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Macroscopic Strings as Heavy Quarks of Large N Gauge Theory and Anti-de Sitter Supergravity [1]

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abstract

Maldacena has put forward large N correspondence between superconformal field theories on the brane and anti-de Sitter supergravity in spacetime. We study some aspects of the correspondence between $\mathcal{N} = 4$ superconformal gauge theory on D3-brane and maximal supergravity on $adS_5 \times S_5$ by introducing macroscopic strings as heavy (anti)-quark probes. The macroscopic strings are semi-infinite Type IIB strings ending on D3-brane world-volume. We first study deformation and fluctuation of D3-brane when a macroscopic BPS string is attached. We find that both dynamics and boundary conditions agree with those for macroscopic string in anti-de Sitter supergravity. As a by-product we clarify how Polchinski's Dirichlet and Neumann open string boundary conditions arise. We then study non-BPS macroscopic string anti-string pair configuration as physical realization of heavy quark Wilson loop. We obtain $Q\bar{Q}$ static potential and again find agreements between the gauge theory and the supergravity results. By turning on Ramond-Ramond zero-form potential, we also study θ vacuum angle dependence of the static potential. We finally discuss dynamical realization of loop equation via turning on local electric field and recoil of heavy quark thereof, and of heavy baryon via of N-prong string junction. Throughout comparisons of the correspondence, we emphasize crucial role played by 'geometric duality' between coordinates perpendicular to D3-brane and parallel ones, hinting a possible explanation of emergence of the entire adS_5 spacetime.

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1 Introduction

With better understanding of D-brane dynamics, new approaches to outstanding problems in gauge theory have become available. One of such problems is on the behavior of $SU(N)$ gauge theory in the large N limit: $N \rightarrow \infty$ with $g_{\rm eff}^2 = g_{\rm YM}^2 N$ fixed. Planar diagram dominance as shown first by 't Hooft is an indicative of certain connection to string theory but it has never been clear how and to what extent the string is related to fundamental string. Recently, built on earlier study of near-horizon geometry of D- and M-branes and their absorption and Hawking emission processes, Maldacena has put forward a remarkable proposal to the large N behavior. According to his proposal, large N limit of d-dimensional conformal field theories with sixteen supercharges is governed in dual description by maximal supergravity theories (chiral or non-chiral depending on d) with thirty-two supercharges that are compactified on adS_{d+1} times internal round sphere. The most tractible example of Maldacena's proposal is four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory with gauge group $U(N)$. The theory is superconformally invariant with vanishing beta function and is realized as the world-volume theory of N coincident D3-branes of Type IIB string theory. The latter produces near horizon geometry of $adS_5 \times S_5$, where $\lambda_{\rm IIB} = g_{\rm YM}^2$, the radius of curvature $\sqrt{g_{\rm eff}}$ and self-dual flux of $Q_5 = \int_{S_5} H_5 = N$ units. By taking $\lambda_{\rm IIB} \rightarrow 0$ while keeping $g_{\rm eff}$ large in the large N limit, the classical Type IIB string theory is approximated by the compactified supergravity.

In this paper, we put Maldacena's proposal to one more test. The spectrum of $d = 4, \mathcal{N} = 4$ super-Yang-Mills theory contains BPS spectra carrying electric and magnetic charges. From D3-brane point of view, the BPS spectra correspond to Type IIB (p, q) string ending on the D3-brane. The Hilbert space of gauge theory may be decomposed into superselection sectors with fixed electric and magnetic charges. The former is the usual electric charge superselection sectors and the latter is the soliton sectors. As such, the correspondence between gauge theory and supergravity should continue to hold in each superselection sectors. In particular, dynamics of BPS states should match between gauge theory and supergravity descriptions. From gauge theory side, the BPS states are realized as solitons on the D3-brane world-volume. Indeed, Callan and Maldacena have found exact solution of the world-volume soliton that corresponds to (p, q) Type IIB string ending on it. In supergravity side, the corresponding configuration may be described by a test open string in the background of D3-brane.

The Type IIB string has two ends: one on the D3-brane and another at the boundary od AdS_5 spacetime.

In due course of the study, we will elaborate more on boundary condition the BI soliton string satisfies at the neck. According to Polchinsk's prescription, string coordinates in perpendicular and parallel directions to D-brane should satisfy Dirichlet and Neumann boundary conditions respectively.

In this paper, we study *dynamical properties* of the open string soliton. More specifically, we examine propagation and scattering of harmonic waves on D-brane and deduce boundary conditions that the macroscopic open string satisfies. Callan and Maldacena have studied already waves with polarization perpendicular to D-brane and have concluded that they satisfy Dirichlet boundary condition. We append their analysis with several further observation and extend to the case that the waves have polarization parallel to D-brane. We undertake the study in two steps. We first consider an open fundamental string in curved supergravity background of D-brane and study propagation of low-energy waves on it. The propagation can be mapped to an analog one-dimensional Schrödinger equation. We find that the perpendicular and longitudinal polarizations corresponds to analog potentials of δ- and δ'-functions respectively.

In due course of our investigation, two technical issues have played important role. The first is the use of tortoise coordinate on the D3-brane world-volume. The second is self-adjoint extension of wave operators, since the fundamental or solitonic string ends on the D3-brane. With these technical addition, we were able to make more precise dynamical study than the steps taken by Callan and Maldacena.

This paper is organized as follows. In Section 2, we study dynamics of Type IIB string in the supergravity background of multiple D3-branes. We will start with In section 3, for D3-brane, we study dynamics of brane waves propagating along the Born-Infeld soliton.

2 Fundamental String in D-Brane Background

We begin with dynamical study of a test fundamental string placed in the background of D3-brane. We will assume that the Ramond-Ramond charge Q_3 of the D3-brane is macroscopically large. In this case, back-reaction of the test fundamental string to the background geometry may be ignored consistenly. Throughout this section, we thus restrict to this test string approximation.

Consider an infinitely planar D3-brane located at $\mathbf{x}_\perp = 0$. In the string frame, spacetime metric of Dp-brane is given by

$$ds^2 = \frac{1}{\sqrt{G}}\left(-dt^2 + d\mathbf{x}_\parallel^2\right) + \sqrt{G}d\mathbf{x}_\perp^2 \tag{1}$$

where

$$G(r) = 1 + gQ_3\left(\frac{\sqrt{\alpha'}}{r}\right)^{7-p} \qquad (r \equiv |\mathbf{x}_\perp|). \tag{2}$$

In addition, Dp-brane exerts a nontrivial dilaton background

$$e^{-2\phi} = \frac{1}{\lambda_{\rm st}^2} = G^{(p-3)/2}, \tag{3}$$

so that string theory becomes strongly coupled near the D-brane for $p < 3$ and weakly coupled for $p > 3$. An exception to this is when $p = 3$, for which string theory can be made weakly coupled everywhere. To make use of such a simplifying feature, in what follows, we concentrate only on $p = 3$.

Consider a semi-infinite open fundamental string attached to the D3-brane extending radially outward. Denote the string coordinate fields $X^\mu(\sigma^a)$. Low-energy dynamics of the test open string is then described by Nambu-Goto Lagrangian density:

$$\mathcal{L}_{\rm NG} = T\ (-\det h)^{1/2}, \tag{4}$$

where $T = 1/2\pi\alpha'$ denotes string tension and h_{ab} denotes the pull-back of spacetime metric to the worldsheet:

$$h_{ab} = g_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu. \tag{5}$$

In spherical coordinates, the spacetime metric reads

$$ds^2_{D3} = \frac{1}{\sqrt{G}}\left(-dt^2 + d\mathbf{x}_\parallel^2\right) + \sqrt{G}\left(dr^2 + r^2 d\Omega_5^2\right). \tag{6}$$

Thus, in the static gauge

$$X^0 = t, \qquad X^r = r, \tag{7}$$

eight transverse coordinates of the test open string is decomposed into

$$\mathbf{X} = \mathbf{X}_\parallel + \mathbf{X}_\perp. \tag{8}$$

Here, $\mathbf{X}_\parallel$ denotes string coordinates longitudinal to the D3-brane. Likewise, $\mathbf{X}_\perp$ denotes coordinates transverse to the D3-brane.

In the background metric Eq.(1), straightforward calculation yields ($\dot{} \equiv \partial_t$, $' \equiv \partial_r$)

$$\begin{aligned} h_{00} &= \sqrt{G}(\dot{\mathbf{X}}_\perp)^2 - \frac{1}{\sqrt{G}}\left(1 - (\dot{\mathbf{X}}_\parallel)^2\right) \\ h_{11} &= \sqrt{G}\left(1 + (\mathbf{X}'_\perp)^2\right) + \frac{1}{\sqrt{G}}(\mathbf{X}'_\parallel)^2 \\ h_{01} &= \frac{1}{\sqrt{G}}\dot{\mathbf{X}}_\parallel \cdot \mathbf{X}'_\parallel + \sqrt{G}\dot{\mathbf{X}}_\perp \cdot \mathbf{X}'_\perp. \end{aligned} \tag{9}$$

Thus, apart from the static energy contribution, the Nambu-Goto Lagrangian is given by:

$$L_{\rm NG} = \frac{T}{2}\int_0^\infty dr \left[\left((\dot{\mathbf{X}}_\parallel)^2 - \frac{1}{G}(\mathbf{X}'_\parallel)^2\right) + \left(G(\dot{\mathbf{X}}_\perp)^2 - (\mathbf{X}'_\perp)^2\right) + \left(\dot{\mathbf{X}}_\parallel \cdot \mathbf{X}'_\perp - \dot{\mathbf{X}}_\perp \cdot \mathbf{X}'_\parallel\right)^2 \right]. \tag{10}$$

Note that the worldsheet coordinate r ranges $[0, +\infty]$, where $r = 0$ is the location the fundamental string ends on the D3-brane. As such, a suitable boundary condition at $r = 0$ should be appended to the action Eq.(10). In the present context, the boundary condition reflects the fact that fundamental string and the D3-branes are coupled each other and renders the test string dynamics essentially self-adjoint.

We find it more convenient to use a tortoise worldsheet coordinate σ:

$$\frac{dr}{d\sigma} = \frac{1}{\sqrt{G}} \equiv \cos\theta(r); \qquad (-\infty < \sigma < +\infty). \tag{11}$$

Physically, θ measures inclination angle between the curved metric Eq.(1) and the conformally flat metric:

$$ds^2_{\rm D3} = \frac{1}{\sqrt{G}}\left(-dt^2 + d\mathbf{x}^2_{\parallel} + d\sigma^2\right) + r^2\sqrt{G}d\Omega^2_5. \tag{12}$$

The Nambu-Goto Lagrangian now reads

$$L_{\rm NG} = \frac{T}{2}\int_{-\infty}^{+\infty} d\sigma \Big[\frac{1}{\sqrt{G}}\left((\partial_t\mathbf{X}_{\parallel})^2 - (\partial_\sigma\mathbf{X}_{\parallel})^2\right) + \sqrt{G}\left((\partial\mathbf{X}_{\perp})^2 - (\partial_\sigma\mathbf{X}_{\perp})^2\right)\Big], \tag{13}$$

which reflects explicitly the conformal metric structure of Eq.(12). Neglecting quartic interaction terms, the equations of motion are

$$\begin{aligned} \left[-\partial_t^2 + \frac{1}{\sqrt{G}}\partial_\sigma\sqrt{G}\partial_\sigma\right]\mathbf{X}_{\perp} &= 0 \\ \left[-\partial_t^2 + \sqrt{G}\partial_\sigma\frac{1}{\sqrt{G}}\partial_\sigma\right]\mathbf{X}_{\parallel} &= 0. \end{aligned} \tag{14}$$

Note that, in the tortoise coordinate Eq.(11, 12), $\sigma \to -\infty$ corresponds to near D3-brane junction $r \to 0$, while $\sigma \to +\infty$ is asymptotic infinity $r \to \infty$. Again, to ensure self-adjointness of fundamental string dynamics, an appropriate boundary condition at $\sigma = -\infty$ needs to be supplemented to Eq.(14).

To identify boundary conditions of $\mathbf{X}_{\parallel,\perp}$, we now analyze scattering process of an incoming wave from $\sigma = +\infty$ inward and identify S-matrices thereof.

2.1 Transverse Coordinates

For a monochromatic wave $\mathbf{X}_{\perp}(\sigma,t) = \mathbf{X}_{\perp}(\sigma)e^{-i\omega t}$, unitary transformation $\mathbf{X}_{\perp}(\sigma) \to G^{-1/4}\mathbf{Y}_{\perp}(\sigma)$ yields Helmholtz equation

$$\omega^2\mathbf{Y}_{\perp}(\sigma) = -\left(G^{-1/4}\partial_\sigma\sqrt{G}\partial_\sigma G^{-1/4}\right)\mathbf{Y}_{\perp}(\sigma). \tag{15}$$

Changing variables to dimensionless ones

$$\sigma \to \sigma/\omega, \quad r \to r/\omega, \quad R \to R/\omega, \quad \epsilon \equiv R\omega, \tag{16}$$

the Helmholtz equation can be turned into a form of Schrödinger equation:

$$[-\partial_\sigma^2 + V(\sigma)]\mathbf{Y}_{\perp}(\sigma) = +1\cdot\mathbf{Y}_{\perp}(\sigma). \tag{17}$$

The potential $V(\sigma)$ is straightforwardly calculated:

$$\begin{aligned} V(\sigma) &= -\frac{1}{16}G^{-3}\left[5(\partial_r G)^2 - 4G(\partial_r^2 G)\right] \\ &= \frac{5\epsilon^4}{(r^2+\epsilon^4 r^{-2})}. \end{aligned} \tag{18}$$

For low-energy scattering, $\epsilon \to 0$, Callan and Maldacena have argued that the potential may be approximated by δ-function. We now elaborate more for justification of their approximation. This analog potential has a maximum at $r = \kappa$. In terms of σ coordinates, this is again at $\sigma \approx \mathcal{O}(\kappa)$. We thus find that the one-dimensional Schrodinger equation has a delta function-like potential. For low energy scattering, the delta function gives rise to Dirichlet boundary condition. Alternative way of understanding this is via taking $\kappa \to 0$ limit. In this case, the distance between $r = 0$ and $r = \kappa$ becomes zero. Therefore, the low-energy scattering may be described by a self-adjoint extension of free Laplacian operator at $r = 0$.

We now turn to fluctuation of F-string parallel to the D3-brane. The equation of motion reads

$$-\frac{1}{\sqrt{G}}\partial_t^2 \mathbf{X}_\parallel + \partial_\sigma\left(\frac{1}{\sqrt{G}}\partial_\sigma \mathbf{X}_\parallel\right) = 0. \tag{19}$$

For stationary scattering state $\mathbf{X}_\parallel(t,\sigma) = \mathbf{X}_\parallel(\sigma)e^{-i\omega t}$, unitary transformation $\mathbf{X}_\parallel = G^{1/4}\mathbf{Y}_\parallel$ yields a one-dimensional, stationary Schrödinger equation:

$$\omega^2 \mathbf{Y}_\parallel = \left[-\frac{d^2}{d\sigma^2} + V(\sigma)\right]\mathbf{Y}_\parallel \tag{20}$$

where

$$\begin{aligned} V(\sigma) &= \frac{1}{16}G^{-3}\left[7(\partial_r G)^2 - 4G(\partial_r^2 G)\right] \\ &= -\frac{(5-2r^{-4})}{(r^2+r^{-2})^3}. \end{aligned} \tag{21}$$

It is straightforward to see that the analog potential has a potential of the type involving two delta functions with opposite coefficients. At low-energy scattering process, hence, the potential may be approximated by delta-prime potential. This yields Neumann boundary condition. An interesting point is that the scattering center is *not* at the brane location $r = 0$ but a distance $\mathcal{O}(\kappa)$ away. In terms of proper distance, this is infinitely far away.

3 Harmonic Fluctuations of D-brane

In this section, we study harmonic fluctuation of D-brane world-volume dynamics. From Born-Infeld Lagrangian, we derive explicit form of low-energy effective Lagrangian for brane waves.

The Born-Infeld Lagrangian is obtained by dimensional reduction from Type I superstring action in flat ten-dimensional spacetime:

$$L_{\rm BI} = -\frac{1}{g_p}\int d^px \left(\det[\eta_{\mu\nu}\otimes g^{ij} + \alpha' F_{\mu\nu} + \partial_\mu X^i]\right)^{1/2} \tag{1}$$

Here, the determinant is over (μ, i)-indices. Using the identity

$$\begin{aligned} \det\begin{pmatrix} N & A \\ -A^{\rm T} & M \end{pmatrix} &= \det M \cdot \det\left(N + A\cdot M^{-1}\cdot A^{\rm T}\right) \\ &= \det N \cdot \det\left(M + A^{\rm T}\cdot N^{-1}\cdot A\right), \end{aligned} \tag{2}$$

the Born-Infeld action may be reduced to

$$L_{\rm BI} = -\frac{1}{g_p}\int d^px \left(\det[\eta_{\mu\nu} + \alpha' F_{\mu\nu} + \partial_\mu X^i g_{ij}\partial_\nu X^j]\right)^{1/2} \tag{3}$$

To study fluctuations of world-volume around a BPS configuration, we expand the Born-Infeld action to quadratic order:

$$\begin{aligned} F_{\mu\nu} &= \overline{F}_{\mu\nu} + \delta F_{\mu\nu} \\ X^i &= \overline{X}^i + \delta X^i \end{aligned} \tag{4}$$

The expansion is tedious but straightforward. In particular, the algebra becomes considerably simplified if the background is BPS state. Quadratic fluctuations come from two parts. The first is from second-order variation of the $(p+1)\times(p+1)$ matrix, viz. the transverse coordinate parts. The gives the Lagrangian for transverse coordinate fluctuation. The second is from the first-order variation of the $(p+1)\times(p+1)$ matrix. Evidently, if the background involves nontrivial transverse coordinate fields, this contribution induces mixing between gauge field fluctuation and transverse coordinate fluctuation.

Keeping only up to quadratic order, the harmonic fluctuation Lagrangian is found as:

$$\begin{aligned} \delta^{(2)}L_{BI} = \frac{1}{2g_p}\int d^px, \Big[\quad & (1+\mathbf{E}^2)(\delta\mathbf{E})^2 - (\delta\mathbf{B})^2 - \mathbf{E}^2\,(\delta\mathbf{E}\cdot\nabla\eta) \\ + \quad & (\partial_t\eta)^2 - (1-\mathbf{E}^2)(\nabla\eta)^2 \\ + \quad & (1+\mathbf{E}^2)(\partial_t\rho)^2 - (\nabla\rho)^2 \Big]. \end{aligned} \tag{5}$$

In this expression, for clarity, we have decomposed the D-brane transverse coordinate fields into Ψ that is perpendicular to both the D-brane world-volume and the Born-Infeld soliton elongation directions and Ξ that is perpendicular to the D-brane world-volume but parallel to the Born-Infeld soliton elongation direction. Recall that the ten-dimensional SO(9,1) is spontaneously broken to SO(p,1)×SO(9-p). The Born-Infeld soliton break the latter further into SO(8-p). By SO(8-p) transformation, each fluctuation of Ψ and Ξ can always brought into a single component. Because of this, we have suppressed any SO(8-p) vectorial indices to these fields. It is straightfoward, however, to reinstate them.

We now integrate out the gauge field fluctuation. This can be done exactly for $p = 1$ case, as shown in REY-YEE paper. Recapitulating the procedure, since the world-volume gauge field in this case consist only of electric field, it can be integrate out explicitly. We find that

$$\mathcal{L}_{\rm eff}^{(p=1)} = \frac{1}{2g_1}\int dx \Big[(\delta\dot{X})^2 - \frac{1}{(1+E^2)}(\delta X')^2 + (1+E^2)(\delta Y)^2 - (\delta Y')^2 \Big] \tag{6}$$

The resulting Lagrangian is completely local in D-string world-volume. While the gauge field fluctuation is quadratic for $p > 1$ as well, hence, can be integrated out explicitly, the resulting Lagrangian is not local any more. This is due to the presence of fluctuation in magnetic field components. We thus make an approximation of truncating the magnetic field fluctuations and integrate out the electric field fluctuation. While we do not have a solid argument, physics-wise, this should not be a big problem. The gauge field fluctuation couples only through field-strength, reflecting the fact that there is no charged elementary excitations. Much the same way as electric dipoles are screened in dielectric medium and rendering only local, renormalized operators, we expects that the approximation employed should work at least for low-energy dynamics. The rest of steps is exactly the same as $p = 1$ case and yields:

$$\begin{aligned}\mathcal{L}_{\rm eff}^{(p)} = \frac{1}{2g_p}\int d^px \Big[& \quad (\partial_t\chi)^2 - \frac{1}{(1+\mathbf{E}^2)}(\nabla\chi)^2 \\ & + \quad (1+\mathbf{E}^2)(\partial_t\rho)^2 - (\nabla\rho)^2 \Big]. \end{aligned} \tag{7}$$

4 D3-Brane Fluctuation via Born-Infeld

In this section, we restrict the fluctuations to S-wave part and study their propagation along the Born-Infeld solitonic string. The world-volume coordinate x is *not* the intrinsic coordinates measured on the D-brane world-volume. Since we are studying fluctuation on the D-brane itself, it is quite crucial to use the intrinsic coordinates. Therefore, we now make a change of variable to the tortoise coordinate σ related to x as

$$\frac{dr}{d\sigma} = \frac{1}{\sqrt{G}} \qquad G(r) = (1 + \frac{A_3}{r^4}) \tag{8}$$

Once this change of variables is made, the effective Lagrangian of Born-Infeld solitonic string becomes

$$\begin{aligned}\mathcal{L}_{\rm eff} = \frac{T_3}{2}\int d\sigma(\sigma)^2 \Big[\quad & \sqrt{G} \quad \Big((\partial_t\Psi)^2 - (\partial_\sigma\Psi)^2\Big) \\ = \quad & \tfrac{1}{\sqrt{G}} \quad \Big((\partial_t\Xi)^2 - (\partial_\sigma\Xi)^2\Big) \,] \end{aligned} \tag{9}$$

We thus find that the soliton string fluctuates along two orthogonal polarization directions with different local tension and mass density even though the propagation velocity is the speed of light everywhere for both. The directional dependence can be understood geometrically from the fact that the proper parallel and orthogonal directions to the D-brane does not coincide with the above decomposition. For the case of D-string, this has been explicitly demonstrated by REY+YEE. We will not do this further geometric transformation in the foregoing analysis, since our analysis does not depend on it.

4.1 Fluctuation of Orthogonal Direction

The harmonic equation of motion along orthogonal radial direction is given by

$$-\partial_t^2\Psi_\perp + \frac{1}{r^2\sqrt{G}}\partial_\sigma\Big(r^2\sqrt{G}\partial_\sigma\Psi_\perp\Big) = 0. \tag{10}$$

For stationary fluctuation modes $\Psi(\sigma,t) = \texttt{Re}\Psi(\sigma)e^{-i\omega t}$, after making redefinition

$$\Psi_\perp = \frac{1}{rG^{1/4}}\tilde{\Psi}_\perp \tag{11}$$

the above fluctuation equation can be brought into the form

$$[-\frac{d^2}{dx^2} + V_\perp(x)]\tilde{\Psi}_\perp = +1\cdot\tilde{\Psi}_\perp, \tag{12}$$

where

$$V_\perp(x) = \frac{5}{(x^2+\kappa^2/x^2)^3}. \tag{13}$$

5 Transmission of Waves Through Junction Point

6 Born-Infeld BPS Spike in Large-N Limit

Maldacena has recently put forward an interesting conjecture on strong coupling dynamics of superconformal field theories. For D3-brane, his conjecture asserts that large N limit of $\mathcal{N}=4, D=4$ Yang-Mills gauge theory as the world-volume description of is most appropriately described by near-horizon supergravity.

In this section, we test Maldacena's conjecture. The strong-weak coupling duality in large N limit $g^2N \to 1/g^2N$ then corresponds to Type IIB SL(2,$\mathbf{Z}$) duality. We now take $N\to\infty$ limit. In the supergravity side, the spacetime metric becomes

$$ds^2 = \alpha'\left[\frac{u^2}{\sqrt{g}N}(-dt^2 + d\mathbf{x}_\parallel^2) + \sqrt{g}N(\frac{du^2}{u^2} + d\Omega_5^2)\right] \tag{14}$$

hence, reduces to $adS_5\times S_5$.

We now show that the fluctuation equation looks exactly the same for gauge theory side and the supergravity side. We claim that the origin of this symmetry is precisely the remnant of Type IIB SL(2,**Z**) duality.

6.1 BPS Spike in Anti-de Sitter Background

Consider a ($\gg 1$) D3-branes. In the large N limit, master field configuration of the world-volume gauge theory is anti-de Sitter supergravity background. If we place a probe D3-brane, which is oriented parallel, then the world-volume gauge theory on the probe is governed by DBI action:

$$S_{\rm DBI} = T_3 \int d^4x \frac{U^2}{\kappa^2} \Big[||\eta_{ab} + \frac{\kappa^2}{U^4}(\partial_a U \partial_b U + U^2 \partial_a \Omega \partial_b \Omega) + \frac{\kappa}{U^4} F_{ab}||^{1/2} - 1 \Big] \tag{15}$$

Evaluating the determinant explicitly, one finds

$$S_{\rm DBI} = T_3 \int d^4x \frac{U^4}{\kappa^2} \Big[\left(1 - \frac{\kappa^2}{U^4}\mathbf{E}^2\right)\left(1 + \frac{\kappa^2}{U^4}(\nabla U)^2\right) + \frac{\kappa^4}{U^8}(\mathbf{E}\cdot\nabla U)^2 - \frac{\kappa^2}{U^4}(\dot{U})^2\Big]^{1/2}. \tag{16}$$

In the weak coupling limit $\kappa \ll 1$, quadratic approximation of the action agrees with the standard $\mathcal{N}=4, d=4$ supersymmetric gauge theory with correct coupling constant normalization. For brevity, let us name the square-root combination as M. Since we will be strictly interested in dynamics in U and gauge fields, we drop the angular field Ω on S_5 from now on. For the gauge and Higgs field U, canonical conjugate momenta are:

$$\begin{aligned} \mathbf{\Pi}_A &= \frac{1}{L_{\rm DBI}}\frac{U^4}{\kappa^2}\left[-\mathbf{E}\left(1 + \frac{\kappa^2}{U^4}(\nabla U)^2\right) + \frac{\kappa^2}{U^4}\nabla U(\mathbf{E}\cdot\nabla U)\right] \\ \mathbf{P}_U &= -\frac{1}{L_{\rm DBI}}\frac{U^4}{\kappa^2}\dot{U}. \end{aligned} \tag{17}$$

We now look for spike BPS soliton solution on the world-volume of D3-brane. For static configuration, the equations of motions read:

$$\begin{aligned} &\nabla\cdot\Big[\frac{1}{L_{\rm DBI}}\frac{U^4}{\kappa^2}\left(\nabla U\left(1-\frac{\kappa^2}{U^4}\mathbf{E}^2\right) + \frac{\kappa^2}{U^4}(\mathbf{E}\cdot\nabla U)\mathbf{E}\right)\Big] = \frac{4U^3}{L_{\rm DBI}}\frac{\kappa^2}{U^4}\Big[(\mathbf{E}\cdot\nabla U)^2 - \mathbf{E}^2(\nabla U)^2\Big]. \\ &\nabla\cdot\Big[\frac{1}{L_{\rm DBI}}\frac{U^4}{\kappa^2}(-\mathbf{E}\left(1+\frac{\kappa^2}{U^4}(\nabla U)^2\right) + \frac{\kappa^2}{U^4}\nabla U(\mathbf{E}\cdot\nabla U))\Big]. \end{aligned} \tag{18}$$

While quite complicated coupled equations, the two equations are solved remarkably by the following BPS equation:

$$\mathbf{E} = \pm\nabla U. \tag{19}$$

In this case, the whole DBI Lagrangian collapses to $L_{\rm DBI} = U^4/\kappa^2$ and nonlinear terms in each equations cancel each other. Furthermore, the compensating term -1 in the AdS DBI

Lagrangian, which were present to ensure vanishing ground-state energy, is absolutely crucial to have the right-hand side of the first equation of motion. The resulting equation is nothing but Gauss' law constraint in linear form:

$$\nabla \cdot \mathbf{E} = \nabla^2 U = 0, \tag{20}$$

where the Laplacian is in terms of conformally flat coordinates. The scalar function satisfies

$$U = U_0 + \frac{Q}{r}. \tag{21}$$

The interpretation is that spike of Higgs field U is a source of world-volume electric field. From Type IIB string theory point of view, the source is nothing but Type IIB fundamental string. Therefore, we now have found that the world-volume spike describes nonsingular electric source and is identified with F-string.

The total energy now reads

$$\begin{aligned} E &= \int d^4x \, -\frac{U^4}{\kappa^2}\left[1 + \frac{\kappa^2}{U^4}(\nabla U)^2\right] + \frac{U^4}{\kappa^2} \\ &= \int d^4x \, (\nabla U)^2 \end{aligned} \tag{22}$$

Thus, the total energy diverges with the radial cut-off as in the weak coupling case.

Since the above spike soliton is a BPS state and has a nonsingular tension the solution remains valid even at strong coupling regime.

7 Non-BPS String Pair

So far, in the previous sections, we have studied BPS dynamics involving a single probe string. In this section, we extend the study to non-BPS configuration. We do this again from Born-Infeld super Yang-Mills and anti-de Sitter supergravity points of view. Among the myriad of non-BPS configurations, the simplest and physically interesting one is a pair of oppositely oriented, semi-infinte strings attached to the D3-brane.

Physically, the above configuration may be engineered as follows. We first prepare a macroscopically large, U-shaped fundamental string, whose tip part is parallel to the D3-brane but the two semi-infinite sides are oriented radially outward. As we move this string toward D3-brane, the tip part will be attracted to the D3-brane and form a non-threshold bound-state. The configuration is still not a stable BPS configuration since the two end points from which semi-infinite sides emanate acts as a pair of opposite charges. They are nothing but $W_\pm$ pairs. As such, the two ends will attract each other (since the bound-state energy on the D3-brane is lowered by doing so) and eventually annihilate into radiations. However, in so far as the string is semi-infinite, the configuration will be energetically stable: inertia of the two open strings is infinite. Since the string length represents the vacuum expectation value of Higgs field, this means that the $W_\pm$ pairs are infinitely heavy. In this way, we have engineered static configuration of a $(Q\bar{Q})$ pair on the D3-brane.

The configuration is of some interest since it may tell us whether the $d = 4, \mathcal{N} = 4$ super Yang-Mills theory exhibits confinement. The theory has vanishing β-function, hence, no dimensionally transmuted mass gap either. As such, one might be skeptical to get any signal of confinement from *gedanken* experiment using the above configuration. While this seems to be the result we are getting, we also find a hint that may be interpreted as a 'confinement'. The interpretation relies on the earlier observation that paralle and perpendicular directions to D3-brane are geometrically dual each other.

7.1 $Q\bar{Q}$ Pair in anti-de Sitter Supergravity

We first construct the aforementioned string configuration corresponding to $q\bar{Q}$ pair on D3-brane in anti-de Sitter supergravity. To find the configuration we find it most convenient to study portions of the string separately. Each of the two semi-infinite portions is exactly the same as a single semi-infinite string studied in the previous section. Thus, we concentrate mainly on the tip portion that is about to bound to the D3-brane. The portion cannot be bound entirely parallel to the D3-brane since it will cause large bending energy near the location we may associate with Q and $\bar{Q}$. The minimum energy configuration would be literally like U-shape. We now show that this is indeed what comes out.

The dynamics of $Q\bar{Q}$ open string segment is described by Nambu-Goto action

$$\begin{aligned} S_{\rm NG} \;\; = \;\; & \int d\tau d\sigma \sqrt{\det h_{ab}} \\ & h_{ab} = G_{MN}(X)\partial_a X^M \partial_b X^N \\ & dS^2 = \frac{1}{\sqrt{G}}\left(-dT^2 + dX_\parallel^2\right) + \sqrt{G}\left(dU^2 + U^2 d\Omega_5^2\right). \end{aligned} \tag{23}$$

In static gauge $T = \tau$, $X_\parallel = \sigma\hat{n}$ for the string oriented along $\hat{n}$ on D3-brane,

$$\begin{aligned} h_{00} &= \frac{1}{\sqrt{G}}\left(1 - \dot{Y}_\parallel^2\right) + \sqrt{G}\left(\dot{U}^2 + U^2\dot{\Omega}_5^2\right) \\ h_{11} &= \frac{1}{\sqrt{G}}\left(1 + Y_\parallel'^2\right) + \sqrt{G}\left(U'^2 + U^2{\Omega_5'}^2\right) \\ h_{01} &= \frac{1}{\sqrt{G}}\dot{Y}_\parallel \cdot Y_\parallel' + \sqrt{G}\left(\dot{U}U' + U^2\dot{\Omega}_5\Omega_5'\right). \end{aligned} \tag{24}$$

This yields the Nambu-Goto Lagrangian

$$\begin{aligned} L_{\rm NG} = \Big[\frac{1}{G}(1 \;\; - \;\; & \dot{Y}_\parallel^2 + Y_\parallel'^2 - (\dot{Y}_\parallel^2 Y_\parallel'^2) - (\dot{Y}_\parallel \cdot Y_\parallel'^2) \\ - \;\; & \dot{U}^2 + U'^2 - U^2\dot{\Omega}_5^2 + U^2{\Omega_5'}^2 - G((\dot{U}U')^2 + U^2(\dot{\Omega}_5 \cdot {\Omega_5}'))\Big]^{1/2} \end{aligned} \tag{25}$$

The Lagrangian exhibits an intriguing fact that the tension associated with string motion within the D3-brane is softened by $1/\sqrt{G}$ compared to fundamental string itself. Softening of string tension and eventual tensionless limit has been noticed previously [**?**, **?**, 4] in the context of (p, q) string bound-state. The configuration under consideration is related to the $(1, N)$ string configuration by T-duality.

For static configuration, the Nambu-Goto Lagrangian simplifies considerably:

$$L_{\rm NG} \rightarrow \frac{1}{\sqrt{G}}\Big[1 + Y_\parallel'^2 + U'^2 + U^2 \Omega_5' \cdot \Omega_5'\Big]^{1/2}. \qquad (26)$$

Clearly, both the $Y_\parallel$ and Ω_5 directions give rise to conserved quantities. The simplest choice is that $Y_\parallel = \Omega_5$ being constant. In this case, the equation of motion for $U(\sigma)$ is given by

$$-\frac{1}{G}U'' + \frac{1}{2}\left(\partial_U \frac{1}{G}\right)\left(2U'^2 + \frac{1}{G}\right) = 0. \qquad (27)$$

From this we obtain first integral of motion:

$$G^2 U'^2 + G = \frac{\kappa^2}{U_*^4}, \qquad (28)$$

where we have parametrized an integration constant as on the right-hand side. Denoting $Z = U_*/U$, the solution in implicit functional form reads:

$$(x_\parallel - d/2) = \pm \frac{\kappa}{\sqrt{2}U_*} F(arccos Z, 1/\sqrt{2}). \qquad (29)$$

Here, $F(\phi, k)$ denotes the elliptic integrals of first kind. It is easy to visualize (on the Mathematica) that the solution describes monotonic lifting of U-direction fluctuation (thus away from the D3-brane plane) and diverges at $x_\parallel = 0$ and d. This prompts to interpret the integration constant d in Eq.(xx) as the inter-quark separation measured in $x_\parallel$ coordinates. The string is bended (roughly in U-shape) symmetrically about $x_\parallel = d/2$. As such, the inter-quark distance measured *along* the string is not the same as d. The proper distance along the string is measured by the U-coordinate. The relation between the coordinate separation and proper separation is obtained easily by integrating over the above Eq.(xx). It gives

$$\begin{aligned} d/2 &= \frac{\kappa}{U_*} F(\pi/2, 1/\sqrt{2}) \\ &= \frac{\kappa}{U_*} \sqrt{\pi}\Gamma(5/4)/\Gamma(3/4). \end{aligned} \qquad (30)$$

This formula implies that the integration constant U_* would be interpreted as the height of the U-shaped tip along U-coordinate. Up to numerical factors, the relation again shows 'geometric duality' between coordinate distance d and proper distance U_*.

Using the first integral of motion, the inter-quark potential is obtained straightforwardly from the Born-Infeld Lagrangian. The proper length of the string is infinite, so we would expect linearly divergent (in U-coordinate) energy. Thus, we first calculate regularized expression of energy by excising out a small neighborhood around $x_\parallel = 0, d$:

$$
\begin{aligned}
E &= \lim_{\epsilon \to 0} \Big[\sqrt{G}_* \int_0^{d/2-\epsilon} dx_\parallel \, G^{-1} \Big] \\
&= \lim_{U \to \infty} \Big[2U_* \int_1^U dt \frac{t^2}{\sqrt{t^4 - 1}} \Big] \\
&= 2U_* \Big[U + \frac{1}{\sqrt{2}} K(1/\sqrt{2}) - \sqrt{2} E(1/\sqrt{2}) + \mathcal{O}(U^{-3}) \Big]. \qquad (31)
\end{aligned}
$$

The last clearly exhibits the infinite energy associate with the infinitely stretched string and indeed is proportional to the proper length $2U$. The remaining, finite part may now be interpreted as the inter-quark potential. Amusing fact is that it is proportional to the inter-quark distance when measured in U-coordinate. One might argue that the inter-quark potential is a Coulomb potential by using the relation Eq.(xxx) but it does *not* have the expected strength of charge-squared. Instead it only has a linear power in N_1. Because of this, we believe the correct interpreation is that the inter-quark potential displays linearly confining potential (as measured by N units of electric fluxes from N static quarks). The tension of flux tube is a constant multiple of fundamental string tension.

The static-quark Wilson loop is

$$
\langle W_{\rm static} \rangle = e^{-E_{\rm reg} T} \sim e^{-\frac{1}{\alpha'}}, \qquad (32)
$$

where we have used the fact that the α'-expansion in anti-de Sitter supergravity corresponds to expansion in $1/\kappa$. It is therefore tempting to interpret that the Wilson loop expectation value of large N gauge theory is proportional to $\exp(-1/g_{\rm YM})$ as envisioned by Shenker or, equivalently, as the world-sheet instanton effect in Type IIB string theory.

7.2 $Q\bar{Q}$ Pair in Gauge Theory

We first construct the

7.3 θ-Dependence of Static Quark Potential

The $d = 4, \mathcal{N} = 4$ super Yang-Mills theory contains two coupling parameters $g^2_{\rm YM}$ and θ, the latter being a coefficient of ${\rm Tr}(\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta})/32\pi^2$. From the underlying Type IIB string theory, they arise from the string coupling parameter $\lambda_{\rm st}$ and Ramond-Ramond zero-form potential C_0. They combine into a holomorphic coupling parameter

$$
\begin{aligned}
\tau &= \frac{\theta}{2\pi} + i \frac{4\pi}{g^2_{\rm YM} N} \\
&= C_0 + i \frac{1}{\lambda_{\rm st}}. \qquad (33)
\end{aligned}
$$

From the gauge theory point of view, one of the interesting question is θ-dependence of the static quark potential. Under $d = 4$ P and CP, the former is odd while the latter is even. Thus, the static quark potential should be an even function of θ.

The θ ranges $(0, 2\pi)$. Then, the periodicity of θ (i.e. T-transformation of $SL(2, \mathbf{Z})$ and invariance of static quark potential under parity transformation dictate immediately that the quark potential should be symmetric under $\theta \to -\theta$ and $\pi - \theta \to \pi + \theta$. This yields cuspy form of the potential. Since the whole physics descends from the $SL(2, \mathbf{Z})$ S-duality, let us make a little calculation in a closely related system: the triple junction network of (p, q) strings. This system will exhibit most clearly the very fact that string tension is reduced most at $\theta = \pi$.

Consider a $(0, 1)$ D-string in the background of Ramond-Ramond zero-form potential. The Born-Infeld Lagrangian reads

$$L_{D1} = \frac{T}{\lambda_{\rm st}} \int dx \sqrt{1 + (\nabla X)^2 - F^2} + C_0 \wedge F \tag{34}$$

Consider a $(1, 0)$ fundamental string attached on D-string at location $x = 0$. The static configuration of the triple string junction is then found by solving the equation of motion. In $A_1 = 0$ gauge,

$$\nabla\Big(\frac{-\nabla A_0}{\sqrt{1 + (\nabla X_9)^2 - (\nabla A_0)^2}} - \lambda_{\rm st} C_0 \Big) = \lambda_{\rm st} \delta(x). \tag{35}$$

The solution is $X_9 = \sqrt{a} A_0$ for a continuous parameter a, where

$$\frac{\nabla A_0}{\sqrt{1 - (1 - a)(\nabla A_0)^2}} = \lambda_{\rm st} \theta(x_1) - \lambda_{\rm st} C_0. \tag{36}$$

Substituting the solution to the Born-Infeld Lagrangian, we find the string tension of D-string:

$$T_D = \begin{cases} \sqrt{\frac{1}{\lambda_{\rm st}^2} + (1 - C_0)}, & x_1 > 0 \\ \sqrt{\frac{1}{\lambda_{\rm st}^2} + C_0^2}, & x_1 < 0. \end{cases} \tag{37}$$

clearly, the tension of $(1, 1)$ string (on which electric field is turned on) attains the minimum when $C_0 = 1/2$, viz. $\theta = \pi$. Moreover, in this case, the D-string bends symmetrically around the junction point $x_1 = 0$, reflecting the fact that P and CP symmetries are restored at $\theta = \pi$.

There is yet another way of understanding the θ-dependence to string tension [2]

8 Multi-Prong Strings as Baryons

Moving a step further, can we manufacture static baryon out of Type IIB strings? The baryon is a gauge singlet configuration obeying N-ality. For the gauge group $U(N)$, this means the

[2] I thank C.G. Callan and K. Intriligator for helpful discussions on this point.

baryon consists of N 'quarks' that carry charges of $[U(1)]^N$ Cartan subalgebra. The string configuration that corresponds to such charge assignment on the D3-brane is the multi-prong string junctions. For example, for gauge group $SU(3)$ realized by three D3-branes, multi-monopole configuration that corresponds to the baryon is triple string junction whose each end is attached to each D3-branes. The N-pronged string junction is a natural generalization of this, as can be checked from counting of multi-monopole states and comparison with the (p,q) charges of Type IIB string theory.

9 Discussion

In this paper, we have explored some aspects of the proposed relation between $d = 4, \mathcal{N} = 4$ supersymmetric gauge theory and maximal supergravity on $adS_5 \times S_5$ using the Type IIB (p,q) strings as probes. From the point of view of D3 brane and gauge theory thereof, semi-infinite strings attached on it are natural realization of quarks and anti-quarks. Whether a given configuration involving quarks and anti-quarks is a BPS configuration or not does depend on relative orientation among the strings (parametrized by angular coordinates on S_5). The physics we have explored, however, did not depend much on it since the quarks and anti-quarks have infinte inertia mass and are nominally stable.

The results we have obtained may be summarized as follows. For a single quark Q (or anti-quark $\bar{Q}$) BPS configuration, near-extremal excitation corresponds to fluctuation of the fundamental string. We have found that the governing equations and boundary conditions do match precisely between the large-N gauge theory and the anti-de Sitter supergravity sides. In due course, we have clarified the emergence of Polchinski's D-brane boundary condition (Dirichlet for perpendicular and Neumann for parallel directions) as the limit $\lambda_{\rm st} \to 0$ is taken. For non-BPS $Q\bar{Q}$ pair configuration, we first have studied inter-quark potential and again have found an agreement between the gauge theory and the anti-de Sitter supergravity results. Measured in units of Higgs expectation value, the potential exhibits linear potential that allows an interpretation of confinement. Because the theory has no mass gap generated by dimensional transmutation, the fact that string tension is measured in units of Higgs expectation value may not be so surprising. We have also explored θ-dependence of the static quark potential by turning on a constant Ramond-Ramond 0-form potential. The $SL(2, \mathbf{Z})$ S-duality of underlying Type IIB string theory implies immediately that the static quark potential exhibits cusp behavior at $\theta = \pi$. The potential strength is the weakest at this point and hints a possible realization of deconfinement transition at $\theta = \pi$. The static baryons $(Q \cdots Q)$ are represented by multi-prong string junctions. As such, it becomes quite complicated to analyze explicitly in either gauge theory or anti-de Sitter supergravity sides.

We think the results in the present paper may be of some help in understanding dynamical issues in the large N limit of superconformal gauge theories. For one thing, it would be very desirable to understand dynamics of dynamical quarks and gauge invariant excitation spectra. While we have indicated that qualitative picture of the excitation spectrum as conjectured by Maldacena would follow from near-extremal excitation of fundamental strings, a definitive answer awaits for a full-fledged study of dynamical quarks.

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10 Appendix: Transmission through Junction

In Section 2, we have studied near-extremal excitation of the DBI string. For harmonic fluctuations, the dynamics was described by one-dimensional Schrödinger equation. In this Appendix, we elaborate self-adjoint extension of the wave equation on a half-line.

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