$ for all t. Also, $\lambda_{U\rightarrow U\ t}$ is given by $\lambda_{U\rightarrow U\ t} = 1 - \lambda_{U\rightarrow S\ t}$ in all periods. Therefore it is sufficient to solve for the following equilibrium sequences: $\{x_t\}_{t=0}^{T}$, $\{V_{St}, V_{Ut}\}_{t=0}^{T}$, $\{L_{St}, L_{Ut}\}_{t=0}^{T}$, $\{w_{St}, w_{Ut}\}_{t=0}^{T}$, $\{\lambda_{U\rightarrow S\ t}\}_{t=0}^{T}$, and $\{n_{St}(S), n_{St}(U), n_{Ut}(U)\}_{t=0}^{T}$. Notice that the state vector consists of productivity and population values, $x_t = \{A_{At}, A_{It}, N_{St}, N_{Ut}\}$.

I will use superscripts to denote iterations. The algorithm starts with an initial guess

$\{x_t^0\}_{t=0}^T$, $\{V_{St}^0, V_{Ut}^0\}_{t=0}^{T+1}$, $\{\lambda_{U\to S\,t}^0\}_{t=0}^T$, and $\{n_{St}^0(S), n_{St}^0(U), n_{Ut}^0(U)\}_{t=0}^T$. The productivity values in $\{x_t^0\}_{t=0}^T$ are already chosen to satisfy $A_{F\,t+1} = \gamma_F A_{F\,t}$ and $A_{I\,t+1} = \gamma_I A_{I\,t}$. We need to guess utility values up to period $T+1$, because they are needed for decisions of adults in period T. The utilities in $T+1$ are updated using the relationship $V_{i\,t+1} = \gamma_I^\sigma V_{i\,t}$ which holds in the growth steady state. Therefore T has to be chosen large enough for the economy to be close to the growth steady state after T periods. Whether this is the case can be checked by computing the growth steady state as described above before computing the transition.

The algorithm proceeds in "nested iterations." At first the sequence $\{\lambda_{U\to S\,t}^0\}_{t=0}^T$ will be held constant, until the other sequences converge. The sequences are updated by computing the optimal decisions of adults, given wages and the utilities of children. Given initial guesses, the decisions of adults lead to new sequences $\{\tilde{x}_t\}_{t=0}^T$, $\{\tilde{V}_{St}, \tilde{V}_{Ut}\}_{t=0}^{T+1}$, and $\{\tilde{n}_{St}(S), \tilde{n}_{St}(U), \tilde{n}_{Ut}(U)\}_{t=0}^T$. Instead of using these sequences directly, I take a linear combination of $\{x_t^0\}_{t=0}^T$, $\{V_{St}^0, V_{Ut}^0\}_{t=0}^{T+1}$, $\{n_{St}^0(S), n_{St}^0(U), n_{Ut}^0(U)\}_{t=0}^T$ and $\{\tilde{x}_t\}_{t=0}^T$, $\{\tilde{V}_{St}, \tilde{V}_{Ut}\}_{t=0}^{T+1}$, $\{\tilde{n}_{St}(S), \tilde{n}_{St}(U), \tilde{n}_{Ut}(U)\}_{t=0}^T$ in order to prevent cycling. Then $\{\lambda_{U\to S\,t}\}_{t=0}^T$ is updated by comparing the utilities of unskilled adults with skilled and unskilled children: If utility from having skilled children is higher in period t, $\lambda_{U\to S\,t}$ is increased, and it is decreased if adults prefer unskilled children. Then the new $\{\lambda_{U\to S\,t}\}_{t=0}^T$ sequence is held constant until the other sequences converge again. Overall convergence is reached once unskilled adults are just indifferent between the two types of children along the equilibrium path. This algorithm is not guaranteed to converge, but it works well in practice.

C Data Sources

Brazil and Korea

Real GDP per capita is from Penn World Tables, Mark 5.6 (see Heston and Summers 1991). Total fertility rates[5] (Figures 2 and 3, Table 2) are from various editions of the World Bank World Tables and Chesnais (1992), Tables A2.6 and A2.7. Inequality data (Figures 2 and 3) is from Deininger and Squire (1996). For both Brazil and Korea, Gini coefficients and income shares are computed using household gross income data (before taxes) from national samples. Adjusted enrollment rates (Table 5) are from Barro and Lee (1993) and World Tables. Illiteracy rates and public expenditures on schooling are from World Tables and various editions of the UNESCO Statistical Yearbook. Average years of schooling (Table 4) are from Barro and Lee (1993). Data on child labor is from various editions of the ILO Year Book of Labor Statistics. Data on differential fertility from Mboup and Saha (1998), Muhuri et al. (1994), United Nations (1987), and United Nations (1995).

England

Real GDP per capita is from Maddison (1992), Table A.2 (real GDP) and Tables B.1–B.4 (population). Total fertility rates (Figure 8) are from Chesnais (1992), Table A2.1, for 1860 until present, and from Lee and Schofield (1981) for the period 1700 to 1840. The numbers by Lee and Schofield are based on family reconstitution data described in Wrigley et al. (1997). Inequality data after 1960 is from Deininger and Squire (1996). Gini coefficients and income shares are computed using personal net income data (after taxes) from national samples. Gini coefficients from 1823–1915 are estimates by Williamson (1985, Table 4.2), based on inhabited house duty tax assessment data. Average years of schooling

[5]Total fertility rates are defined as the sum of age specific fertility rates in a given year. That is, if n_t^a is the average number of children born to women of age a in year t, the total fertility rate in year t is given by $TFR_t = \sum_{a=15}^{5} 0 n_t^a$. The total fertility rate can be interpreted as the total number of children a woman can expect to have over her life time if age-specific fertility rates remain at the same level.

by birth cohort (Table 8) is from Matthews et al. (1982), Table E.1. Differential fertility data is from Jones (1982). Literacy data is from Mitch (1992). Child labor data is from Nardinelli (1990).

D Model Parameters

All simulations are based on the same parameter values. The only difference is the timing of the change in education subsidies and child labor restrictions. For the Brazil, there is no change at all. In Korea, the change occurs in the third period of the simulation (interpreted as 1950), and for England the change occurs in the eighth period (interpreted as 1875). In Korea the change therefore coincides with the beginning of industrialization, while in Britain there is a lag of about 100 years.

Parameter	Value
θ_S	.1
θ_U	.6
α	.1
γ_F	1.03
γ_I	1.2
σ	.5
β	.5
ϵ	.5
ϕ	.48
ϕ_S	.1
ϕ_U	.18
ρ	.1
δ	.5
A_{F0}	4
A_{I0}	4
N_{S0}	.03
N_{U0}	.28

Table 1: Parameter Values for Simulations

E Data for Brazil and Korea

Year	Brazil	Korea
1950-55	6.2	5.2
1960-65	6.2	6.0
1970	5.8	4.2
1977	4.2	2.8
1987	3.1	1.7
1992	2.8	1.7

Source: Chesnais (1992), World Tables.

Table 2: Total Fertility Rates 1950–1992

	1970		1987 / 88	
Quintile	Brazil	Korea	Brazil	Korea
1	3.2	7.3	2.7	7.4
2	9.5	19.6	8.4	19.6
3	19.3	36	19	35.9
4	38.3	58.4	38	57.7
5	100	100	100	100

Source: Deininger and Squire (1995).

Table 3: Cumulative Quintile Income Shares in Percent

Year	Brazil	Korea
1960	2.6	3.2
1965	2.6	4.4
1970	2.9	5.6
1975	2.8	5.9
1980	3.0	6.8
1985	3.5	7.8

Source: Barro and Lee (1993).

Table 4: Average Years of Schooling of Population over Age 25

	Primary		Secondary	
Year	Brazil	Korea	Brazil	Korea
1960	.95	.94	.11	.27
1965	1.0	1.0	.16	.35
1970	.72	1.0	.26	.42
1975	.88	1.0	.26	.56
1980	.99	1.0	.34	.76
1985	1.0	.96	.35	.95
1990	1.0	1.0	.39	1.0

Source: Barro and Lee (1993), World Tables.

Table 5: Adjusted Enrollment Ratios

	Brazil			Korea		
Year	Total	Male	Female	Total	Male	Female
1950	50.6	45.2	55.8			
1955				23.2	12.6	33.3
1960	39.3	35.6	42.6	29.4	16.6	41.8
1970	33.8	30.6	36.9	12.4	5.6	19.0
1980	25.5	23.7	27.2			
1991	20.1	19.9	20.3			
1995	16.7	10.7	20.3	2.0	.7	3.3

Source: UNESCO Statistical Yearbook, various editions.

Table 6: Adult (15+) Illiteracy Rates

F Data for England

Year	TFR
1700	4.4
1750	4.3
1800	5.1
1840	4.9
1860	4.8
1880	4.9
1900	3.6
1920	2.4
1940	1.8
1960	2.5
1980	1.9
1992	1.8

Source: Lee and Schofield (1981), Chesnais (1992), World Tables.

Table 7: Total Fertility Rate in England 1700–1992

Cohort	Years of Schooling
1806–1815	2.7
1816–1825	3.6
1846–1851	5.0
1872–1876	6.6
1887–1896	8.3
1907–1916	9.4
1927–1931	9.7
1942–1946	10.3

Source: Matthews et al. (1982).

Table 8: Average Years of Schooling of Males by Birth Cohort

G Figures

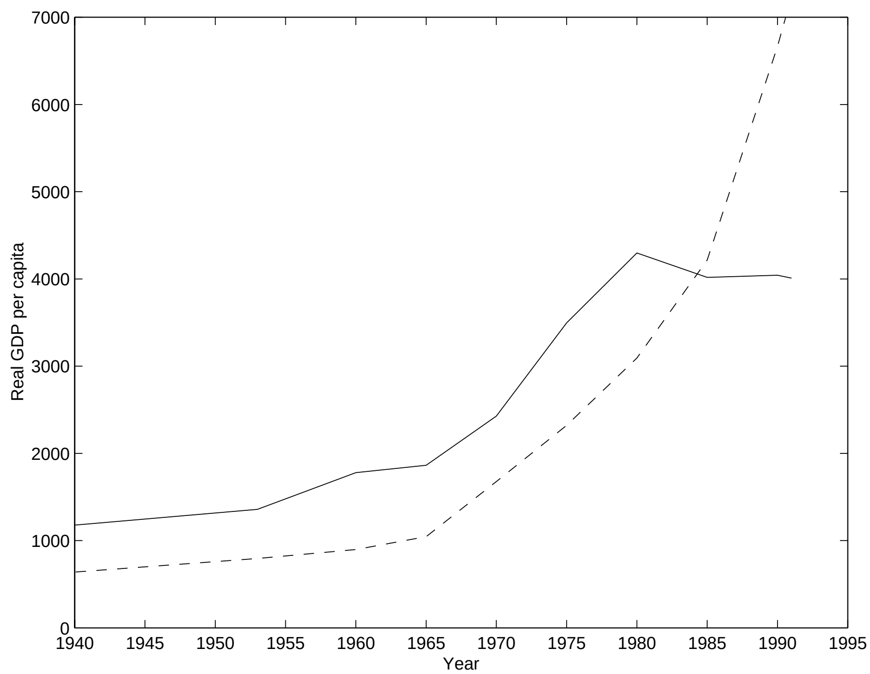

Figure 1: Real GDP in Brazil (solid line) and Korea (dotted line), 1950-1992

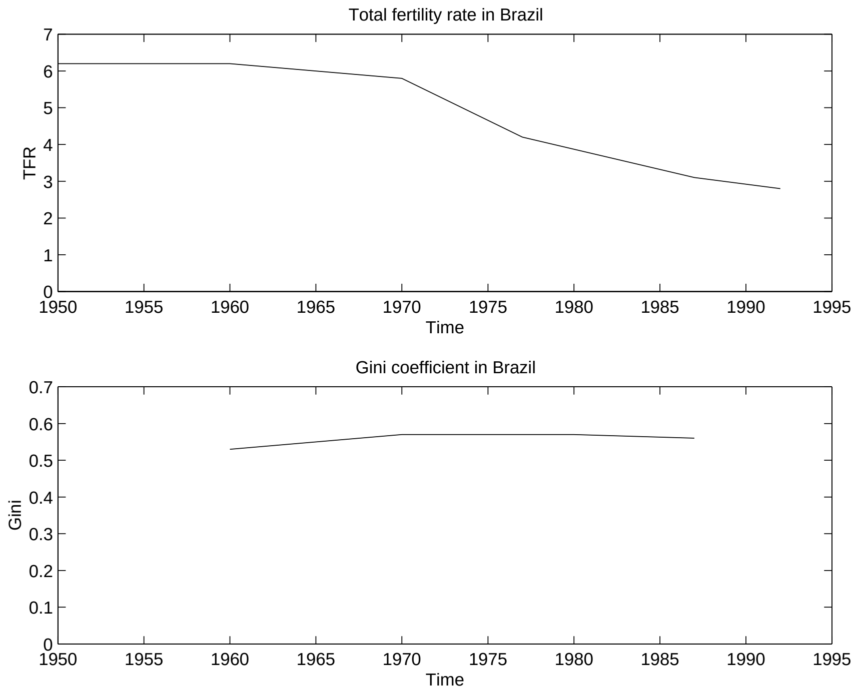

Figure 2: Actual Total Fertility Rate and Gini Coefficicent for Brazil

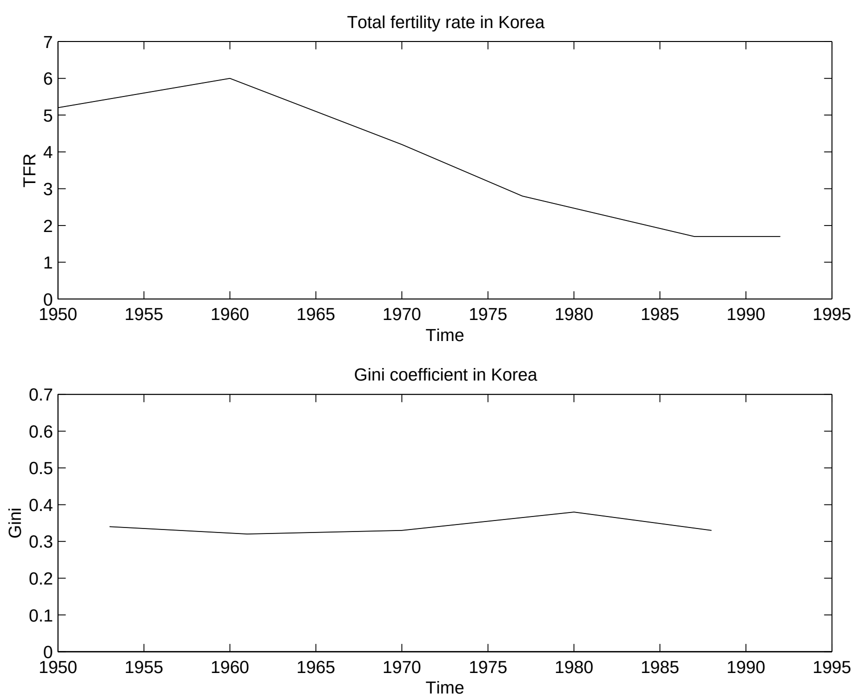

Figure 3: Actual Total Fertility Rate and Gini Coefficicent for Korea

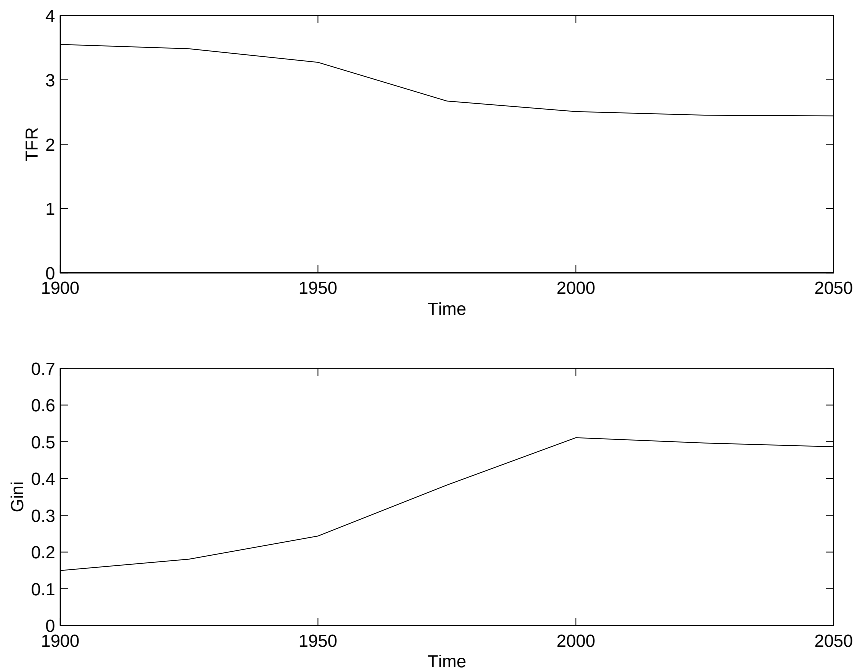

Figure 4: Simulated Total Fertility Rate and Gini Coefficicent for Brazil

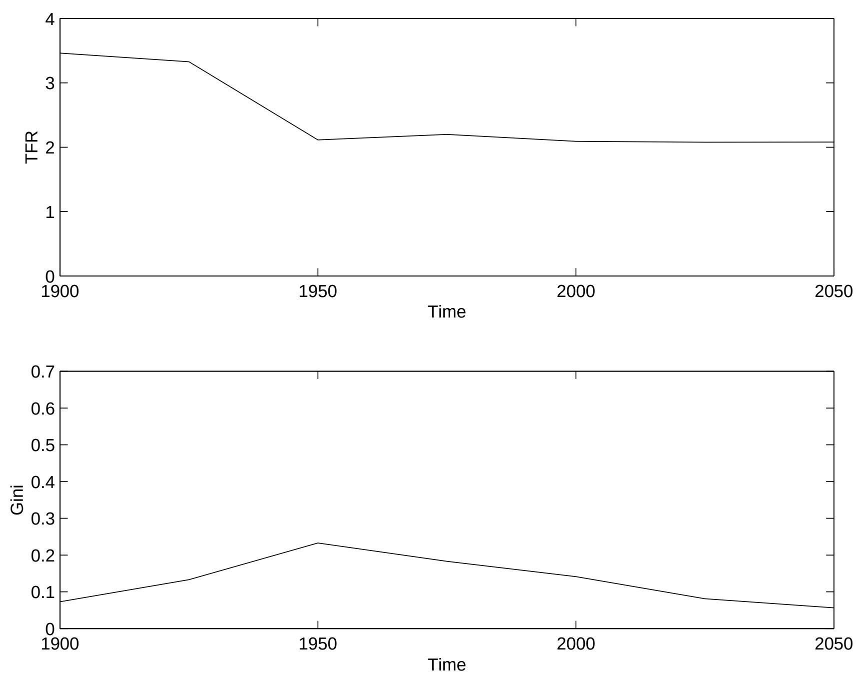

Figure 5: Simulated Total Fertility Rate and Gini Coefficicent for Korea

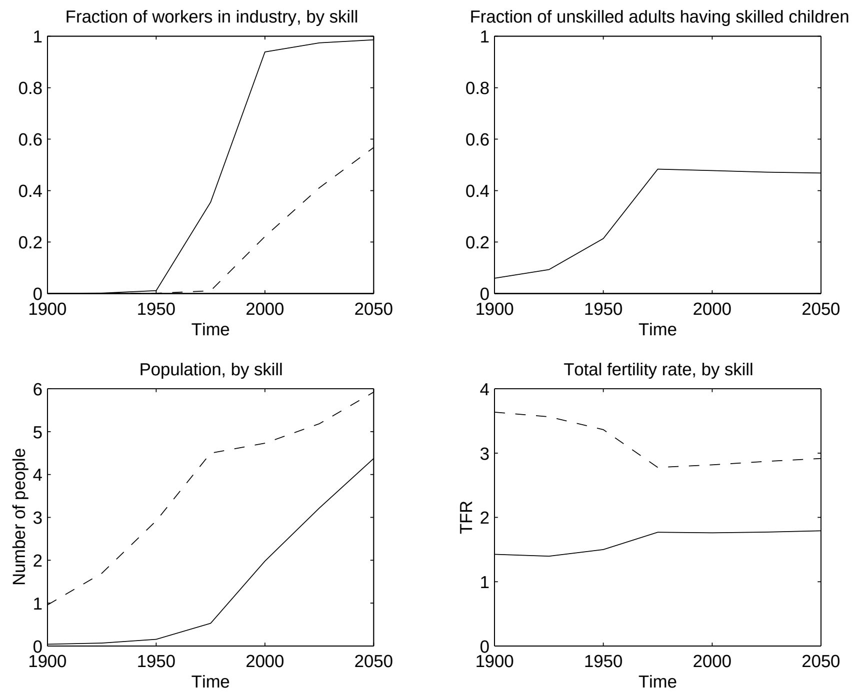

Figure 6: Simulations for Brazil (solid lines: skilled, dotted lines: unskilled)

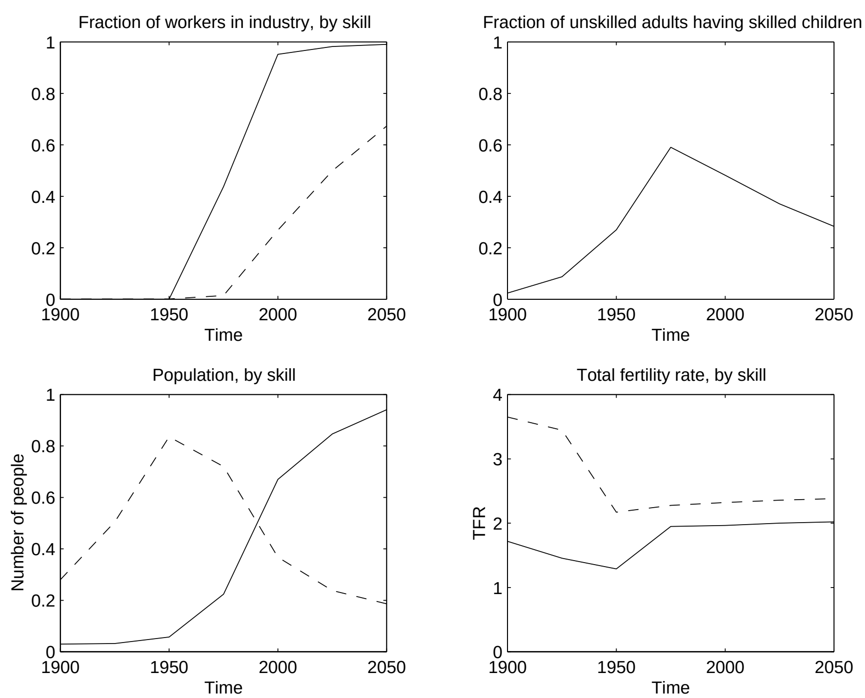

Figure 7: Simulations for Korea (solid lines: skilled, dotted lines: unskilled)

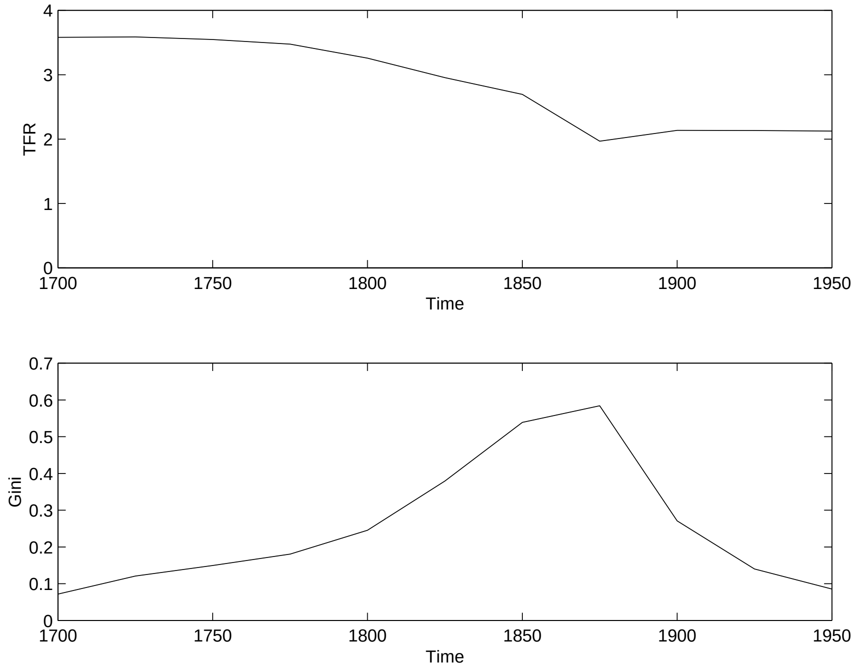

Figure 8: Simulated Total Fertility Rate and Gini Coefficicent for England

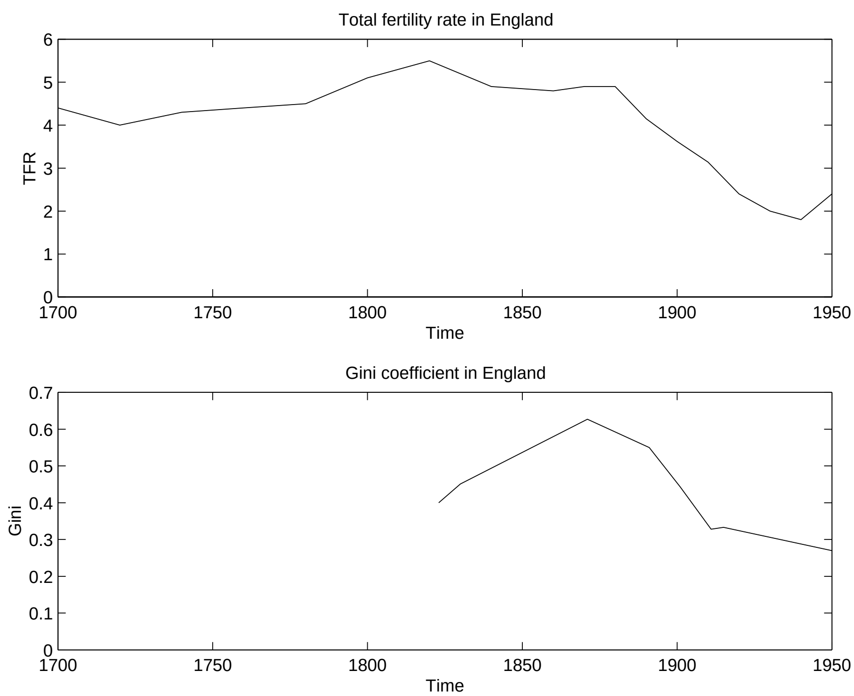

Figure 9: Actual Total Fertility Rate and Gini Coefficicent for England

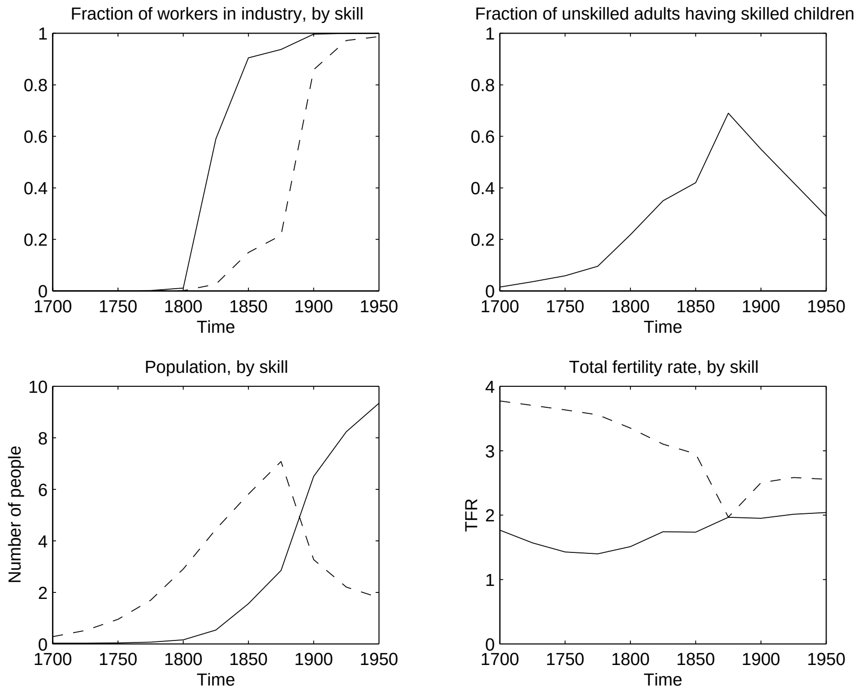

Figure 10: Simulations for England (solid lines: skilled, dotted lines: unskilled)

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