Metastable Quivers in String Compactifications

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Abstract

We propose a scenario for dynamical supersymmetry breaking in string compactifications based on geometric engineering of quiver gauge theories. In particular we show that the runaway behavior of fractional branes at del Pezzo singularities can be stabilized by a flux superpotential in compact models. Our construction relies on homological mirror symmetry for orientifolds.

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1. Introduction

It has been observed in [1–3] that nonperturbative effects induce dynamical supersymmetry breaking in certain quiver gauge theories. These quiver gauge theories can be engineered in terms of fractional branes at Calabi-Yau threefold singularities. Therefore this setup is a natural candidate for a supersymmetry breaking mechanism in string theory.

However it has been shown in [4] (see also [2, 5, 6]) that these models give rise to runaway behavior in the Kähler moduli space when embedded in string compactifications. More precisely, in the absence of a moduli stabilization mechanism, the closed string Kähler parameters – which couple to brane world-volume actions as FI terms – can take arbitrary values. Therefore the D-flatness conditions are automatically satisfied, and one does not obtain a metastable non-supersymmetric vacuum. This problem is present in the F-theory models developed in [5], in which case the Kähler moduli should be stabilized by Euclidean D3-brane instanton effects [7] by analogy with [8–13]. In the present context such instantons develop extra zero modes as a result of their interaction with the fractional branes, which can in principle lead to cancellations in the effective superpotential [14]. Therefore one cannot rule out the existence of runaway directions in the Kähler moduli space without a more thorough analysis. Some progress in this direction has been recently made in [15–17] (see also [18] for a discussion of D-brane instanton effects in IIA models.)

In this paper we propose an alternative embedding scenario of supersymmetry breaking quivers in string compactifications. The starting point of our construction is the observation that quiver gauge theories typically occur in nongeometric phases in the Kähler moduli space. These are regions of the $N = 2$ Kähler moduli space where the quantum volumes of certain holomorphic cycles become of the order of the string scale. As a result, the dynamics of IIB $N = 1$ orientifold models is very hard to control in this regime, since the supergravity approximation is not valid.

It has been long known that nongeometric phases in $N = 2$ compactifications become more tractable in the mirror description of the models. Mirror symmetry identifies the Kähler moduli space of a Calabi-Yau threefold X with the complex structure moduli space of the mirror threefold Y. The nongeometric phases of X are mapped to certain regions in the complex structure moduli space of Y. One can maintain at the same time the volume of the mirror threefold Y large, obtaining therefore a large radius compactification, where the supergravity approximation is valid. This idea has been implemented in the context of $N = 1$ string vacua in [19].

Following the same strategy, in section two we propose a construction of supersymmetry breaking quivers in terms of D6-brane configurations in IIA Calabi-Yau orientifolds. This construction relies on homological mirror symmetry for orientifold models, but does not require an extension of mirror symmetry to nonzero flux. Note that a different dynamical supersymmetry breaking mechanism in IIA toroidal orientifolds has been proposed in [20].

In addition to solving the problem of small quantum volumes, this scenario also offers a natural moduli stabilization superpotential induced by background fluxes. In particular we will show in section three that the potential runaway directions in the moduli space can be very efficiently stabilized by turning on IIA NS-NS flux. The IIA vacuum structure in the presence of fluxes has been previously investigated in [21–31]. Note that in this picture

closed string moduli stabilization occurs at a much higher scale than the typical quiver gauge theory energy scale. The low energy effective dynamics of the system reduces to open string dynamics in a fixed closed string background, and is dominated by strong infrared effects in the Yang-Mills theory. A similar hierarchy of scales occurs in the construction of de Sitter vacua in string theory proposed in [10].

In section four we construct concrete compact models satisfying the general conditions formulated in section three. We exhibit a landscape of vacua equipped with supersymmetry breaking quivers in a specific example, which is very similar to the ones discussed in section 6 of [5]. An interesting problem for future work is whether the present construction of supersymmetry breaking vacua is compatible with the fractional brane realization of the MSSM [32, 33]. In particular, it would be very interesting to find stabilized string vacua which can accommodate both supersymmetry breaking quivers and MSSM fractional brane configurations. This would be a concrete realization of the ideas outlined in [5].

2. Supersymmetry Breaking and del Pezzo Surfaces in String Compactifications

In this section we review the construction of supersymmetry breaking quivers in terms of fractional branes at del Pezzo singularities, and propose an embedding strategy in IIA string compactifications. Since the gauge theory dynamics has been thoroughly analyzed in [1–3], we will focus only on the relevant string theory aspects.

2.1. Fractional Branes and Supersymmetry Breaking Quivers

The first examples of supersymmetry breaking quivers [1–3] were realized in terms of fractional D5-branes at a dP_1 singularity. Recall that the first del Pezzo surface dP_1 is a one point blow-up of the projective plane $\mathbb{P}^2$. The Picard group of dP_1 is generated by the hyperplane class h and the class of the exceptional divisor e. Under Calabi-Yau/Landau-Ginzburg correspondence, the fractional branes at a dP_1 singularity are in one to one correspondence with the following collection of bundles[1] on dP_1 (see for example [34])

$$E_1 = \mathcal{O}_{dP_1}, \qquad E_2 = \overline{\mathcal{O}_{dP_1}(h-e)}, \qquad E_3 = \overline{\mathcal{O}_{dP_1}(e)}, \qquad E_4 = \mathcal{O}_{dP_1}(h). \qquad (2.1)$$

The quiver gauge theory is engineered by a brane configuration of the form

$$E_1^{\oplus(N+M)} \oplus E_2^{\oplus(N+3M)} \oplus E_3^{\oplus N} \oplus E_4^{\oplus(N+2M)} \qquad (2.2)$$

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[1]Here we denote by $\overline{E}$ the anti-brane of a D-brane with Chan-Paton bundle E. This notation is not quite rigorous from the point of view of the derived category, but it will suffice for our purposes.

where N, M are positive integers. This gives rise to the quiver represented in fig. 1. In terms
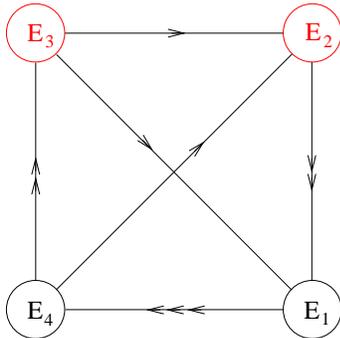

Figure 1: Quiver gauge theory associated to a dP_1 singularity.

of large radius charges, one can show that this configuration has N units of D3-brane charge and M units of D5-brane charge wrapped on a two-cycle Γ orthogonal to the canonical class K_{dP_1}. The net D7-brane charge of the configuration (2.2) is zero.

For the purpose of embedding in string compactifications it is important to note that the quiver theory in fig. 1 can be alternatively engineered using other dP_k singularities with various values of $1 \leq k \leq 8$. For example according to section 4.1 of [5], one can obtain the same quiver gauge theory from fractional branes at dP_8 or dP_5 singularities. Similar constructions can be carried out for dP_6 and dP_7 singularities as well. We will not need the details of these constructions in the following. The essential point for us is that in each case the quiver gauge theory will be engineered by a collection of fractional branes of the form

$$\oplus_{a=1}^{s} E_a^{\oplus N_a} \tag{2.3}$$

where $E_1, \ldots, E_s$ are exceptional bundles on the del Pezzo surface, and N_a are positive integers. The bundles E_a are stable and have no infinitesimal deformations (they are usually called spherical objects in derived category language.)

2.2.   Strategy For Embedding in String Compactifications

Let us now present our strategy for embedding the local del Pezzo models in string compactifications. As explained in the introduction, we will construct IIA orientifold models related by mirror symmetry to IIB compactifications with fractional branes at del Pezzo singularities.

We start with a few preliminary remarks on del Pezzo singularities in IIB Calabi-Yau compactifications. Suppose we have a Calabi-Yau threefold X and a contraction map

$$c : X \to \widehat{X}$$

which collapses a smooth del Pezzo surface S to a singular point of $\widehat{X}$. Moreover, suppose that this degeneration occurs on a codimension k wall in the Kähler cone of X. This means that we have to tune k Kähler moduli of X in order to reach the singularity i.e. the image of the restriction map

$$H^{1,1}(X) \to H^{1,1}(S)$$

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has rank k. Note that in local models X is isomorphic to the total space of the canonical bundle $K_S \to S$, and $k = h^{1,1}(S)$.

As explained earlier in this section, the fractional branes associated to a del Pezzo singularity are related by analytic continuation to an exceptional collection $E_1, \ldots, E_s$ on the collapsing del Pezzo surface. The quiver gauge theory occurs along a subspace of the Kähler moduli space where the $N = 2$ central charges of the branes $E_1, \ldots, E_s$ are aligned. We will refer to this subspace as the quiver subspace, or the quiver locus, in the following. A detailed analysis of this locus in a local del Pezzo model can be found in [35]. We will perform a similar computation in a compact model in section four.

Note that the central charges are aligned, but nonzero, along the quiver locus, therefore the underlying $N = 2$ SCFT is not singular. In other words there are no massless states or tensionless extended objects associated to the collapsing del Pezzo surface since the quantum volumes of the stable branes wrapping various cycles therein do not vanish. From this point of view the quiver subspace is analogous to a Landau-Ginzburg phase, as opposed to a conifold singularity. It follows however that world-sheet instanton corrections are important near the quiver locus of the moduli space, and the supergravity approximation is not valid. This can be a serious drawback in the context of $N = 1$ compactifications, especially compactifications with background flux since the dynamics is under control only in the large radius regime.

This problem can be avoided employing the strategy developed in [19]. Namely, using homological mirror symmetry, we map the IIB fractional brane configuration to a configuration of D6-branes wrapping special lagrangian cycles in a IIA compactification on the mirror threefold Y. The dynamics can be kept under control by working near the large complex limit point in the complex structure moduli space of X, which corresponds to the large radius limit in the Kähler moduli space of Y. We stress that this statement relies only on the standard homological mirror symmetry conjecture, and eventually its extension to orientifold models. We do not need to invoke any form of mirror symmetry in the presence background flux, which would be a more delicate issue. As in [19], the type IIB description of the theory is mainly used here as a convenient mathematical tool which makes certain geometric aspects more transparent. Once we have established the existence of a suitable IIA D6-brane configuration in a large volume compactification, the IIB theory ceases to play any role. Flux dynamics and moduli stabilization will be discussed only in terms of the IIA model, in which the supergravity approximation is valid. In principle one could formulate the whole discussion in IIA variables from the beginning, but then it would be more difficult to establish the existence of a Calabi-Yau threefold Y with the desired properties.

Homological mirror symmetry predicts the existence of a collection $(M_1, V_1), \ldots, (M_s, V_s)$ of special lagrangian three-cycles[2] $M_1, \ldots, M_s$ equipped with flat unitary bundles V_a corre-

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[2]In principle one may wonder if the mirror **A**-branes of the del Pezzo fractional branes may be stable coisotropic branes [36, 37] rather than special lagrangian three-cycles. The role of such objects in homological mirror symmetry and their phenomenological implications have been discussed in [36–38] and [39] respectively. Although we cannot rigorously rule out this possibility, we would like to point out that it is rather unlikely for del Pezzo fractional branes. For local models it was found for example in [40] that del Pezzo fractional branes are indeed related to special lagrangian three-cycles by mirror symmetry. Since local models are obtained in fact from compact ones by taking a certain (analytic) limit [41–43], the same conclusion will conjecturally hold in compact models as well. We thank S. Kachru for a discussion on these issues.

sponding to the exceptional objects $E_1, \ldots, E_s$. Since the later are spherical objects in the derived category of X, $M_1, \ldots, M_s$ must be lagrangian three-spheres in Y. In particular the bundles V_a will be topologically trivial. Moreover, since the fractional branes are indecomposable objects in the derived category of X, the bundles V_a must have rank 1. Therefore each object E_a corresponds to a single D6-brane wrapping a special lagrangian three-sphere in Y. The central charge of such a brane is

$$Z_a = \int_{M_a} \Omega_Y. \tag{2.4}$$

As explained above, the quiver locus is a special subspace in the complex structure moduli space of Y where all Z_a are aligned.

Since X, Y are compact threefolds, the model should also include an orientifold projection in order for tadpole cancellation conditions to be satisfied. We will impose a IIB orientifold projection of the form

$$(-1)^{F_L} \Omega \sigma \tag{2.5}$$

where $\sigma : X \to X$ is a holomorphic involution so that the fixed point set X^σ consists of zero and two complex dimensional components. This defines a mixed O3/O7 IIB orientifold model. We will also require several additional conditions to be satisfied.

(i) The threefold X contains a pair (S, S') of disjoint del Pezzo surfaces which do not intersect the fixed locus X^σ so that $\sigma : S \to S'$ is an isomorphism. We will refer to such a pair as conjugated del Pezzo surfaces. Although most considerations in this paper hold for arbitrary del Pezzo surfaces S, S' we will restrict ourselves to models in which $S, S' \simeq dP_n$, $n = 6, 7, 8$. This restriction will facilitate the analysis of the critical locus of the flux superpotential in the next section.

(ii) The holomorphic involution should be compatible with the large complex structure limit in the complex structure moduli space of X. This means we should be able to find a family of threefolds X equipped with involutions σ passing arbitrarily close to the large complex structure limit point. This condition is needed in order for the mirror IIA theory to be a large radius compactification.

Assuming these conditions satisfied, orientifold mirror symmetry [44–47] implies that a pair (X, σ) is related to a pair (Y, η) where $\eta : Y \to Y$ is an anti-holomorphic involution of Y. The D6-brane configuration (M_a, V_a) is mapped to a conjugate configuration (M'_a, V'_a) so that $\eta : M_a \to M'_a$ is an isomorphism and $\eta^*(V'_a) = V_a$. If the complex structure of X is near the large complex structure point, Y will determine a large volume IIA compactification. This is the desired embedding of del Pezzo quiver theories in string compactifications. Next we address the issue of moduli stabilization.

3. IIA moduli stabilization

In this section we analyze the effective four dimensional supergravity theory of the IIA compactification using the formalism of [47]. Our goal is to argue for the existence of

stabilized supersymmetric IIA vacua so that the complex structure moduli of Y are fixed on the quiver locus.

The Kähler moduli of Y can be stabilized by turning on generic RR flux $F = F_0 + F_2 + F_4 + F_6$ on X. This gives rise to a superpotential of the form [48, 49]

$$W_K = \int_Y F \wedge e^{-J_Y} \tag{3.1}$$

where J_Y is the Kähler class of Y. For generic fluxes, this superpotential depends on all Kähler moduli, therefore we expect all flat directions in the Kähler moduli space to be lifted.

Complex moduli stabilization is a more complicated problem in the present situation. In order to fix the complex structure moduli of Y we have to turn on the IIA NS-NS flux H [21–31, 47]. In the presence of D6-branes the flux is constrained however by the Freed-Witten anomaly cancellation condition [50], which states that

$$\int_M H = 0 \tag{3.2}$$

for any three-cycle M supporting a D6-brane. This can in principle hinder moduli stabilization by disallowing a sufficiently generic H flux. This was previously observed in the context of IIA flux vacua in [29], and a supergravity interpretation of this constraint has been developed in [51]. In particular, we have to enforce condition (3.2) for all cycles $M \subset Y$ supporting D6-branes related by mirror symmetry to fractional branes.

In order to address this problem we need a detailed description of the effective supergravity action of IIA orientifolds, which has been given in [47]. Following [47], let us choose a symplectic basis of three-cycles $(\alpha_i, \alpha_\lambda; \beta^i, \beta^\lambda)$ on Y so that $(\alpha_i, \beta^\lambda)$ span $H^3_+(Y)$ and $(\alpha_\lambda, \beta^i)$ span $H^3_-(Y)$. The indices (i, λ) take values $i = 0, \ldots, I$ and $\lambda = I + 1, \ldots, h^{1,2}$ for some integer I, $0 \leq I \leq h^{1,2}_+$. The only nontrivial intersection numbers are

$$\int_Y \alpha_i \wedge \beta^j = \delta_i^j \qquad \int_Y \alpha_\rho \wedge \beta^\lambda = \delta_\rho^\lambda. \tag{3.3}$$

In terms of this basis of cycles, the holomorphic three-form ω_Y has the following expansion

$$\Omega_Y = Z^i \alpha_i + i Z^\lambda \alpha_\lambda - \mathcal{F}_\lambda \beta^\lambda - i \mathcal{F}_i \beta^i. \tag{3.4}$$

The orientifold projection imposes the following constraints on the periods

$$\mathrm{Im}(C Z^i) = \mathrm{Re}(C \mathcal{F}_i) = 0 \qquad \mathrm{Re}(C Z^\lambda) = \mathrm{Im}(C \mathcal{F}_\lambda) = 0 \tag{3.5}$$

where C is the compensator field introduced in [47]. For completeness recall that

$$C = e^{-\Phi - i\theta} e^{K^{cs}/2}$$

where Φ is the four-dimensional dilaton, $e^{i\theta}$ is a phase defined by

$$\eta^* \Omega_Y = e^{2i\theta} \overline{\Omega}_Y$$

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and K^{cs} is the Kähler potential of the underlying $N = 2$ theory evaluated on the invariant subspace under the orientifold involution.

In the spirit of homological mirror symmetry, we can choose the symplectic basis so that Z^0 is the $N = 2$ central charge of a point-like brane on X and (Z^i, Z^λ), $i > 0$, are central charges of branes wrapping holomorphic curves in X. $\mathcal{F}^0$ can be similarly taken to be the central charge of a D6-brane wrapping X and $(\mathcal{F}_i, \mathcal{F}_\lambda)$ can be identified with central charges of branes wrapping holomorphic divisors in X. Note that there must be a correlation between the transformation properties of holomorphic branes under the holomorphic involution $\sigma : X \to X$ and the reality properties (3.5) of the corresponding periods. Invariant brane configurations should correspond to real periods while anti-invariant brane configurations should correspond to purely imaginary periods. In particular, since the IIB orientifold preserves point-like branes on X, the fundamental period at the IIA large complex structure limit must be real with respect the anti-holomorphic involution $\eta : Y \to Y$. This implies that the phase $e^{i\theta}$ must equal 1 in such models. Then equations (3.5) reduce to

$$\text{Im}(Z^i) = \text{Re}(\mathcal{F}_i) = 0 \qquad \text{Re}(Z^\lambda) = \text{Im}(\mathcal{F}_\lambda) = 0. \tag{3.6}$$

The holomorphic coordinates on the $N = 1$ complex structure moduli space are defined by the periods

$$N^i = \frac{1}{2} \int_Y \Omega_Y^c \wedge \beta^i$$
$$T_\lambda = i \int_Y \Omega_Y^c \wedge \alpha_\lambda \tag{3.7}$$

where Ω_Y^c is a linear combination of the RR three-form field $C^{(3)}$ and the (real part of) holomorphic three-form $C\Omega_Y$

$$\Omega_Y^c = C^{(3)} + 2i\text{Re}(C\Omega_Y).$$

Note that the periods (3.7) actually parameterize an $h^{1,2}(X)+1$ moduli space, which includes the expectation value of the dilaton field.

We can make a more specific choice of symplectic basis if we impose additional constraints on the model. More specifically we will add the following two conditions to the list below equation (2.5)

(iii) The natural push-forward maps

$$H_2(S) \to H_2(X) \qquad H_2(S') \to H_2(X) \tag{3.8}$$

have rank one.

(iv) The anti-invariant subspace $H_-^{1,1}(X)$ is one dimensional and spanned by the difference $S - S'$ between the divisor classes of the conjugate del Pezzo surfaces S, S'.

Note that condition (iii) can be reformulated as

(iii') The natural restriction maps

$$H^2(X) \to H^2(S) \qquad H^2(X) \to H^2(S') \tag{3.9}$$

have rank one.

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Since the second homology of a del Pezzo surface is generated by effective curves, condition (iii) implies that we can pick two effective curve classes Σ, Σ' on X which generate the images of the maps (3.8). Moreover, these curve classes can be chosen so that

$$
\begin{aligned}
\Sigma \cdot S &= -1 & \Sigma \cdot S' &= 0 \\
\Sigma' \cdot S &= 0 & \Sigma' \cdot S &= -1
\end{aligned}
\tag{3.10}
$$

Note that we have the following triple intersection numbers in X

$$
S^3 = (S')^3 = 9 - n
\tag{3.11}
$$

for $S, S' \simeq dP_n$. Recall that according to condition (i) below (2.5) n takes values $6, 7, 8$ in our models. Therefore the self-intersections S^2 and $(S')^2$ must be nontrivial curve classes on X which have nonzero intersection numbers with S, S' respectively. Then we can take Σ, Σ' so that

$$
S^2 = (n-9)\Sigma \qquad (S')^2 = (n-9)\Sigma'.
\tag{3.12}
$$

Using condition (iv), we can also choose a system of generators $\{J_A\}$ of the Kähler cone of X so that $J_1 = S - S'$ generates $H^{1,1}_-(X)$ and J_A, $A = 2, \dots, h^{1,1}(X)$ generate $H^{1,1}_+(X)$. Moreover, we can make this choice so that

$$
\begin{aligned}
(\Sigma - \Sigma') \cdot J_1 &= 1, & (\Sigma - \Sigma') \cdot J_A &= 0, & A &= 2, \dots, h^{1,1}(X) \\
(\Sigma + \Sigma') \cdot J_2 &= 1, & (\Sigma + \Sigma') \cdot J_A &= 0, & A &= 1, \dots, h^{1,1}(X), \ A \neq 2.
\end{aligned}
\tag{3.13}
$$

This follows from (3.10) observing that the restriction of any divisor class D on X to S and S' respectively must be a multiple of the canonical classes K_S, $K_{S'}$.

Another consequence of condition (iv) is that the linear space spanned by purely imaginary periods of the form $\mathcal{F}_i$ is two dimensional, i.e. $I = 1$ and the index i takes only two values $i = 0, 1$. We can choose the generators $\mathcal{F}_0, \mathcal{F}_1$ to be the period corresponding to a D6-brane wrapping X and the period corresponding to the anti-invariant divisor class $J^1 = S - S'$. We can also choose the remaining basis elements so that the real periods $\mathcal{F}_\lambda$, $\lambda = 1, \dots, h^{1,2}(Y)$ correspond to the divisor classes J^A, $A = 2, \dots, h^{1,1}(X)$. In particular, $\mathcal{F}_2$ corresponds to the Kähler cone generator J^2 singled out above equation (3.13).

Then, according to the orientifold constraints (3.6), the dual periods Z^i, $i = 0, 1$, must be real and the remaining periods Z^λ must be imaginary. We take Z^0 to be the period associated to a point-like brane on X. Taking into account the intersection numbers (3.13), we can choose the basis of cycles so that Z^1, Z^2 are associated to the curve classes $\Sigma - \Sigma'$, $\Sigma + \Sigma'$. This is consistent with the fact that the O3 orientifold projection maps a D5-brane supported on Σ to an anti-D5-brane supported on Σ'.

As discussed in the previous section, the type IIB fractional associated to a collapsing del Pezzo surface in X are related by mirror symmetry to a collection of D6-branes wrapping special lagrangian cycles in Y. The IIB fractional branes associated to the del Pezzo surface S carry D3, D5 and D7 charges. We have to work out the corresponding D6-brane charges on the mirror threefold Y in terms of the basis of cycles $(\alpha_i, \alpha_\lambda; \beta^i, \beta^\lambda)$.

Note that in the compact model we have two fractional brane configurations mapped to each other by the orientifold projection. Suppose we have a fractional brane E_a on S with topological charges

$$
r_a S + n_a \Sigma + m_a \omega,
\tag{3.14}
$$

where ω is the class of a point on X. The orientifold projection will map it to a fractional brane E'_a supported on S' with charge vector

$$r'_a S' + n'_a \Sigma' + m'_a \omega \tag{3.15}$$

where

$$r'_a = r_a \qquad n'_a = -n_a \qquad m'_a = m_a.$$

Since the del Pezzo surfaces S, S' are conjugated under the holomorphic involution, $S + S'$ is an invariant divisor class on X. Therefore it corresponds by mirror symmetry to an invariant period $\mathcal{F}_+$ which can be written as linear combination

$$\mathcal{F}_+ = \sum_{\lambda=2}^{h^{1,2}(Y)} s^\lambda \mathcal{F}_\lambda \tag{3.16}$$

with integral coefficients. Since the anti-invariant combination $S - S'$ is related by mirror symmetry to $\mathcal{F}^1$, and Z^1, Z^2 are related to the curve classes $\Sigma \pm \Sigma'$, we find

$$\begin{aligned} Z(E_a) &= \frac{1}{2} r_a (\mathcal{F}_+ + \mathcal{F}^1) + \frac{1}{2} n_a (Z^2 + Z^1) + m_a Z^0 \\ Z(E'_a) &= \frac{1}{2} r_a (\mathcal{F}_+ - \mathcal{F}^1) - \frac{1}{2} n_a (Z^2 - Z^1) + m_a Z^0 \end{aligned} \tag{3.17}$$

Note that the linear combinations (3.17) have half integral coefficients because the basis of cycles $(\alpha_i, \alpha_\lambda; \beta^i, \beta^\lambda)$ we have constructed above generates $H^3(Y)$ over $\mathbb{Q}$, but not over $\mathbb{Z}$. In fact in the models considered here it is impossible to find a basis over $\mathbb{Z}$ consisting of invariant and anti-invariant cycles.

Equations (3.17) determine the charges of the D6-brane configurations on Y related by mirror symmetry to the fractional branes (E_a, E'_a). We have

$$\begin{aligned} M_a &= \frac{1}{2} r_a (s^\lambda \alpha_\lambda + \alpha_1) + \frac{1}{2} n_a (\beta^2 + \beta^1) + m_a \beta^0 \\ M'_a &= \frac{1}{2} r_a (s^\lambda \alpha_\lambda - \alpha_1) - \frac{1}{2} n_a (\beta^2 - \beta^1) + m_a \beta^0 \end{aligned} \tag{3.18}$$

We will be interested in fractional brane configurations of the form $\oplus_{a=1}^s E_a^{\oplus N_a}$ which give rise to supersymmetry breaking quivers. Note that the total D7-charge of such a configuration vanishes

$$\sum_a N_a r_a = 0. \tag{3.19}$$

The total D5-brane charge is represented by a curve class Γ on S which is orthogonal to the canonical class K_S i.e.

$$(\Gamma \cdot K_S)_S = 0. \tag{3.20}$$

Condition (iii) above implies then that Γ must be contained in the kernel of the pushforward map $H_2(S) \to H_2(X)$, hence Γ is homologically trivial in X, and the total D5-brane charge vanishes as well. This is also valid for the conjugate fractional brane configuration supported

on S'. The total D3-brane may be nonzero, depending on the choice of fractional brane configuration. More precisely, the total D3-brane charge for the supersymmetry breaking quiver represented in fig. 1 is an integer N, which is constrained by tadpole cancellation and supersymmetry. The total D3-brane charge, that is the charge N of the fractional branes plus the charge N_{D3} of additional D3-branes on X must equal the absolute value $|N_{O3}|$ of the total charge of the O3-planes. Moreover, we do not allow anti-D3-branes on X in order to preserve tree level supersymmetry. In addition we may also have D7-branes wrapping the divisors in X if the holomorphic involution σ has codimension one fixed loci.

Note that this situation is different from the D-brane landscape constructed in [19]. The model constructed in [19] included a D5 anti-D5 pair wrapping conjugate curves in a Calabi-Yau threefold. This eventually led to tree level supersymmetry breaking while in the present case supersymmetry breaking occurs as a result of strong infrared dynamics at a much lower scale.

In IIA compactifications, complex structure moduli can be stabilized by turning on NS-NS flux. The most general flux compatible with the orientifold projection is given by [47]

$$H = q^\lambda \alpha_\lambda - p_k \beta^k \tag{3.21}$$

where q^λ, p_k are integers. According to [47], the flux superpotential is

$$W_H = -2N^k p_k - iT_\lambda q^\lambda. \tag{3.22}$$

Such a superpotential would generically fix all complex structure moduli and the dilaton.

However in the presence of branes the flux (3.21) is subject to the Freed-Witten anomaly cancellation condition (3.2). In our situation, the three-cycles supporting fractional brane charges can be read off from equations (3.18). We obtain the following constraints

$$\int_X H \wedge \gamma = 0, \qquad \gamma = \beta^0, \beta^1, \beta^2, \alpha_1, s^\lambda \alpha_\lambda. \tag{3.23}$$

Using the intersection numbers (3.3), we find that the conditions (3.23) are equivalent to

$$p_1 = q^2 = 0. \tag{3.24}$$

Therefore the superpotential (3.22) does not depend on the moduli fields N^1, T_2, but it has nontrivial dependence on all the other complex structure moduli and the dilaton.

In principle, we can have additional constraints corresponding to background D3 and D7-branes on X supported away from the del Pezzo surfaces S, S'. Any D3-brane on X is mapped by mirror symmetry to a D6-brane with charge $\beta^0 \in H^3(Y)$. Therefore the corresponding Freed-Witten constraint has already been taken into account in (3.24). A D7-brane wrapping an invariant divisor D is mapped by mirror symmetry to D6-brane wrapping a special lagrangian cycle M in Y so that $b_1(M) = h^{0,2}(D)$. Since D is invariant under the holomorphic involution, the class of the mirror cycle M will be a linear combination of basis elements of the form

$$M = \sum_{\lambda=2}^{h^{1,2}(Y)} d^\lambda \alpha_\lambda. \tag{3.25}$$

10

Taking into account the intersection numbers (3.3) and equation (3.21), it follows that the presence of D6-branes wrapping M does not yield additional Freed-Witten constraints on the background flux H.

One may worry however about open string moduli corresponding to normal deformations of the special lagrangian cycle M in Y. General supersymmetry considerations suggest that these moduli should be lifted in the presence of background flux. In the present case, this effect has been confirmed by a detailed mathematical analysis in [52]. Similar results in IIB theory and M-theory include [53–55]. In fact, as long as the cycle M is not very close to the cycles M_a supporting the fractional D6-branes, there will be no low energy couplings between these brane configurations. Therefore dynamical supersymmetry breaking on the fractional D6-branes will not be affected by the presence of open string moduli on the second stack of D6-branes.

Summarizing the above discussion, the total superpotential for the closed string moduli reads

$$W = W_F + W_H \tag{3.26}$$

where W_F is given by (3.1) and

$$W_H = -2N^0 p_0 - i \sum_{\lambda=3}^{h^{1,2}} T_\lambda q^\lambda. \tag{3.27}$$

This yields the following supergravity F-flatness equations

$$
\begin{aligned}
D_{t^\alpha} W = 0 \qquad & D_{N^0} W = 0 \qquad & \partial_{N^1} \mathcal{K} = 0 \\
& \partial_{T_2} \mathcal{K} = 0 \qquad & D_{T_\lambda} W = 0 \qquad \lambda \geq 3.
\end{aligned}
\tag{3.28}
$$

where $\alpha = 1, \ldots, h^{1,1}(Y)$, t^α are holomorphic coordinates on the Kähler moduli space, and $\mathcal{K}$ denotes the Kähler potential for the complex structure moduli space.

The number of equations in (3.28) equals the number of variables, hence one may be tempted to naively conclude that this system will generically have isolated solutions in the moduli space. However this conclusion would not be reliable without a more detailed analysis. A typical problem in such situations is that a nongeneric superpotential can leave some flat directions unlifted in the moduli space. This is well known for IIB flux compactifications [10], which exhibit a no-scale Kähler potential on the $\mathcal{N} = 1$ Kähler moduli space, in the absence of instanton corrections to the superpotential.

In our case the F-flatness equations

$$\partial_{N^1} \mathcal{K} = 0 \qquad \partial_{T_2} \mathcal{K} = 0 \tag{3.29}$$

may cause in principle a similar problem due to the absence of superpotential terms. In order to address this question, we will rewrite these two equations using some identities proved in [47]. Equation (B.9) in [47] yields

$$\partial_{T^2} \mathcal{K} = -2e^{2\Phi} \mathrm{Im}(C Z^2), \tag{3.30}$$

where Φ is the four dimensional dilaton. Moreover, combining equations (B.9) and (C.12) in [47], we obtain

$$\partial_{N^1} \mathcal{K} = 8e^{2\Phi} \mathrm{Im}(C \mathcal{F}_1). \tag{3.31}$$

11

Since in our case C is real, the flatness equations (3.29) reduce to

$$\mathrm{Im}(Z^2) = 0 \qquad \mathrm{Im}(\mathcal{F}_1) = 0. \tag{3.32}$$

These are the defining equations of the intersection locus between the totally real subspace of the $N = 2$ moduli space fixed by the anti-holomorphic involution and the subspace

$$Z^2 = 0 \qquad \mathcal{F}_1 = 0. \tag{3.33}$$

We claim that the locus cut out by equations (3.33) and the quiver locus intersect at least along a codimension two subspace of the complex structure moduli space. Note that this is a very important feature of the F-flatness equations (3.28), since it shows that low energy dynamical supersymmetry breaking is generic in these models. If the subspace cut by equations (3.33) intersected the quiver locus along a higher codimension locus, dynamical supersymmetry breaking would be non-generic. In that case, one would have to fine tune the flux parameters in order to obtain flux vacua endowed with supersymmetry breaking quivers. Moreover, such a fine tuning may not be ultimately possible since the flux parameters are discrete.

The claim made below equation (3.33) follows from certain universal properties of contractible del Pezzo surfaces in $\mathcal{N} = 2$ compactifications. Recall that we have restricted ourselves to del Pezzo surfaces $S, S' \simeq dP_n$, $n = 6, 7, 8$ embedded in the Calabi-Yau threefold X so that the restriction maps $H^2(X) \to H^2(S)$, $H^2(X) \to H^2(S')$ have rank one. Then the properties of del Pezzo fractional branes – in particular the relations between their central charges – are captured by a universal one parameter local model derived in [41, 42, 56]. The complete solutions of these models for $n = 6, 7, 8$, including analytic continuation to the quiver loci, have been worked out in [57]. Based on the results obtained there, and the universal character of local models, we conjecture the following properties of the compact models.

(a) The quiver locus contains the codimension two subspace of the moduli space where the quantum volumes of the curves Σ, Σ' vanish[3].

(b) The central charges of all fractional branes supported on a collapsing del Pezzo surface are equal to the central charge of a pointlike brane along the locus specified at (a).

We will confirm properties (a), (b) by direct computations for a concrete compact model in the next section.

Note that the subspace where the quantum volumes of Σ, Σ' vanish can be equivalently characterized by the equations

$$Z^1 = 0 \qquad Z^2 = 0 \tag{3.34}$$

in the symplectic basis of periods adapted to the orientifold projection. Moreover property (b) above implies that the central charge of a D-brane wrapping S and the central charge of

---

[3]This condition may seem at odds with our previous claim that the SCFT is nonsingular along the quiver locus. In fact there is no contradiction here since although the central charges of Σ, Σ' vanish, there are no stable massless states along this subspace of the moduli space. The stable supersymmetric D-branes wrapping Σ, Σ' undergo decay into fractional branes along a marginal stability wall which separates the LCS point from the quiver locus [58].

a D-brane wrapping S' will be both equal to the central charge of a pointlike brane on X along the subspace (3.34). By construction, the period $\mathcal{F}_1$ can be written as the difference

$$\mathcal{F}_1 = Z_S - Z_{S'} \tag{3.35}$$

between the $N = 2$ central charges of a D-brane wrapping S and a D-brane wrapping S'. Therefore $\mathcal{F}_1$ vanishes identically along the subspace (3.34). This concludes our argument.

4. Concrete Examples

In this section we construct examples of Calabi-Yau threefolds X satisfying conditions $(i) - (iv)$ imposed in the previous section and check the properties $(a), (b)$ of the quiver locus in a specific model.

4.1. An Elliptic Fibration

Consider a smooth Weierstrass model $\widetilde{X}$ over the del Pezzo surface $B = dP_2$, which is the two-point blow-up of the projective plane $\mathbb{P}^2$. Let p and p' denote the centers of the blow-ups on $\mathbb{P}^2$, and let e and e' denote the exceptional curves. If the fibration is generic, the restriction of the elliptic fibration to any (-1) curve C on B is isomorphic to a rational elliptic surface with 12 I_1 fibers, usually denoted by dP_9. Therefore, taking C to be each of the two exceptional curves, we obtain two rational elliptic surfaces denoted by D and D'. The exceptional curves e, e' can be naturally identified with sections of the rational elliptic surfaces D, D'. We can also naturally regard them as $(-1, -1)$ curves on $\widetilde{X}$, by embedding B in $\widetilde{X}$ via the section of the Weierstrass model.

Now, we can perform a flop on the $(-1, -1)$ curves e, e' in $\widetilde{X}$, obtaining an elliptic fibration $\pi : X \to \mathbb{P}^2$ with two complex dimensional components in the fiber. More precisely, the fibers over the points p, p' have two components: a rational $(-1, -1)$ curve obtained by flopping one of the curves e, e', denoted by C and C' respectively, and a dP_8 del Pezzo surface. We will denote the dP_8 components by S and S'. It is then possible to contract the del Pezzo surfaces in the fiber, obtaining a singular elliptic fibration $\widehat{X}$ over $\mathbb{P}^2$, which can be eventually smoothed out by complex structure deformations [59].

The Calabi-Yau X has Hodge numbers $(h^{1,1}(X), h^{2,1}(X)) = (4, 214)$. The Mori cone of X is given by:

$$
\begin{array}{c|cccccccc}
 & W_1 & W_2 & W_3 & W_4 & W_5 & Z & X & Y \\
\hline
\Sigma : & 1 & 0 & 1 & -1 & 0 & 0 & 2 & 3 \\
\Sigma' : & 0 & 1 & 1 & 0 & -1 & 0 & 2 & 3 \\
C : & -1 & 0 & -1 & 1 & 0 & 1 & 0 & 0 \\
C' : & 0 & -1 & -1 & 0 & 1 & 1 & 0 & 0 \\
h : & 1 & 1 & 1 & 0 & 0 & -3 & 0 & 0,
\end{array}
\tag{4.1}
$$

where W_4 and W_5 are the rays in the toric fan of the ambient space which correspond to the del Pezzo dP_8 surfaces S and S' respectively. h denotes the hyperplane class of $\mathbb{P}^2$ and Σ, Σ' are the generators of the images of the restriction maps (3.9). The disallowed locus is given by

$$\{W_3 = W_4 = 0\} \cup \{W_2 = W_5 = 0\} \cup \{W_4 = W_5 = 0\} \cup \{W_1 = W_2 = W_3 = 0\}. \tag{4.2}$$

13

Note that the Mori cone is not simplicial and we have to chose a simplicial subcone in order to write down the mirror Picard-Fuchs equations. This choice is not essential, since it corresponds to a choice of large complex limit coordinates on the moduli space. In the following we will work with the simplicial subcone generated by (Σ, Σ', C, h).

The IIB orientifold is defined by the following involution:

$$\sigma : (W_1, W_2, W_3, W_4, W_5, Z, X, Y) \rightarrow (W_1, W_3, W_2, W_5, W_4, Z, X, Y), \tag{4.3}$$

which does preserve the large complex structure limit. Taking into account the toric data (4.1), the fixed point set equations are given by

$$
\begin{aligned}
&W_1 = \lambda_1 \lambda_2 \lambda_3^{-1} \lambda_4 W_1, \quad W_2 = \lambda_1 \lambda_3^{-1} \lambda_4 W_3, \quad W_3 = \lambda_2 \lambda_4 W_2, \\
&W_4 = \lambda_1^{-1} \lambda_3 W_5, \qquad W_5 = \lambda_2^{-1} W_4, \qquad Z = \lambda_3 \lambda_4^{-3} Z, \\
&X = \lambda_1^2 \lambda_2^2 X, \qquad\quad Y = \lambda_1^3 \lambda_2^3 Y,
\end{aligned}
\tag{4.4}
$$

where $\lambda_i \in \mathbb{C}^*$, $i = 1, \ldots, 4$. The second and third equations in (4.4) yield

$$W_2 W_3 = \lambda_1 \lambda_2 \lambda_3^{-1} \lambda_4^2 W_2 W_3 \tag{4.5}$$

while the fourth and fifth equation in (4.4) give

$$W_4 W_5 = \lambda_1^{-1} \lambda_2^{-1} \lambda_3 W_4 W_5. \tag{4.6}$$

But W_4 and W_5 are not allowed to vanish simultaneously, therefore we must have

$$\lambda_1^{-1} \lambda_2^{-1} \lambda_3 = 1. \tag{4.7}$$

Then, the fourth and fifth equations in (4.4) reduce to

$$W_4 = \lambda_2 W_5. \tag{4.8}$$

Substituting (4.7) in (4.5) and taking into account the first equation in (4.4), we obtain

$$W_1 = \lambda_4 W_1 \qquad W_2 W_3 = \lambda_4^2 W_2 W_3. \tag{4.9}$$

Since W_1, W_2, W_3 are not allowed to vanish simultaneously, taking also into account (4.8), we obtain two cases

$$
\begin{aligned}
\lambda_4 = 1 &\Rightarrow W_3 = \lambda_2 W_2, \ W_4 = \lambda_2 W_5 \\
\lambda_4 = -1 &\Rightarrow W_3 = -\lambda_2 W_2, \ W_1 = 0, \ W_4 = \lambda_2 W_5.
\end{aligned}
\tag{4.10}
$$

We can eliminate λ_2 from the above equations, obtaining the following cases:

$$
\begin{aligned}
\lambda_4 = 1 &\Rightarrow W_2 W_4 = W_3 W_5 \\
\lambda_4 = -1 &\Rightarrow W_2 W_4 = -W_3 W_5, \ W_1 = 0.
\end{aligned}
\tag{4.11}
$$

If $\lambda_4 = 1$, the remaining equations in (4.4) yield

$$X = \lambda_3^2 X, \quad Y = \lambda_3^3 Y, \quad Z = \lambda_3 Z. \tag{4.12}$$

and therefore the whole divisor D given by the equation $W_2 W_4 - W_3 W_5 = 0$ in the Calabi-Yau threefold X is fixed by the holomorphic involution σ. Note that D does not intersect the del Pezzo surfaces S and S'.

If $\lambda_4 = -1$, we obtain

$$X = \lambda_3^2 X, \quad Y = \lambda_3^3 Y, \quad Z = -\lambda_3 Z, \tag{4.13}$$

which can only be satisfied if $Z = 0$ or $Y = 0$. Taking into account that in this case we also have $W_1 = 0$ and $W_2 W_4 + W_3 W_5 = 0$, we conclude that this component of the fixed locus consists of a finite set of points. Note that these points are away from D as well as the del Pezzo surfaces S and S'.

Next we will show that the generic hypersurface X preserved by the involution (4.3) is smooth. It suffices to show that $\widetilde{X}$ is nonsingular, since X, $\widetilde{X}$ are related by a flop. $\widetilde{X}$ is a hypersurface in a toric fourfold P which is a $\mathbb{P}^2_{[1,2,3]}$-bundle over dP_2. This is closely related to the $\mathbb{P}^2_{[1,2,3]}$-bundle P' over $\mathbb{P}^2$, which is defined by the following toric data

$$
\begin{array}{cccccc}
W_1 & W_2 & W_3 & Z & X & Y \\
1 & 1 & 1 & -3 & 0 & 0 \\
0 & 0 & 0 & 1 & 2 & 3.
\end{array}
\tag{4.14}
$$

The transition from P' to P can be accomplished by adding two extra homogeneous variables W_4, W_5 and modifying the torus action accordingly. Equivalently, we can work over the base $\mathbb{P}^2$ (i.e. on the fourfold P') but require that all sections vanish to appropriate order at the two blown-up points

$$p_1 = [0, 1, 0], \quad p_2 = [0, 0, 1].$$

Thus, for example the anticanonical bundle has 10 sections on $\mathbb{P}^2$, namely the cubic monomials in the W_i, $i = 1, 2, 3$, but only 8 of them survive on dP_2 - eliminate W_2^3 and W_3^3. The most general Calabi-Yau hypersurface in P' is given by a combination of monomials of the form:

$$Y^2, \ a_3(W) XYZ, \ a_9(W) YZ^3, \ X^3, \ a_6 X^2 Z^2, \ a_{12}(W) XZ^4, \ a_{18}(W) Z^6.$$

Here a_{3i} is a monomial of degree $3i$, $i = 1, \ldots, 6$ in W_1, W_2, W_3. The analogous statement for P is obtained by taking a_{3i} to be a monomial of degree $3i$ which vanishes to order i at p_1 and p_2, $i = 1, \ldots, 6$. The "Weierstrass" sublinear system is generated only by the polynomials:

$$Y^2 - X^3, \ a_{12}(W) XZ^4, \ a_{18}(W) z^6. \tag{4.15}$$

These hypersurfaces have a zero section: $Z = 0$, $X = 1$, $Y = 1$. As explained above, we want to consider only the sublinear system consisting of Weierstrass hypersurfaces which are also invariant under the involution (4.3). Since Z, X and Y are invariant, this simply means that the coefficients a_{12}, a_{18} are symmetric in W_2 and W_3. Bertini's theorem allows us to conclude that a generic hypersurface in a linear system is smooth without actually exhibiting such a smooth hypersurface. It asserts that a generic hypersurface can be singular only at points of the base locus. In order to apply Bertini in our case, we need the base locus of the monomials (4.15). This is clearly just the zero section: the three symmetric polynomials

$$(W_1^{18}) Z^6, \ (W_2^{18} + W_3^{18}) Z^6, \ (W_2 W_3 Z)^6$$

already cut out the divisor $Z = 0$, which together with $Y^2 - X^3$ gives the zero section. So a generic σ-invariant Weierstrass hypersurface is smooth away from its zero section. But since sections of an elliptic fibration cannot pass through singular points, it is smooth at points of the zero section as well.

Now, we claim that the model (X, σ) constructed above satisfies conditions $(i), (ii)$ below (2.5) as well as conditions $(iii), (iv)$ below (3.7). Condition (i) follows easily from the definition of the holomorphic involution (4.3), which interchanges the defining equations $W_4 = 0$ and $W_5 = 0$ of the two del Pezzo surfaces respectively. Condition (ii) follows from the fact that the holomorphic involution σ is a symmetry of the toric polytope of X. The toric polytope of X is identified with the dual toric polytope of Y under the monomial-divisor map [60,61]. It is clear that the large radius limit in the complexified Kähler moduli space can be reached by increasing the sizes of the toric divisors of Y in an invariant fashion with respect to the given involution. Then the monomial-divisor map implies that the large complex structure of X can be reached in a similar fashion in the complex structure moduli space. Conditions (iii) and (iv) follow from a simple moduli count. We know that $h^{1,1}(X) = 4$ and we have at least two independent divisor classes – a vertical divisor class $\pi^*(h)$ and the section class B – on X. The two del Pezzo surfaces S, S' can be contracted independently on X (this can be achieved by toric contractions) therefore they provide two additional independent generators of the Picard group. Taking into account the disallowed locus (4.2), it is straightforward to check that the only generator of the Picard group which restricts nontrivially to S is S itself. The same statement holds for the conjugate del Pezzo surface S'. By construction, the divisor classes $\pi^*(h), B, S + S'$ are invariant under the holomorphic involution σ while $S - S'$ is anti-invariant. Therefore conditions $(iii), (iv)$ are indeed satisfied.

Checking properties $(a), (b)$ of the quiver locus listed above (3.34) requires a more involved computation. In particular we have to solve the Picard-Fuchs equations for the mirror threefold Y and identify the region of the moduli space where the quantum volumes of the curves Σ, Σ' vanish. Using the simplicial subcone of the Mori cone generated by (Σ, Σ', C, h), we find that the Kähler form of X is given by

$$J = t_1(3W_1 + 2W_4 + 3W_5 + Z) - t_2 W_5 + t_3(3W_1 + 3W_4 + 3W_5 + Z) + t_4(W_1 + W_4 + W_5).$$

The large radius prepotential on the Kähler moduli space of X reads

$$\mathcal{F}_{\mathcal{K}}^X = \frac{4}{3}t_1^3 - \frac{1}{6}t_2^3 + \frac{3}{2}t_3^3 + \frac{9}{2}t_1^2 t_3 + \frac{9}{2}t_1 t_3^2 + \frac{3}{2}t_1^2 t_4 + \frac{1}{2}t_1 t_4^2 + \frac{3}{2}t_3^2 t_4 + \frac{1}{2}t_3 t_4^2 + 3t_1 t_3 t_4 - \frac{23}{6}t_1 + \frac{5}{12}t_2$$
$$- \frac{3}{2}t_4 - \frac{17}{4}t_3 + \frac{105i}{4\pi^3}\zeta(3) + \mathcal{O}(q_i),$$

(4.16)

where $q_i = e^{t_i/2\pi i}$, $i = 1, \ldots, 4$. In the vicinity of the large complex structure point, the fundamental period is given by:

$$X^0 = \sum_{n_1, n_2, n_3, n_4 \geq 0} \frac{z_1^{n_1} z_2^{n_2} z_3^{n_3} z_4^{n_4}}{\Gamma(1 + n_1 - n_3 + n_4)\Gamma(1 + n_2 + n_4)\Gamma(1 + n_1 + n_2 - n_3 + n_4)\Gamma(1 - n_1 + n_3)}$$
$$\times \frac{\Gamma(1 + 6n_1)\Gamma(1 + 6n_2)}{\Gamma(1 - n_2)\Gamma(1 + n_3 - 3n_4)\Gamma(1 + 2n_1 + 2n_2)\Gamma(1 + 3n_1 + 3n_2)},$$

(4.17)

where $z_1, \ldots, z_4$ are algebraic coordinates on the moduli space centered at the large complex structure limit point. The quantum volumes of Σ, Σ' are given by the logarithmic periods

$$
\begin{aligned}
X^1 &= \frac{1}{2\pi i} X^0 \ln z_1 + \frac{1}{2\pi i} g_1, \\
X^2 &= \frac{1}{2\pi i} X^0 \ln z_2 + \frac{1}{2\pi i} g_2,
\end{aligned}
\tag{4.18}
$$

where

$$
\begin{aligned}
g_1 &= \sum_{n_1,n_2,n_3,n_4} \Big[h_0 (6S_{n_1} - S_{n_1-n_3+n_4} - S_{n_1+n_2-n_3+n_4} + S_{-n_1+n_3} - 2S_{2n_1+2n_2} - 3S_{3n_1+3n_2}) \\
&\quad - h_1 - h_2 + h_3 \Big], \\
g_2 &= \sum_{n_1,n_2,n_3,n_4} \Big[h_0 (6S_{n_2} - S_{n_2+n_4} - S_{n_1+n_2-n_3+n_4} + S_{n_2} - 2S_{2n_1+2n_2} - 3S_{3n_1+3n_2}) - h_2 + h_4 \Big].
\end{aligned}
\tag{4.19}
$$

In the above expressions we have used the notation $S_n = \sum_{k=1}^{n} \frac{1}{k}$; the expressions of the functions $h_i(z_1, \ldots, z_4)$, $i = 0, \ldots, 4$ are presented in appendix A. The periods Z^1, Z^2 defined in section three are related to X^1, X^2 by

$$
Z^1 = X^1 - X^2, \quad Z^2 = X^1 + X^2.
$$

We claim that the periods X^1, X^2 vanish simultaneously along the subspace $\mathcal{Q}$ in the complex structure moduli space of Y defined by $z_1 = z_2 = \infty$. This locus lies outside the domain of convergence of the large complex structure periods, therefore we have to perform analytic continuation. Following the prescription of [62], the good algebraic coordinates in a neighborhood of $\mathcal{Q}$ are

$$
w_1 = \frac{1}{z_1}, \quad w_2 = \frac{1}{z_2}, \quad w_3 = z_1 z_3, \quad w_4 = z_4.
$$

Next, note that the fundamental period X^0 does not actually require analytical continuation. The above change of variables yields a convergent series

$$
\begin{aligned}
\widetilde{X}^0 &= \sum_{p_3 \geq p_1 \geq 0, p_4 \geq 0} \frac{w_1^{p_1} w_3^{p_3} w_4^{p_4}}{\Gamma(1-p_1+p_4)^2 \Gamma(1+p_4)\Gamma(1+p_1)\Gamma(1+p_3-p_4)\Gamma(1-2p_1+2p_3)} \\
&\quad \times \frac{\Gamma(1-6p_1+6p_3)}{\Gamma(1-3p_1+3p_3)}.
\end{aligned}
\tag{4.20}
$$

Note also that $\widetilde{X}^0$ has no zeroes on $\mathcal{Q}$ since the zero-th order term in the expansion (4.20) is 1.

The logarithmic periods however require analytic continuation. Representing the periods as Barnes integral and deforming the integration contour, we obtain, using the prescription

of [62]:

$$
\widetilde{X}^1 = -\frac{1}{2\pi i}\left[\frac{\Gamma\left(-\frac{1}{6}\right)\Gamma\left(\frac{1}{6}\right)}{6\sqrt{\pi}\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{5}{6}\right)}w_1 + \frac{\Gamma\left(-\frac{7}{6}\right)\Gamma\left(\frac{7}{6}\right)}{6\sqrt{\pi}\Gamma\left(-\frac{1}{6}\right)\Gamma\left(\frac{2}{3}\right)}w_1^2 w_3 + \cdots\right],
$$
$$
\widetilde{X}^2 = -\frac{1}{2\pi i}\left[\frac{64\Gamma\left(-\frac{1}{6}\right)\Gamma\left(\frac{1}{6}\right)}{\sqrt{\pi}\Gamma\left(\frac{8}{3}\right)\Gamma\left(\frac{5}{6}\right)}w_2 w_3 + 160 w_2^3 w_3 + \cdots\right].
$$

(4.21)

In order to prove property (b) we also have to compute the central charges of D4-branes wrapped on the del Pezzo surfaces S and S' in a neighborhood of $\mathcal{Q}$. This requires analytic continuation of the quadratic logarithmic periods near the large radius limit. Using (4.16), we can write a basis for the quadratic periods of the form

$$
\mathcal{G}_1 = -\frac{X^0}{4\pi^2}\left[4(\ln z_1)^2 + \frac{9}{2}(\ln z_3)^2 + \frac{1}{2}(\ln z_4)^2 + 9\ln z_1\ln z_3 + 3\ln z_1\ln z_4 + 3\ln z_3\ln z_4\right]
$$
$$
-\frac{23}{6}X^0 + \dots,
$$
$$
\mathcal{G}_2 = \frac{X^0}{8\pi^2}(\ln z_2)^2 + \frac{5}{12}X^0 + \dots,
$$
$$
\mathcal{G}_3 = -\frac{X^0}{4\pi^2}\left[\frac{9}{2}(\ln z_1)^2 + \frac{9}{2}(\ln z_3)^2 + \frac{1}{2}(\ln z_4)^2 + 9\ln z_1\ln z_3 + 3\ln z_1\ln z_4 + 3\ln z_3\ln z_4\right]
$$
$$
-\frac{17}{4}X^0 + \dots,
$$
$$
\mathcal{G}_4 = -\frac{X^0}{4\pi^2}\left[\frac{9}{2}(\ln z_1)^2 + \frac{9}{2}(\ln z_3)^2 + 3\ln z_1\ln z_3 + \ln z_1\ln z_4 + \ln z_3\ln z_4\right] - \frac{3}{2}X^0 + \dots,
$$

(4.22)

where the dots stand for holomorphic terms. Near the large radius limit, the central charges $Z_S, Z_{S'}$ have an expansion

$$
Z_S = Z^0\left[\int_X e^J \mathrm{ch}(\mathcal{O}_S)\sqrt{\mathrm{Td}(X)} + \text{ instanton corrections}\right]
$$
$$
Z_{S'} = Z^0\left[\int_X e^J \mathrm{ch}(\mathcal{O}_{S'})\sqrt{\mathrm{Td}(X)} + \text{ instanton corrections}\right]
$$

(4.23)

where J is the Kähler form on X expressed in terms of the flat coordinates $t_1, \dots, t_4$. Note that for Calabi-Yau threefolds

$$
\sqrt{\mathrm{Td}(X)} = 1 + \frac{1}{24}c_2(X).
$$

For the present model, the second Chern class is given by

$$
c_2(X) = 102W_1^2 + 137W_1W_4 + 45W_4^2 + 137W_1W_5 + 45W_5^2 + 92W_4W_5 + 69W_1Z + 46W_4Z + 46W_5Z + 11Z^2.
$$

Taking into account the triple intersection numbers

$$
\begin{aligned}
W_1^3 &= -2 & W_4^3 &= 1 & W_5^3 &= 1 \\
W_1^2 W_4 &= 1 & W_1^2 W_5 &= 1 & W_1^2 Z &= 1 \\
W_1 W_4^2 &= -1 & W_1 W_5^2 &= -1 & W_1 Z^2 &= -3 & Z^3 &= 9,
\end{aligned}
$$

(4.24)

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we obtain

$$Z_S = -\mathcal{G}_1 + \mathcal{G}_3 + \frac{X^1}{2} + X^0,$$
$$Z_{S'} = -\mathcal{G}_2 + \frac{X^2}{2} + X^0. \tag{4.25}$$

These expressions can be analytically continued to a neighborhood of $\mathcal{Q}$ using again the prescription of [62]. We obtain in the leading order

$$\widetilde{Z}_S = \widetilde{X}^0 - \frac{i\Gamma\left(-\frac{1}{6}\right)\Gamma\left(\frac{1}{6}\right)}{24\pi^{3/2}\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{5}{6}\right)}w_1 + \cdots,$$
$$\widetilde{Z}_{S'} = \widetilde{X}^0 - \frac{i\Gamma\left(-\frac{1}{6}\right)\Gamma\left(\frac{1}{6}\right)}{24\pi^{3/2}\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{5}{6}\right)}w_2 + \cdots. \tag{4.26}$$

Now we can conclude the proof of assertions (a) and (b) in section three. Expressions (4.21) show that indeed the quantum volumes of Σ, Σ' vanish identically along $\mathcal{Q}$. Moreover, according to equation (4.26), $\widetilde{Z}_S, \widetilde{Z}_{S'}$ are equal to $\widetilde{X}^0$ along $\mathcal{Q}$ as claimed in section three.

To conclude this section, note that one can similarly construct elliptic fibration with involution which admit dP_7 and dP_6 contractions respectively. These models can be obtained by replacing the ambient $\mathbb{P}^2_{[1,2,3]}$-toric fibration employed in the above construction by $\mathbb{P}^2_{[1,1,2]}$ and $\mathbb{P}^2_{[1,1,1]}$ fibrations respectively, according to [41].

4.2. A Quintic Model

Next we construct a different model involving quotients of quintic threefolds by holomorphic involutions.

Let Z be the blow-up of $\mathbb{P}^4$ at two distinct points p_1, p_2. For concreteness we will choose homogeneous coordinates $[s, t, z^1, z^2, z^3]$ on $\mathbb{P}^4$ so that the two points are given by

$$p_1 = [1, 0, 0, 0, 0] \qquad p_2 = [0, 1, 0, 0, 0].$$

Let E_1, E_2 denote the exceptional divisors on Z; E_1, E_2 are isomorphic to $\mathbb{P}^3$. The anticanonical class of Z is $-K_Z = 5H - 3E_1 - 3E_2$, where H denotes the pull-back of the hyperplane class on $\mathbb{P}^4$ to Z. Note that if X is a smooth divisor in the anticanonical linear system $|-K_Z|$, then X is Calabi-Yau and contains two del Pezzo surfaces $S_1, S_2 \simeq dP_6$ obtained by intersecting X with the two exceptional divisors E_1, E_2. We will obtain a new model if we can also find an antiholomorphic involution of X which maps S_1 isomorphically to S_2, having at the same time codimension 1 or 3 fixed loci. Therefore our goal is to show that the generic anticanonical divisor on Z is smooth, and exhibit a family of such smooth divisors equipped with holomorphic involutions.

Let us first prove that the generic anticanonical divisor on Z is smooth. Let $p : Z \to \mathbb{P}^4$ denote the blow-up map, which contracts the exceptional divisors E_1, E_2. Given any anticanonical divisor X on Z, the image of X under p is a quintic hypersurface X' in $\mathbb{P}^4$ with at least two cubic singularities at p_1, p_2. Conversely, the strict transform of any such quintic hypersurface X' is an anticanonical divisor on X. Therefore we have a one-to-one

correspondence between anticanonical divisors X on Z and quintic hypersurfaces X' with at least cubic singularities at p_1, p_2.

Now let us analyze the sub-linear system of quintic hypersurfaces X' in $\mathbb{P}^4$. The generic hypersurface in this sub-linear system has an equation of the form

$$\sum_{\substack{m,n=0 \\ m+n+r_1+r_2+r_3=5}}^{2} \sum_{r_1,r_2,r_3=0}^{5} a_{m,n,r_1,r_2,r_3} s^m t^n (z^1)^{r_1} (z^2)^{r_2} (z^3)^{r_3} = 0. \tag{4.27}$$

The base locus of this sub-linear system is the line $L \subset \mathbb{P}^4$ cut by the equations

$$z^1 = z^2 = z^3 = 0. \tag{4.28}$$

Indeed it is easy to check that given a point $p \in \mathbb{P}^4 \setminus L$, one can always find a quintic in the sub-linear system (4.27) not passing through p.

What we actually need is the base locus of the linear system of hypersurfaces X in Z. From the above it follows that this is contained in the total transform of the line L, which equals the proper transform of L plus the two exceptional divisors E_1, E_2. But in fact, the base locus is just the proper transform of L: for any point q of E_i, $i = 1, 2$, other than its intersection point q_i with L, we can find an X not passing through q. This can be achieved, e.g., by taking the quintic[4] $X' = GA_1A_2$ to consist of a generic hyperplane $G(z^1, z^2, z^3) = 0$ through L plus two generic quadratic cones A_i, with vertices at the p_i, $i = 1, 2$. The corresponding X meets E_i in a generic plane through q_i plus a generic quadratic surface. The intersection of all such is clearly just the point q_i.

As in the previous case, according to Bertini's theorem a generic hypersurface can be singular only at points of the base locus. In our case, this is the proper transform of L. The X corresponding to the above $X' = GA_1A_2$ is singular at the two points q_i, $i = 1, 2$, but this is easy to remedy. We consider a smooth cubic surface in the first exceptional divisor E_1, which can be taken of the form:

$$S_1 = F(z^1, z^2, z^3) + t^2 G(z^1, z^2, z^3),$$

where F is a smooth cubic in the z plane (e.g. the Fermat) and G is a general linear polynomial. Then the product $X_1 = s^2 S_1$ is in our linear system and meets E_1 in the smooth cubic surface S_1. This quintic is still singular at points (including q_2) of the second exceptional divisor E_2, but the combination

$$X' := X_1 + X_2 = (s^2 + t^2)F + 2s^2t^2 G,$$

where X_2 is obtained from X_1 by interchanging s and t, lifts to an X which is smooth at all points of L. Bertini's theorem now tells us that a generic X in our linear system is smooth everywhere.

---

[4]Abusing notation, in the following we will use the same notation for a hypersurface and its defining polynomial.

Having gone this far, we can actually write down an explicit smooth X, thus avoiding the need to use Bertini. Let us consider the following family of hypersurfaces of the form (4.27)

$$P \equiv (s^2 + t^2)F(z^1, z^2, z^3) + s^2 t^2 G(z^1, z^2, z^3) + Q(z^1, z^2, z^3) = 0 \qquad (4.29)$$

where

$$F(z^1, z^2, z^3) = \sum_{i=1}^{3} a_i (z^i)^3$$

$$G(z^1, z^2, z^3) = \sum_{i=1}^{3} b_i z^i$$

$$Q(z^1, z^2, z^3) = \sum_{i=1}^{3} c_i z_i^5$$

and (a_i, b_i, c_i), $i = 1, 2, 3$ are generic coefficients. Again, one can check that the base locus of this family is L, therefore the generic hypersurface of the form (4.29) may be singular only at points on L. Note that

$$\partial_{z^i} P(s, t, 0, 0, 0) = s^2 t^2 b_i, \quad i = 1, 2, 3$$

Therefore if at least one of the b_i, $i = 1, 2, 3$ is nonzero, the singularities of P are located at $s = 0$ or $t = 0$. Therefore for generic b_i, $i = 1, 2, 3$ the hypersurfaces (4.29) have singularities only at p_1, p_2.

Next, let us check that the exceptional divisors S_1, S_2 on the strict transform X of a generic hypersurface of the form (4.29) are smooth. It suffices to prove this statement for only one of the exceptional divisors, say S_1, since the computations are identical. We will work in the affine coordinate chart $s \neq 0$. Abusing notation we will denote the affine coordinates in this chart by (t, z^1, z^2, z^3). Then the equations (4.29) become

$$(1 + t^2)F(z^1, z^2, z^3) + t^2 G(z^1, z^2, z^3) + Q(z^1, z^2, z^3) = 0. \qquad (4.30)$$

The blow-up Z is locally isomorphic to the one point blow-up of the affine chart $s \neq 0$ at p_1. Let $[\rho, \lambda^1, \lambda^2, \lambda^3]$ denote homogeneous coordinates on $\mathbb{P}^3$. Then Z is locally described by the following equations

$$t\lambda^i = \rho z^i, \qquad i = 1, 2, 3$$
$$z^i \lambda^j = z^j \lambda^i, \qquad i, j = 1, 2, 3, \ i \neq j$$

in $\mathbb{C}^4 \times \mathbb{P}^3$. Taking into account (4.30), the exceptional divisor S_1 is the cubic hypersurface

$$\sum_{i=1}^{3} a_i (\lambda^i)^3 + \rho^2 \sum_{i=1}^{3} b_i \lambda^i = 0. \qquad (4.31)$$

The singular points of the cubic hypersurface (4.31) are determined by

$$3a_i (\lambda^i)^2 + \rho^2 b_i = 0, \qquad i = 1, 2, 3 \qquad (4.32)$$

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in addition to equation (4.31). Assuming $a_i \neq 0$, for $i = 1, 2, 3$, let ω_i be a square root of $-b_i/3a_i$, $i = 1, 2, 3$. Then the solutions of the equations (4.32) are of the form $\lambda^i = \omega_i \rho$, and S_1 is singular if and only if the following relation holds

$$\sum_{i=1}^3 b_i \omega_i = 0 \tag{4.33}$$

Equation (4.33) is not satisfied for generic values of (a_i, b_i), $i = 1, 2, 3$, therefore the generic cubic is smooth.

Next, we have to exhibit a subfamily of smooth anticanonical divisors X on Z equipped with holomorphic involutions $\sigma : X \to X$ which exchange S_1, S_2 and have codimension 1 or 3 fixed loci. Note that it suffices to construct hypersurfaces X' in the sub-linear system (4.27) equipped with holomorphic involutions $\sigma' : X' \to X'$ so that $\sigma'(p_1) = p_2$ and the fixed locus $(X')^{\sigma'}$ has components of dimension zero or two supported away from p_1, p_2. Any such involution lifts to an involution $\sigma : X \to X$ on the strict transform of X' with the desired properties.

Consider the holomorphic involution

$$\tau : \mathbb{P}^4 \to \mathbb{P}^4, \qquad \tau[s, t, z^1, z^2, z^3] = [t, s, -z^1, -z^2, -z^3].$$

Note that τ preserves equation (4.29), therefore it induces a holomorphic involution $\sigma' : X' \to X'$ on each hypersurface X' in the family (4.29). The fixed locus of σ' consists of the point $p = [1, 1, 0, 0, 0]$, and the divisor cut by the equation $s + t = 0$. Moreover, σ' obviously exchanges p_1, p_2. Since τ preserves equation (4.29) for arbitrary values of (a_i, b_i, c_i), it follows that the strict transform X is a smooth threefold on Z equipped with a holomorphic involution $\sigma : X \to X$ which maps S_1 isomorphically to S_2.

In order to conclude our construction, let us show that the generic hypersurface X equipped with a holomorphic involution σ satisfies conditions $(i) - (iv)$ formulated in section three. Condition (i) is clearly satisfied. Condition two follows from the fact that the dominant monomial $stz^1z^2z^3$ in the large complex structure limit is odd under the involution τ. Since the right hand side of equation (4.30) is also odd, it follows that we can perturb the family (4.30) by a term of the form $\mu stz^1z^2z^3$, with arbitrary μ. The resulting hypersurfaces are still smooth if the coefficients (a_i, b_i, c_i) are generic. Conditions (iii) and (iv) follow from the fact that $H^{1,1}(X)$ is generated by the divisor classes S_1, S_2, H, while $H^{2,2}(X)$ is generated by the curve classes L, Σ_1, Σ_2, where $\Sigma_i = (S_i)_X^2$.

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A. Complements on periods

For completeness we provide explicit expressions for the functions $h_i(z_1, \ldots, z_4)$ in the expressions (4.18) of the periods X^1, X^2.

$$
\begin{aligned}
h_0 =& \frac{z_1^{n_1} z_2^{n_2} z_3^{n_3} z_4^{n_4}}{\Gamma(1+n_1-n_3+n_4)\Gamma(1+n_2+n_4)\Gamma(1+n_1+n_2-n_3+n_4)\Gamma(1-n_1+n_3)\Gamma(1-n_2)} \\
& \times \frac{\Gamma(1+6n_1)\Gamma(1+6n_2)}{\Gamma(1+n_3-3n_4)\Gamma(1+2n_1+2n_2)\Gamma(1+3n_1+3n_2)}, \\
h_1 =& \frac{(-z_1)^{n_1} z_2^{n_2}(-z_3)^{n_3}(-z_4)^{n_4}}{\Gamma(1+n_2+n_4)\Gamma(1+n_1+n_2-n_3+n_4)\Gamma(1-n_1+n_3)\Gamma(1-n_2)\Gamma(1+n_3-3n_4)} \\
& \times \frac{\Gamma(-n_1+n_3-n_4)\Gamma(1+6n_1)\Gamma(1+6n_2)}{\Gamma(1+2n_1+2n_2)\Gamma(1+3n_1+3n_2)}, \\
h_2 =& \frac{(-z_1)^{n_1}(-z_2)^{n_2}(-z_3)^{n_3}(-z_4)^{n_4}}{\Gamma(1+n_1-n_3+n_4)\Gamma(1+n_2+n_4)\Gamma(1-n_1+n_3)\Gamma(1-n_2)\Gamma(1+n_3-3n_4)} \\
& \times \frac{\Gamma(-n_1-n_2+n_3-n_4)\Gamma(1+6n_1)\Gamma(1+6n_2)}{\Gamma(1+2n_1+2n_2)\Gamma(1+3n_1+3n_2)}, \\
h_3 =& \frac{(-z_1)^{n_1} z_2^{n_2}(-z_3)^{n_3} z_4^{n_4}}{\Gamma(1+n_1-n_3+n_4)\Gamma(1+n_2+n_4)\Gamma(1+n_1+n_2-n_3+n_4)\Gamma(1-n_2)\Gamma(1+n_3-3n_4)} \\
& \times \frac{\Gamma(n_1-n_3)\Gamma(1+6n_1)\Gamma(1+6n_2)}{\Gamma(1+2n_1+2n_2)\Gamma(1+3n_1+3n_2)}, \\
h_4 =& \frac{z_1^{n_1}(-z_2)^{n_2} z_3^{n_3} z_4^{n_4}}{\Gamma(1+n_1-n_3+n_4)\Gamma(1+n_2+n_4)\Gamma(1+n_1+n_2-n_3+n_4)\Gamma(1-n_1+n_3)} \\
& \times \frac{\Gamma(n_2)\Gamma(1+6n_1)\Gamma(1+6n_2)}{\Gamma(1+n_3-3n_4)\Gamma(1+2n_1+2n_2)\Gamma(1+3n_1+3n_2)}.
\end{aligned}
\tag{A.1}
$$

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