Extended MLSE Receiver for the Frequency-Flat, Fast Fading Channel

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***Abstract* - This paper develops an MLSE receiver for the frequency-flat, fast fading channel corrupted by additive Gaussian noise, when linear modulations (*M*-ASK, *M*-PSK, *M*-QAM) are employed. The paper extends Ungerboeck's derivation of the Extended MLSE receiver for the purely frequency-selective channel to the time-selective channel. Although the new receiver's structure and metric assume ideal channel state information (CSI) at the receiver, the receiver structure can be used wherever high-quality CSI is available. The receiver is Maximum Likelihood for a variety of channels, including Ricean, Rayleigh, log-normal and AWGN channels. Bounds on the receiver's BER are deduced for ideal and pilot tone CSI for fast Rayleigh fading. A crude lower bound is developed on the BER of predictor-based receivers for the same channel. The paper offers insight into matched filtering and receiver processing for the fast fading channel, and shows how pilot symbols and tones should be exploited.**

I. Introduction

The optimal filtering arrangement for the AWGN channel is a square-root Nyquist filter at the transmitter, and its time-reversed complex conjugate as a matched filter at the receiver. Many researchers [1-5] have used the same fixed filters at transmitter and receiver in the time-selective channel of figure 1, although the channel properties are quite different except in very slow fading. In fast fading, the pulse distortion due to the multiplicative channel leads to non-root-Nyquist pulses, ISI, and thus a BER floor. Others [7,8] have employed full-Nyquist pulse shaping at the transmitter, and a zonal filter at the receiver. These receivers are effective, in that there is no ISI-induced error floor. However they are not optimum, since they only take one sample per symbol and this is below the Nyquist sampling rate. In fact they fail to exploit the implicit Doppler diversity of fast fading channels [9].

This paper derives an optimal receiver configuration that assumes ideal channel state information (CSI) is available at the receiver but not the transmitter. The derivation follows Ungerboeck's Extended MLSE receiver for the frequency-selective channel [10] and is known as the "Extended MLSE receiver for the time-selective channel," or EMLSE-*t*. Although the derivation assumes ideal CSI, the receiver operates successfully whenever high quality CSI is available. Suitable systems for estimating CSI include pilot tones [1,2] and pilot symbols (when the transmitter pulse shaping is full Nyquist) [4]. The need for CSI is not restrictive, since any high performance receiver must accurately estimate the channel.

In the absence of pilot signals, the MLSE receiver structure uses predictors in a Per-Survivor-Processing (PSP) structure [7,11,14,15]. These jointly estimate the channel and make decisions. For ease of analysis, these can be reformulated as two stage receivers, where the first stage uses predictors and PSP to provide CSI rather than to make data decisions, and the second stage is an EMLSE-*t* receiver. Then the BER techniques for the EMLSE-*t* receiver do apply crudely to the PSP receivers, under some idealising assumptions, and this leads to a lower bound on their performance.

In Section II, we derive the maximum likelihood path and branch metrics for an arbitrary noise autocorrelation. In Section III, we narrow our focus to white noise. In Section IV, the metrics are translated into a finite complexity receiver structure. In Section V, the BER of the proposed receiver is analyzed for fast Rayleigh fading and white noise.

II. Receiver Derivation

The transmitter maps a binary source to a sequence of M-ary complex phasors, $\{\alpha_{ir}\}$, where i indexes time and r is an integer constant, present to align this notation with the discrete-time notation used later. The transmission interval is assumed to extend sufficiently far into the past and future that the end points can be assumed to be $\pm\infty$ without penalty. In complex baseband, the transmitter sends

$$a(t) = \rho_a \sum_i \alpha_{ir} h(t - irT_r) \tag{1}$$

where ρ_a^2 is the power fraction allocated to the signal; $h(t)$ is the transmitter pulse shape, normalized to energy, T; with T being the symbol period; and $T_r = T/r$. In a discrete time receiver, r is the number of samples per symbol and T_r is the receiver sampling period. The phasors are normalized so that, averaged across the constellation, $E\left[\left|\alpha_{ir}\right|^2\right] = 1$.

Using complex baseband notation, the channel multiplicatively distorts the signal by a complex fading process, $z(t)$, and complex Gaussian noise, $n(t)$, is added, resulting in the received signal (c.f. figure 1),

$$y(t) = a(t)z(t) + n(t) \tag{2}$$

The receiver requires CSI, which we represent as the estimate,

$$\hat{z}(t) = \rho_z z(t) + e(t) \tag{3}$$

where $e(t)$ is the CSI estimation error, and ρ_z^2 is the normalized power of the CSI. For ideal CSI, $\rho_a = \rho_z = 1$ and $e(t) = 0$. For pilot-based systems, the transmitted power is balanced between the signal and the pilot, as $\rho_z^2 = 1 - \rho_a^2$. For PSP receivers, all the transmitted power is allocated to the signal, which is also used for CSI estimation, so $\rho_a = \rho_z = 1$. For pilot tones and PSP receivers, $e(t) \neq 0$. In our derivation, ideal CSI is assumed, but the notation is generalized to allow for the other cases. Strictly speaking, the receiver is optimal for ideal CSI only.

The MLSE receiver searches all allowed symbol sequences in the transmission interval and chooses the one with maximum likelihood. Conditioned on the symbol sequence and the fading process (i.e. ideal CSI), the log-likelihood of $y(t)$ matches the log-likelihood of the complex, zero-mean, Gaussian noise,

$$\begin{aligned} l_{y|\alpha,z}\left(y|\alpha,z\right) &= l_n(n) \\ &\sim -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{n}(t_1) R_{nn}^{-1}(t_1 - t_2) n(t_2)\, dt_1 dt_2 \end{aligned} \tag{4}$$

where the overbar denotes complex conjugation. The noise autocovariance is given by

$$R_{nn}(\tau) = \tfrac{1}{2} E\left(n(t)\bar{n}(t+\tau)\right) \tag{5}$$

and $R_{nn}^{-1}(\tau)$ is the limiting form of a matrix inverse [16], so it must satisfy

$$R_{nn}(\tau) * R_{nn}^{-1}(\tau) = \delta(\tau) \tag{6}$$

where $*$ denotes convolution. Substituting (1) and (2) into (4) leads to

$$\begin{aligned} &l_{y|\alpha,z}\left(y|\alpha,z\right) \sim \\ &-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\bar{y}(t_1) - \rho_a \sum_i \bar{\alpha}_{ir} \bar{h}(t_1 - irT_r)\bar{z}(t_1)\right) R_{nn}^{-1}(t_1 - t_2) \\ &\qquad \times \left(y(t_2) - \rho_a \sum_k \alpha_{kr} h(t_2 - krT_r) z(t_2)\right) dt_1 dt_2 \end{aligned} \tag{7}$$

Interchanging the order of summation and integration, and neglecting terms independent of the symbol sequence, the log-likelihood can be rendered as

$$l_{y|\alpha,z}\left(y|\alpha,z\right) \sim \sum_i 2\Re\{\bar{\alpha}_{ir} m_{ir}\} - \sum_l \sum_k \bar{\alpha}_{lr} s_{lr,kr} \alpha_{kr} \tag{8}$$

where m_{ir} and $s_{lr,kr}$ are defined as

$$\begin{aligned} m_{ir} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_a y(t_2) R_{nn}^{-1}(t_1 - t_2) \bar{h}(t_1 - irT_r) \bar{z}(t_1)\, dt_1 dt_2 \\ &= \rho_a \left[\left(y(t) * R_{nn}^{-1}(t)\right)\bar{z}(t)\right] * \bar{h}(-t)\Big|_{t = irT_r} \\ s_{lr,kr} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_a^2 \bar{h}(t_1 - lrT_r) \bar{z}(t_1) R_{nn}^{-1}(t_1 - t_2) \times \\ &\qquad h(t_2 - krT_r) z(t_2)\, dt_1 dt_2 \end{aligned} \tag{9}$$

As with Ungerboeck's EMLSE-*f* receiver [10], there is no need for a noise-whitening filter.

The conjugate symmetry of $s_{lr,kr}$, $s_{lr,kr} = \bar{s}_{kr,lr}$, can exploited by properly grouping the double summation of (8) as,

$$\sum_{l=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\bar{\alpha}_{lr}s_{lr,kr}\alpha_{kr} = 2\Re\left\{\sum_{l=-\infty}^{\infty}\sum_{k=-\infty}^{k-1}\bar{\alpha}_{lr}s_{lr,kr}\alpha_{kr}\right\} + \sum_{l=-\infty}^{\infty}\left|\alpha_{lr}\right|^2 s_{lr,lr} \tag{10}$$

leading to the iterative metric,

$$l_{y|\alpha,z}\left(y|\alpha,z\right) \sim \sum_{i=-\infty}^{\infty} l_{ir} = \sum_{i=-\infty}^{\infty}2\Re\left\{\bar{\alpha}_{ir}\left(m_{ir} - \sum_{k=-\infty}^{i-1}s_{ir,kr}\alpha_{kr}\right)\right\} - \left|\alpha_{ir}\right|^2 s_{ir,ir} \tag{11}$$

which can be calculated more efficiently. This additive metric leads to a straightforward implementation using the Viterbi algorithm.

The metric acts in the following way. All signal information is combined in the m_{ir} term. This includes ISI from adjacent symbols. The ISI from past symbols is ideally cancelled or equalized by the summation, $\sum_k s_{ir,kr}\alpha_{kr}$. The $s_{ir,kr}$ term calculates the interfering, faded pulse tails exactly. Assuming the interfering symbols are chosen correctly, the factor in round brackets contains only the faded *i*th symbol, ISI from future symbols and additive noise. This is correlated with the hypothesised *i*th symbol. The ISI from future symbols is progressively removed by future branch metrics.

Having assumed ideal CSI, the statistics of the fading process are irrelevant to the metric. The analysis thus applies to all noise limited, frequency-flat channels, including Rayleigh, Ricean, log-normal and AWGN. When ideal CSI is not available, the performance of this receiver leads to a lower bound on a practical receiver's performance.

III. Receiver Behaviour In White Noise

For the important case of white noise with a two-sided RF power spectral density of $N_0/2$, we have in complex baseband $R_{nn}(\tau) = \frac{1}{2}E\left(n(t)\bar{n}(t+\tau)\right) = N_0\delta(\tau)$. Since the metric of (11) is used for comparisons only, the scale factor of N_0 can be neglected. Then using $R_{nn}^{-1}(\tau) = \frac{\delta(\tau)}{N_0} \propto \delta(\tau)$ simplifies m_{ir} and $s_{ir,kr}$ to

$$\begin{aligned} m_{ir} &= y(t)\bar{z}(t) * \bar{h}(-t)\Big|_{t=irT_r} \\ &= \int_{-\infty}^{\infty}\rho_a y(t_1)\bar{h}(t_1 - irT_r)\bar{z}(t_1)dt_1 \\ &= \sum_{k=-\infty}^{\infty}\alpha_{kr}\int_{-\infty}^{\infty}\rho_a^2\left|z(t_1)\right|^2\bar{h}(t_1 - irT_r)h(t_1 - krT_r)dt_1 \\ &\quad + \int_{-\infty}^{\infty}\rho_a n(t_1)\bar{z}(t_1)\bar{h}(t_1 - irT_r)dt_1 \\ s_{ir,kr} &= \int_{-\infty}^{\infty}\rho_a^2\left|z(t_1)\right|^2\bar{h}(t_1 - irT_r)h(t_1 - krT_r)dt_1 \end{aligned} \tag{12}$$

We note that m_{ir} can be considered as the symbol-rate sampled output, $t = irT_r = iT$, of a modified matched filter for the time-selective channel (MF-*t*) which includes the effect of the fading process. Assuming ideal CSI, the MF-*t* completely removes the channel's phase distortion by premultiplying the signal by the conjugated fading process. Furthermore the expected peak symbol power to noise power ratio of the *i*th symbol is maximized by summing the signal weighted according to the instantaneous fading and pulse power, $\left|z(t_1)h(t_1 - irT_r)\right|^2$. It is important to understand that this in itself does not deal with ISI, and in fact may make it worse. The complete EMLSE-*t* receiver of (11) deals with the ISI by cancelling or equalising it by means of $s_{lr,kr}$. This avoids an error floor.

In very slow fading, it is typically assumed that the fading process is constant over a symbol duration. Therefore it can be shifted outside the $s_{lr,kr}$ integral in (12), and, assuming a square-root Nyquist pulse, $s_{lr,kr}$ equal zero for all $l \neq k$. In this case, the receiver comprises three stages only: phase recovery, a MF-*f* and a hard decision device. Thus in slow fading, this receiver reduces to the optimal receiver for the AWGN channel. However, even for a square-root Nyquist pulse, a fast fading process introduces ISI, since in general the channel's complex gain changes along the pulse's length, and the pulse is not simply a scaled version of itself. Reference [19] addresses the boundaries between slow and fast fading. In the time-invariant channel, the ISI is fixed, so Ungerboeck's EMLSE-*f* receiver is able to precompute $s_{lr,kr}$. This is not possible in the time-varying channel, since the ISI changes as the fading process changes.

For the special case of pulses of less than one symbol in duration, there can be no ISI, the receiver makes hard, symbol-by-symbol decisions, and the branch metric reduces to a Euclidean distance,

$$l_{ir} \sim \left|m_{ir} - \alpha_{ir} s_{ir,ir}\right|^2 \tag{13}$$

Exact BERs can be calculated for this special case. Here, the EMLSE-*t* receiver's decisions have the same geometric interpretation as the decisions of a receiver for the AWGN channel, except that the constellation points and decision boundaries vary with the depth of the fade, according to $s_{ir,ir}$.

IV. Receiver Structure

The MLSE receiver employs exhaustive comparison to find the maximum likelihood sequence. This task is undertaken efficiently by a Viterbi processor, which has finite complexity provided the ISI extends over only a finite number of symbols. This condition is satisfied if the pulse shape has a finite duration, $h(t) = 0$, $t < 0$, $LT \le t$, where L is an integer. Then the ISI term, $s_{ir,kr}$, in (12) is zero for $|k - i| \ge L$, and the branch metric is given by

$$l_{ir} \sim 2\Re\left\{\overline{\alpha}_{ir} m_{ir} - \sum_{k=i-L+1}^{i-1} \overline{\alpha}_{ir} s_{ir,kr} \alpha_{kr}\right\} - \left|\alpha_{ir}\right|^2 s_{ir,ir} \tag{14}$$

This is a function of L symbols, $\left(\alpha_{(i-L+1)r} \cdots \alpha_{ir}\right)$, known as the hypothesis vector. A partially-connected trellis is built up, with M^{L-1} states, labelled by the first L-1 symbols of the vector, $\left(\overbrace{\hat{\alpha}_{(i-L+1)r} \cdots \hat{\alpha}_{(i-1)r}}^{State}, \hat{\alpha}_{ir}\right)$, where the last symbol labels the current branch.

During the ith symbol period, the receiver forms one value of m_{ir} and L values of $s_{ir,kr}$ according to (9) or (12). In a practical receiver, ideal CSI is unavailable, so the CSI estimate, $\hat{z}(t)/\rho_z$, replaces $z(t)$ in the m_{ir} and $s_{ir,kr}$ expressions. The M^L branch metrics are calculated according to (14), and then applied to a standard Viterbi processor. The EMLSE-*t* structure is shown in Figure 1.

V. Performance Evaluation

We seek the receiver's BER for fast Rayleigh fading channels. The analysis of trellis-based receivers is not straightforward, since the errors are not independent and appear as error events. An error event occurs whenever a sequence exists that has a greater likelihood than the transmitted sequence. We may arbitrarily align the first symbol error with time $i = 0$, so an erroneous sequence can be written as $\{\alpha^{u,v,w}\} = \{\alpha^{u,v}\} + \{\varepsilon^{u,v,w}\}$,

$$\left\{\ldots, \alpha_{-r}^{u,v}, \alpha_0^{u,v} + \varepsilon_0^{u,v,w}, \ldots \alpha_{ur}^{u,v} + \varepsilon_{ur}^{u,v,w}, \alpha_{(u+1)r}^{u,v}, \ldots\right\} \tag{15}$$

where the sequence $\{\varepsilon^{u,v,w}\}$ is the difference in the phasor plane between the erroneous sequence, $\{\alpha^{u,v,w}\}$, and the actual transmitted sequence, $\{\alpha^{u,v}\}$. This notation identifies the sequence by the length, u, of the error event under consideration and an index, v, that enumerates the symbol sequences. Each transmitted sequence can be confused with several others, so the erroneous sequence, $\{\alpha^{u,v,w}\}$, has an additional index, w, to enumerate the error events that deviate from the transmitted sequence, v, of length u.

ε_{ir} is zero for $i < 0$ and $i > u$, where u+1 is the length of the error event. During the event, the receiver cannot choose L-1 correct symbols in a row, because this returns the receiver to the correct state. We assume no channel coding, but the channel's Doppler spread leads to implicit diversity.

An upper bound on the BER can then be deduced from a union bound of error event probabilities as

$$BER \le \sum_{u=0}^{\infty} \sum_{v=1}^{M^{u+2L-1}} \sum_{\alpha^{u,v,w}} \frac{P\left(\alpha^{u,v}\right) P\left(\alpha^{u,v} \to \alpha^{u,v,w}\right) e\left(\alpha^{u,v} \to \alpha^{u,v,w}\right)}{\log_2 M} \tag{16}$$

The union bound considers all transmitted sequences and possible error events, weighted by their transmission probability, $P\left(\alpha^{u,v}\right) = \frac{1}{M^{u+2L-1}}$, the probability, $P\left(\alpha^{u,v} \to \alpha^{u,v,w}\right)$, that an incorrect sequence has a larger metric than the correct sequence (the pairwise probability of error), and the number of bit errors, $e\left(\alpha^{u,v} \to \alpha^{u,v,w}\right)$, the error event introduces. Error events can begin in any symbol interval, during which $\log_2 M$ bits are sent. At high SNR the likelihood of overlapping error events is low enough to neglect.

In practice, only the dominant short error events are considered, where up to E symbols may be in error. If the fading is fast and the SNR high, the dominant error events are short, so the truncated bound can validly neglect long error events. However, most errors occur in deep fades, when the instantaneous SNR is below about 5dB. If the fading is very slow then a deep fade lasts hundreds of symbols at reasonable SNRs, and so does an error event. Thus the upper bound is tight and easily calculated for fast fading and high SNR only. For slow fading, the conventional slow-fading approximations are valid [1,2,4,13].

For BPSK and QPSK, the union of E = 1 error events due the nearest neighbours of the errored symbol only is particularly useful. Clearly, the BER of this nearest neighbours union is asymptotically correct at high SNR's. For slow fading or low SNR, this nearest neighbours union also leads to the same BER as predicted by the slow-fading approximations. Thus the nearest neighbours union is accurate at low and high SNR, so we expect it to be a good estimate of the BER for all SNR.

With some manipulation, the pairwise probability of error for each erroneous sequence can be calculated from (8) and (14) in the form

$$\begin{aligned}
&P\left(\alpha^{u,v} \to \alpha^{u,v,w}\right) = \\
&P\left(l_{y|\alpha^{u,v,w},z}\left(y \middle| \alpha^{u,v,w}, z\right) > l_{y|\alpha^{u,v},z}\left(y \middle| \alpha^{u,v}, z\right)\right) \\
&= P\left(\begin{array}{l}
\sum_{i=-\infty}^{\infty} 2\Re\left\{\begin{array}{l}\left(\overline{\alpha}_{ir}^{u,v} + \overline{\varepsilon}_{ir}^{u,v,w}\right) m_{ir}^{u,v} \\ - \sum_{k=i-L+1}^{i-1} \left(\overline{\alpha}_{ir}^{u,v} + \overline{\varepsilon}_{ir}^{u,v,w}\right) s_{ir,kr} \left(\alpha_{kr}^{u,v} + \varepsilon_{kr}^{u,v,w}\right)\end{array}\right\} \\
\qquad - \left|\alpha_{ir}^{u,v} + \varepsilon_{ir}^{u,v,w}\right|^2 s_{ir,ir} \\
> \sum_{i=-\infty}^{\infty} 2\Re\left\{\overline{\alpha}_{ir}^{u,v} m_{ir}^{u,v} - \sum_{k=i-L+1}^{i-1} \overline{\alpha}_{ir}^{u,v} s_{ir,kr} \alpha_{kr}^{u,v}\right\} \\
\qquad - \left|\alpha_{ir}^{u,v}\right|^2 s_{ir,ir} +
\end{array}\right) \\
&= P\left(\sum_{i=0}^{u} \Re\left\{\begin{array}{l} 2\overline{\varepsilon}_{ir}^{u,v,w} m_{ir}^{u,v} - 2\sum_{k=i-L+1}^{i+L-1} \overline{\varepsilon}_{ir}^{u,v,w} s_{ir,kr} \alpha_{kr}^{u,v} - \\ \sum_{k=\max\left\{0,\, i-L+1\right\}}^{\min\left\{u,\, i+L-1\right\}} \overline{\varepsilon}_{ir}^{u,v,w} s_{ir,kr} \varepsilon_{kr}^{u,v,w} \end{array}\right\} > 0\right)
\end{aligned} \tag{17}$$

Modern digital receivers operate in discrete time, and it is convenient to so analyze them. The receiver first applies a zonal anti-aliasing filter to the received signal, with transfer function, $rect\left(fT_r\right)$. The filtered signal is sampled at $t = lT_r$, so the receiver takes $r \ge 1$ samples per symbol period. Subscripts are used to denote time, so that $y_l = y(lT_r)$, $z_l = z(lT_r)$, $n_l = n(lT_r)$, $e_l = e(lT_r)$, $h_l = h(lT_r)$. The integrals of (12) are replaced by summations, resulting in

$$\begin{aligned}
m_{ir}^{u,v} &= \sum_{l=ir}^{ir+Lr-1} y_l \frac{\overline{\hat{z}}_l}{\rho_z} \overline{h}_{l-ir} \\
&= \sum_{k=i-L+1}^{i+L-1} \sum_{l=\max\{ir,kr\}}^{\min\{ir,kr\}+Lr-1} \frac{\rho_a^2}{\rho_z} \alpha_{kr}^{u,v} \overline{h}_{l-ir} h_{l-kr} z_l \overline{\hat{z}}_l \\
&\quad + \sum_{l=ir}^{ir+Lr-1} \frac{\rho_a}{\rho_z} \overline{h}_{l-ir} n_l \overline{\hat{z}}_l \\
s_{ir,kr} &= \sum_{l=\max\{ir,kr\}}^{\min\{ir,kr\}+Lr-1} \frac{\rho_a^2}{\rho_z^2} \overline{h}_{l-ir} h_{l-kr} \left|\hat{z}_l\right|^2
\end{aligned} \tag{18}$$

where the factor of T_r is neglected since the metric is used for comparisons only.

The absolute bandwidth of a time-limited pulse is infinite, so strictly speaking a sampling rate of $1/T_r$ is inadequate. However, we can normally use another bandwidth definition (such as the -40dB bandwidth), add the maximum Doppler spread, and sample accordingly with negligible aliasing. Employing (18) in (17), the pairwise probability of error then reduces to

$$P\left(\alpha^{u,v} \to \alpha^{u,v,w}\right) = P\left(0 < \Re\left\{ 2\frac{\rho_a}{\rho_z}\sum_{i=0}^{u}\sum_{l=ir}^{ir+Lr-1}\bar{\varepsilon}_{ir}^{u,v,w}\bar{h}_{l-ir}n_l\bar{\hat{z}}_l + 2\frac{\rho_a^2}{\rho_z}\sum_{i=0}^{u}\sum_{k=i-L+1}^{i+L-1}\sum_{l=max\{ir,kr\}}^{min\{ir,kr\}+Lr-1}\bar{\varepsilon}_{ir}^{u,v,w}\alpha_{kr}^{u,v}\bar{h}_{l-ir}h_{l-kr}\left(z_l - \frac{\hat{z}_l}{\rho_z}\right)\bar{\hat{z}}_l - \frac{\rho_a^2}{\rho_z^2}\sum_{i=0}^{u}\sum_{k=max\left\{0,\ i-L+1\right\}}^{min\left\{u,\ i+L-1\right\}}\sum_{l=max\{ir,kr\}}^{min\{ir,kr\}+Lr-1}\bar{\varepsilon}_{ir}^{u,v,w}\varepsilon_{kr}^{u,v,w}\bar{h}_{l-ir}h_{l-kr}\left|\hat{z}_l\right|^2 \right\}\right) \tag{19}$$

The left hand side of the inequality is a Gaussian quadratic form, so the probability can be calculated easily. However, for non-ideal CSI, the probability depends on the sequence of u+2L-1 transmitted symbols, and u+1 errored symbols. The BER bound is tedious to calculate, since it requires $\sum_{u=0}^{E-1} M^{u+2L-1}(M-1)^{u+1}$ pairwise probabilities. For ideal CSI, (19) reduces to

$$P\left(\alpha^{u,v} \to \alpha^{u,v,w}\right) = P\left(-\sum_{i=0}^{u}\sum_{k=max\left\{0,\ i-L+1\right\}}^{min\left\{u,\ i+L-1\right\}}\sum_{l=max\{ir,kr\}}^{min\{ir,kr\}+Lr-1}\bar{\varepsilon}_{ir}^{u,v,w}\varepsilon_{kr}^{u,v,w}\bar{h}_{l-ir}h_{l-kr}\left|z_l\right|^2 + \sum_{i=0}^{u}\sum_{l=ir}^{ir+Lr-1}\left(\bar{\varepsilon}_{ir}^{u,v,w}\bar{h}_{l-ir}n_l\bar{z}_l + \varepsilon_{ir}^{u,v,w}h_{l-ir}\bar{n}_l z_l\right) > 0 \right) \tag{20}$$

which is not explicitly dependent on the transmitted symbol sequence. There is still an implicit dependence however, since the allowed values of $\{\varepsilon\}$ depend on the position of each transmitted symbol within the constellation. Neglecting constellation-specific symmetries, only $\sum_{u=0}^{E-1} M^{u+1}(M-1)^{u+1}$ pairwise probabilities must be calculated and $P\left(\alpha^{u,v}\right) = \frac{1}{M^u}$.

Define $\mathbf{g}$ as

$$\mathbf{g} = \begin{bmatrix} \mathbf{z} \\ \hline \mathbf{n} \\ \hline \mathbf{e} \end{bmatrix} \tag{21}$$

where $\mathbf{z}$, $\mathbf{n}$ and $\mathbf{e}$ are column vectors of all the fading samples, z_l, additive noise samples, n_l, and CSI error samples, e_l, in (19) or (20), ordered chronologically. Define $\kappa^{u,v,w}$ as the left-hand-side of the inequalities in (19) or (20), so it can be written explicitly as a Gaussian quadratic form,

$$\kappa^{u,v,w} = \mathbf{g}^H \mathbf{G}^{u,v,w}\mathbf{g} \tag{22}$$

where the kernel, $\mathbf{G}^{u,v,w}$, is a Hermitian symmetric matrix, defined by (19) or (20). The covariance matrix of $\mathbf{g}$, $\mathbf{R_{gg}}$, is given by

$$\mathbf{R_{gg}} = \tfrac{1}{2}E\left(\mathbf{g}\mathbf{g}^H\right) = \left[\begin{array}{c|c|c} \mathbf{R_{zz}} & \mathbf{0} & \mathbf{R_{ze}} \\ \hline \mathbf{0} & \mathbf{R_{nn}} & \mathbf{R_{ne}} \\ \hline \mathbf{R_{ez}} & \mathbf{R_{en}} & \mathbf{R_{ee}} \end{array}\right] \tag{23}$$

An isotropic scattering model is assumed [12], so the fading autocorrelation matrix is the usual Bessel function,

$$\left(\mathbf{R_{zz}}\right)_{ik} = \tfrac{1}{2}E\left(z_i\bar{z}_k\right) = J_0\left(2\pi f_D T\left|i-k\right|/r\right) \tag{24}$$

where f_D is the one-sided fading bandwidth and $f_D T$ is the fractional Doppler spread. For ideal CSI and pilot tone CSI, the noise autocorrelation matrix is controlled by the zonal filter, so that

$$\left(\mathbf{R_{nn}}\right)_{ik} = \tfrac{N_0}{T_r}\delta_{ik} \tag{25}$$

For ideal CSI, $\mathbf{R_{ze}} = \mathbf{R_{ez}}^H = \mathbf{0}$, $\mathbf{R_{ne}} = \mathbf{R_{en}}^H = \mathbf{0}$, and $\mathbf{R_{ee}} = \mathbf{0}$. For pilot tone CSI, $\mathbf{R_{ze}} = \mathbf{R_{ez}}^H = \mathbf{0}$, $\mathbf{R_{ne}} = \mathbf{R_{en}}^H = \mathbf{0}$. Assuming an ideal brick-wall filter for the pilot tone,

$$\left(\mathbf{R_{ee}}\right)_{ik} = 2f_D T_r \tfrac{N_0}{T_r} sinc\left(2f_D T_r\left|i-k\right|\right) \tag{26}$$

The operation of PSP receivers as channel estimators must be idealized for convenient analysis. Since the PSP receiver of [11] is maximum likelihood for rectangular pulses only, we restrict our attention to undersampled multi-sinc pulses, as described in the Appendix. For general pulse shapes, the CSI error is time-varying and data dependent.

The PSP receivers estimate the fading process using predictors. The predictors are fed with the received sampled, y_l, and multiplied by the hypothesised signal, $\bar{\hat{a}}_l$, according to $\zeta_l = \bar{\hat{a}}_l y_l$. A Euclidean metric selects the most likely hypothesis, by comparing the shifted prediction with the received signal,

$$l_{ir} = \sum_{l=ir}^{(i+1)r-1} \left| y_l - \hat{a}_l \sum_{k=1}^{B} b_k \zeta_{l-k} \right|^2 \tag{27}$$

where there are B tap weights, $\{b_k\}$, in the predictor.

We assume the PSP channel estimator always selects the correct channel estimate, $\{\zeta_l\}$, corresponding to the correct (transmitted) symbol sequence (not the maximum likelihood sequence, which is the PSP channel estimator's selection in practice). This assumption is essential for tractability, and it is reasonable at high SNR. It ensures phase lock is kept between transmitter and receiver, so this idealized channel estimator outperforms any practical implementation, and its BER is a lower bound on the actual BER.

For the analysis of predictor receivers, the one-sided IF filter bandwidth must be widened to r/T, to ensure white noise. The additive noise autocorrelation is then

$$(\mathbf{R_{nn}})_{ik} = 2\tfrac{N_0}{T_r}\delta_{ik} \tag{28}$$

The fading process prediction can be expanded as

$$z_l + e_l = \sum_{k=1}^{B} b_k z_{l-k} + \sum_{k=1}^{B} b_k \bar{a}_{l-k} n_{l-k} \tag{29}$$

since $\rho_z = 1$. The estimation error, $e_l = \sum_{k=1}^{B} b_k z_{l-k} - z_l + \sum_{k=1}^{B} b_k \bar{a}_{l-k} n_{l-k}$, comprises a fading process prediction error term and a noise term. For reasonable sampling rates and predictor lengths, the prediction error comes mainly from the noise term, so that $\sum_{k=1}^{B} b_k z_{l-k} - z_l$ can be neglected, as $\mathbf{R_{ze}} = \mathbf{R}_{\mathbf{ez}}^H = \mathbf{0}$. The estimation error, e_l, is correlated with the signal noise, n_l, through the predictor tap weights,

$$\begin{aligned}(\mathbf{R_{ne}})_{ik} &= (\mathbf{R}_{\mathbf{en}}^H)_{ik} = \tfrac{1}{2}E(n_i \bar{e}_k) \\ &= \tfrac{1}{2}E\left(n_i \sum_{l=1}^{B} b_l \bar{n}_{k-l}\right) = 2\tfrac{N_0}{T_r} b_{k-i}\end{aligned} \tag{30}$$

where $b_k = 0$ for $k < 1$. $\mathbf{R_{ee}}$ is given by

$$\begin{aligned}(\mathbf{R_{ee}})_{ik} &= \tfrac{1}{2}E(e_i \bar{e}_k) \\ &= \tfrac{1}{2}E\left(\left(\hat{a}_i \sum_{l=1}^{B} b_l \bar{\hat{a}}_{i-l} n_{i-l}\right)\left(\bar{\hat{a}}_k \sum_{m=1}^{B} \bar{b}_m \hat{a}_{k-m} \bar{n}_{k-m}\right)\right) \\ &= 2\tfrac{N_0}{T_r} \sum_{l=1}^{B} b_l \bar{b}_{l+k-i}\end{aligned} \tag{31}$$

where the last line follows from $\hat{a}_x \bar{\hat{a}}_y = 1$ if x and y are samples from the same symbol period.

With these definitions, $\mathbf{R_{gg}}$ is completely described. The characteristic function of a Gaussian quadratic form, $\kappa^{u,v,w}$, is given by [21]

$$P_\xi^{u,v,w}(\xi) = \frac{1}{\left|\mathbf{I} - j2\xi \mathbf{R_{gg}} \mathbf{G}^{u,v,w}\right|} \tag{32}$$

which is an inverse polynomial in ξ, and can expanded as simple partial fractions to obtain

$$P_\xi^{u,v,w}(\xi) = \sum_i \frac{r_{-1,cf,i}}{\xi - p_{cf,i}} \tag{33}$$

where $p_{cf,i}$ is the ith pole of (32), and $r_{-1,cf,i}$ is its residue. Locating the poles is equivalent to finding the eigenvalues of $j2\mathbf{R_{gg}}\mathbf{G}^{u,v,w}$. The pdf can be deduced from the inverse transform,

$$\begin{aligned}&p_{u,v,w}(\kappa^{u,v,w}) = \\ &-j \sum_{i,\Im\{p_{cf,i}\}<0} r_{-1,cf,i} e^{-jp_{cf,i}\kappa^{u,v,w}} \qquad \kappa^{u,v,w} \ge 0\end{aligned} \tag{34}$$

and the pairwise error probability is the probability that $\kappa^{u,v,w}$ exceeds zero, namely

$$P(\alpha^{u,v} \to \alpha^{u,v,w}) = -\sum_{i,\Im\{p_{cf,i}\}<0} \frac{r_{-1,cf,i}}{p_{cf,i}} \tag{35}$$

The average bit energy to noise spectral density is given by

$$\frac{E_b}{N_0} = \frac{\frac{1}{2} E \int_{-\infty}^{\infty} \left| h(t_1) z(t_1) \right|^2 dt_1}{N_0 log_2 M} = \frac{T}{N_0 log_2 M} \qquad (36)$$

VI. RESULTS

We first consider the case of ideal CSI. The only deficiency in the analysis is the truncation of the union bound. Figure 3 plots the union bounds due to error events of one, two and three symbols in length. At low SNR, the longer events dominate, and truncation is not valid. Furthermore, the bound is very loose. At high SNR, the shortest error event dominates, and the bound is both valid and effective. The analysis uses a pulse with a 50% excess bandwidth square-root raised cosine spectrum, but windowed in the time domain by a Kaiser window with $\beta = 2.5$ to $L = 5$ symbol periods.

Figure 4 compares the performance of different QPSK systems. Curves 1a and 1b are bounds on the EMLSE-*t* performance for a fast fading channel. The lower bound considers only one error event. The upper, union bound considers all error events up to $E = 3$ symbols. By sampling at the Nyquist rate, T_r^{-1}, the diversity inherent in a changing channel is fully exploited.

By comparison, a receiver that takes only one sample per symbol can exploit only one order of diversity. This is shown as Curve 3, with

$$BER = \frac{1+\beta+2f_D T}{2\left(\begin{array}{l} E_b/N_0 + 1 + \beta + 2 f_D T + \\ \sqrt{\left(E_b/N_0\right)^2 + \left(1+\beta+2f_D T\right) E_b/N_0} \end{array} \right)} \qquad (37)$$

where here β is the excess bandwidth of the pulse. The BER is inversely proportional to the SNR at asymptotic SNRs. The exact BER can be calculated here, since there is no ISI and the Viterbi processor reduces to a hard decision device. The noise bandwidth is the same as the Doppler-spread signal bandwidth, and $\frac{1}{2} E\left(\left|n_l\right|^2\right) = N_0\left(\frac{1+\beta}{T} + 2 f_D\right)$. Standard residue calculus completes the analysis.

On a physical level the EMLSE-*t* receiver improves performance through space/time diversity. The receiver moves through the multipath interference pattern at some speed, converting the spatial variation of the field into an apparent temporal change. The faster the motion, the more signal averaging there is, and the chance of a catastrophic null is thereby reduced. Mathematically this diversity effect is seen through the Doppler spread, and is called “implicit Doppler diversity.”

The number of samples per pulse equals the ultimate diversity order. However, the power in each additional diversity branch is substantially smaller, so the weakest diversity branches are unhelpful until unreasonably high SNR’s. The breakpoint SNRs, where new diversity branches emerge, are governed by $f_D T$. The faster the fading, the more powerful the additional diversity branch, and the smaller the breakpoint SNR. For practical values of $f_D T$, only the first and second diversity orders are significant [9].

In very slow fading, the receiver is not moving over the pulse duration and thus there is no useful implicit Doppler diversity at reasonable SNRs. Curve 2 in Figure 4 is inversely proportional to the SNR at asymptotic SNRs.. Again, there is also no ISI and the Viterbi processor simplifies to a hard decision device. The exact BER is a standard result [13],

$$BER = \frac{1}{2\left(E_b/N_0 + 1 + \sqrt{\left(E_b/N_0\right)^2 + E_b/N_0}\right)} \qquad (38)$$

which is the optimal performance in slow fading. The pulse’s excess bandwidth does not affect the BER, since proper matched filtering is performed.

Figure 5 compares the performance of QPSK with 16-QAM in fast fading. The power penalty of the more spectrally efficient constellation is approximately 4dB at 10^{-4} BER. The union bound is particularly loose for 16-QAM due to the large constellation. For 16-QAM, the lower bound considers two disjoint error events. In slow fading, the power penalty is 3dB [17].

Finally, we compare the performance of systems with and without pilot tones. When pilot tones are transmitted, they are exploited; when there is no pilot information, a predictor-based PSP receiver is employed. As a baseline, the ideal CSI curves are also presented.

The convenient analysis of the predictor-based receiver in [11] requires sub-Nyquist rate sampling, so white noise is only achieved by widening the IF filter cutoff frequency to r/T without doubling the sampling rate. However, only the analysis is affected: the receiver can accommodate any noise autocorrelation. When the same pulses are processed by the pilot tone CSI and ideal CSI receivers at the same sampling rate, coloured noise arises again. For a fair comparison, the IF filter bandwidth is doubled for these receivers also, thus doubling the noise spectral density. For pilot tone CSI, we arbitrarily choose $\rho_a = \frac{1}{\sqrt{1+\sqrt{2 f_D T}}}$ which is asymptotically optimum for $r = 1$ sample per symbol [20]. For the predictor-based receivers, four curves are plotted in Figures 6 and 7, namely

- the analytic lower bound, assuming coherent QPSK.
- twice the analytic lower bound, to approximate the effect of coherent demodulation/ differential decoding.
- a simulation, with a PSP receiver providing the EMLSE-*t* second-stage with predicted CSI estimates. This arrangement is most related to the analysis.
- a simulation, with the PSP receiver performing joint channel estimation and data detection. This is the maximum likelihood arrangement of [7,11]. The analysis used in computing the first two curves attempts to bound and to approximate respectively this receiver's performance.

The simulated transmitter employed an $r = 3$ multi-sinc pulse, sending differentially encoded QPSK. The fading channel was simulated using filtered white noise, passed through a 192-tap FIR filter, with tap weights [18], $f_l = \frac{J_{\frac{1}{4}}\left(2\pi f_D T_r |l|\right)}{|lT_r|^{\frac{1}{4}}} \times$ Hanning(lT_r). The analytic curves also used these windowed taps in calculating $\mathbf{R_{zz}}$, $\mathbf{R_{ze}}$ and $\mathbf{R_{ee}}$. White noise was added to the sampled signal, obviating the need for an IF filter. The PSP receiver makes a hard decision each symbol interval (i.e. one state [11]), with $B = 10$ fixed, MMSE predictor taps. Each BER estimate is based on at least 200 bit errors.

From Figures 6 and 7, the performance of practical receivers gets progressively worse than the ideal as the fading bandwidth increases. Neglecting the penalty of power amplifier linearization, systems with pilot tones outperform MLSE predictor-based receivers that lack pilot tones. This can be clearly seen from the lower bound on the performance of predictor-based receivers. Twice the lower bound approximates the PSP receivers' performance. In slow fading, there is little difference between predictors, pilot tones and the ideal performance.

VII. Conclusions

An MLSE receiver for the time-selective channel has been derived, assuming ideal CSI and linear constellations, such as *M*-ASK, *M*-PSK or *M*-QAM. As expected, the matched filter for the time-selective channel (MF-*t*) differs from the conventional matched filter for the frequency-selective channel (MF-*f*). In the time-invariant channel, for non-zero delay spread, there is ISI which must be explicitly dealt with to avoid an error floor. There is a dual property in the time-selective channel: for non-zero Doppler, there is ISI, which must be explicitly dealt with to avoid an error floor.

A BER analysis indicates that the receiver's performance improves with faster fading, by exploiting the channel's implicit Doppler diversity. The new receiver can operate satisfactorily using non-ideal CSI with finite complexity, but it is not optimal. The performance is better when there are pilot tones for an EMLSE-*t* receiver, than when there are no pilot tones and a predictor-based receiver must be employed.

Appendix

Lodge and Moher[14] explore receivers for the flat fading channel, assuming no channel knowledge and CPM. The constant envelope of CPM signals assists their analysis. Vitetta and Taylor [11] apply predictors to PSK signals, but the receivers are maximum likelihood only for white noise and rectangular pulses of period, T. However, the signal is not close to bandlimited, IF filtering must be fairly wideband, the noise bandwidth is greater than half the sampling rate, so the noise is aliased and typically not white. Furthermore, it is difficult to relate the RF noise spectral density to the noise spectral density after IF filtering and undersampling.

The situation is clarified by considering multi-Nyquist pulses. For any Nyquist pulse, $g(t)$, a multi-Nyquist pulse,

$$h(t) = \sum_{i=0}^{r-1} g(tr - iT) \quad (39)$$

appears rectangular, if sampled at $t = lT_r$

$$h(lT_r) = \begin{cases} 1 & 0 \le l < r \\ 0 & otherwise \end{cases} \quad (40)$$

The multi-Nyquist pulse provides r ISI-free sampling points. The bandwidth of $h(t)$ is r times the bandwidth of $g(t)$, although the signal power at the band edges diminishes with increasing r. In the limit as $r \to \infty$, the multi-Nyquist pulses converge on $rect(t/T)$.

Using $g(t) = sinc(t/T)$, the one-sided Doppler-spread bandwidth of the received signal is $r/2T+f_D$. The Dam, Vitetta and Taylor[6,11] receivers take r/T samples per second, so the aliasing is small for slow fading or large r. The error floor of these receivers found for extremely fast fading and low values of r is due to this aliasing.

Undersampling folds the noise down at the band edges, doubling the noise spectral density at high frequencies. A noise whitening filter can whiten the noise, but the pulses are no longer rectangular at the sampling points. The only way to get white noise and maintain rectangular pulses for a tractable analysis is to extend the one sided IF bandwidth to $1/T$ Hz. However, the noise spectral density is doubled across the band and the total noise power is $2N_0r/T$.

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Figure 1: Time-selective channel model of the signal path.

Figure 2: Structure of the EMLSE-*t* receiver.

Figure 3: Comparison of union bounds of length 1, 2 and 3 symbol error events (u = 0, 1, 2 respectively) for a truncated pulse with a square root raised cosine spectrum and 50% excess bandwidth. Ideal CSI, $f_D T$ = 0.1 and QPSK.

Figure 4: BER comparison for QPSK and ideal CSI. Curves 1a and 1b are upper and lower bounds on the optimal EMLSE-*t* receiver for the square-root pulse. The upper, union bound considers all error events up to E = 3 symbols. Curve 2 is the exact BER for slow fading, as a reference. Curve 3 is the exact BER for a system where all pulse shaping is located at the transmitter, and the receiver only applies a zonal, minimum-bandwidth noise-limiting filter and takes one sample per symbol. The pulse has 50% excess bandwidth.

Figure 5: BER comparison of QPSK and 16-QAM, using the EMLSE-*t* receiver. The union bound only considers E = 1 error events. Ideal CSI, $f_D T$ = 0.1. The square root-Nyquist pulse is used.

Figure 6: BER comparison of ideal CSI (QPSK), pilot tone CSI (QPSK) and predictor CSI (DQPSK) for $f_D T$ = 0.01. r = 3 samples per symbol, multi-Nyquist pulse with widened IF filter. The predictor receiver used B = 10 predictor taps.

Figure 7: BER comparison of ideal CSI (QPSK), pilot tone CSI (QPSK) and predictor CSI (DQPSK) for $f_D T$ = 0.1. r = 3 samples per symbol, multi-Nyquist pulse with widened IF filter. The predictor receiver used B = 10 predictor taps.