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Proof of vanishing cohomology at the tachyon vacuum

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Abstract

We prove Sen's third conjecture that there are no on-shell perturbative excitations of the tachyon vacuum in open bosonic string field theory. The proof relies on the existence of a special state A, which, when acted on by the BRST operator at the tachyon vacuum, gives the identity. While this state was found numerically in Feynman-Siegel gauge, here we give a simple analytic expression.

1 Introduction

Following Sen's famous three conjectures [1, 2], there has been an intensive effort to study the physics of tachyon condensation in Witten's cubic open string field theory [3]. The power of open string field theory (OSFT) over conventional CFT methods is that OSFT is an off-shell formulation of open string interactions. Many questions about open string vacua, which must be understood using indirect arguments in CFT, can be rephrased in OSFT as questions about the classical solutions of the OSFT equations of motion.

Unfortunately, finding solutions to the OSFT equations of motion is non-trivial. Indeed, in the standard oscillator basis, these equations become an infinite number of coupled non-linear differential equations and, until recently, much of the work in OSFT has been numerical.

In spite of the approximate nature of the analysis, it has been found that OSFT has a rich structure. Starting from perturbative vacuum on the D25-brane, one can find classical solutions to the equations of motion representing lower-dimensional branes [4, 5, 6, 7] as well as the tachyon vacuum [8, 9, 10, 11], in which there are no branes present. In each case, the energy of these solutions precisely matches the energy of the relevant brane configuration, beautifully demonstrating Sen's first and second conjectures.

Having found solutions representing various vacua, one can attempt to find the spectrum of perturbative states around each solution. In particular, Sen's third conjecture states that around the tachyon vacuum, which represents the absence of any brane at all, there should be no physical states. This conjecture has been checked in two complementary ways. First, the kinetic terms and gauge transformations of certain low-mass excitations were computed to verify that, indeed, there were no on-shell states [12, 13]. Second, it was argued that the full spectrum of states was empty using a trick, which we now describe [14].

The physical states around a given vacuum are given by the cohomology of a BRST operator Q_Ψ. It turns out that the cohomology of Q_Ψ vanishes – meaning that there are no physical states – if and only if there exists a state A such that $Q_\Psi A = \mathcal{I}$, where $\mathcal{I}$ is the identity of the star algebra. Hence, the problem of showing that Q_Ψ has vanishing cohomology reduces to determining whether there is a solution to a single linear equation. This makes the problem amenable to numerical analysis and it was found in [14] that, within the level-truncation approximation, one could find such a state A.

Recently, one of us found an analytic solution to the OSFT equations of motion representing the tachyon vacuum [15]. This solution has now been checked to satisfy the equations of motion, even when contracted with itself [16, 17], and has the correct energy [15], giving an analytic proof of Sen's first conjecture. This solution opens up the possibility that other questions in OSFT, which previously had only been understood numerically, may have nice analytic solutions.

Indeed, in this paper we give a simple proof of Sen's third conjecture. We do this following the method described above: Given the analytic solution Ψ, we find an analytic expression for a state A that satisfies $Q_\Psi A = \mathcal{I}$.

The organization of this paper is as follows: In section 2, we review the relevant aspects of OSFT. In section 3 we present the recently found analytic solution to the equations of motion, Ψ. Next, in section 4, we define a new string field A, which we then prove satisfies $Q_\Psi A = \mathcal{I}$. Finally, in section 5, we discuss the fact that the tachyon vacuum is a limit of a family of pure-gauge solutions and show how this does not spoil our cohomology arguments.

2 Review of OSFT

We begin with a review of some basic aspects of Witten's cubic open string field theory. Since there are many excellent reviews of OSFT [18, 9, 19, 20], we will only touch on some of the more relevant points. The action is given by [3]

$$S = \frac{1}{2}\int \Phi * Q_B\Phi + \frac{1}{3}\int \Phi * \Phi * \Phi. \tag{2.1}$$

The classical field, Φ, is an element of the free string Fock space. For example, for OSFT on a D25-brane background, it has an expansion,

$$\Phi = \int dp \left\{t(p) + A_\mu(p)\alpha^\mu_{-1} + \psi(p)c_0 + \dots\right\} c_1|p\rangle, \tag{2.2}$$

where $t(p)$ is the tachyon, $A_\mu(p)$ is the gauge field and $\psi(p)$ is a ghost field.

The action (2.1) has a large gauge invariance, which makes solving the equations of motion in the non-gauge-fixed theory difficult[1];

$$\Phi \to \Phi + Q_B\Lambda + [\Phi, \Lambda]. \tag{2.3}$$

Globally, fixing a gauge is a subtle issue [21]. Around the perturbative vacuum, however, a suitable choice is Feynman-Siegel gauge;

$$b_0\Phi = 0. \tag{2.4}$$

Most of the numerical work in OSFT was performed in this gauge. However, as we will discuss shortly, there is a different gauge which is more suitable for analytic analysis.

The equations of motion of (2.1) are given by

$$Q_B\Psi + \Psi * \Psi = 0. \tag{2.5}$$

[1] The commutator is taken using the star product and is graded by ghost number. Explicitly,

$$[\Phi_1, \Phi_2] = \Phi_1 * \Phi_2 - (-1)^{\mathrm{gh}(\Phi_1)\mathrm{gh}(\Phi_2)}\Phi_2 * \Phi_1$$

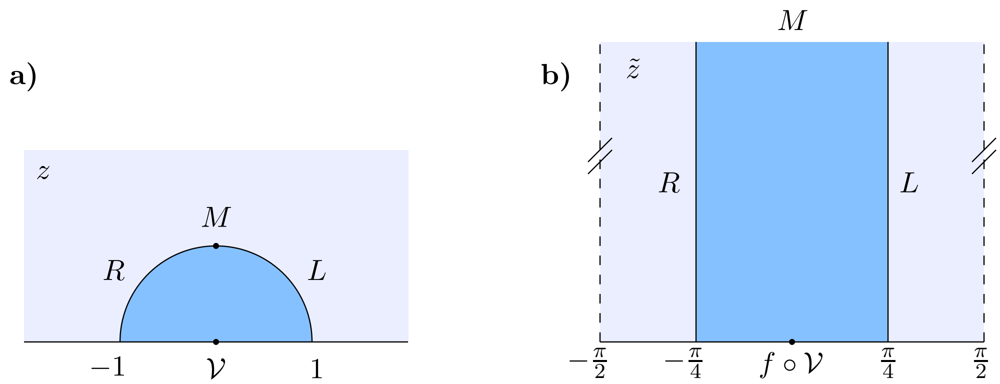

Figure 1: The string field as seen by two coordinate systems. In a) the standard description on upper half plane is illustrated. A vertex operator $\mathcal{V}$ generates a state on the unit circle. The right half, left half and midpoint of the string are labeled as viewed from infinity. Diagram b) gives the same state in the $\tilde{z} = \arctan(z)$ coordinate. The left and right sides of the figure are identified to give a cylinder. The left/right half of the string now lies along the line $\Re(\tilde{z}) = \pm\pi/4$. The midpoint of the string is mapped to infinity.

Given a solution, Ψ, one can re-expand the action around the new vacuum;

$$S(\Psi + \Phi) = \frac{1}{2}\int \Phi * Q_\Psi \Phi + \frac{1}{3}\int \Phi * \Phi * \Phi + \text{constant}. \tag{2.6}$$

The new action takes the same form as the old action: the cubic term is left completely invariant, while the kinetic term is only modified by a change in the BRST operator, $Q_B \to Q_\Psi$, where

$$Q_\Psi \Lambda = Q_B \Lambda + [\Psi, \Lambda]. \tag{2.7}$$

It is straightforward to check that $Q_\Psi^2 = 0$ using the equations of motion of Ψ. Just as the spectrum around the perturbative vacuum was given by the cohomology of Q_B, the spectrum around the new vacuum is given by the cohomology of Q_Ψ.

2.1 OSFT in the $\arctan(z)$ coordinate system

Most of the difficulty in working with OSFT arises from the complexity of the star product. It was one of the key realizations of [22, 15], however, that the star product simplifies when written in a different coordinate frame.

The standard method for specifying states in open string theory is by putting a vertex operator, $\mathcal{V}$, on the boundary of the upper half plane at the point $z = 0$. By the operator-state correspondence we can associate with $\mathcal{V}$ a state $|\mathcal{V}\rangle$ in the string Fock space that lives on the unit circle.

However, there was no reason why we had to choose the upper half plane to define our states. It turns out to be useful to work instead in the coordinate $\tilde{z} = f(z) = \arctan(z)$. Under

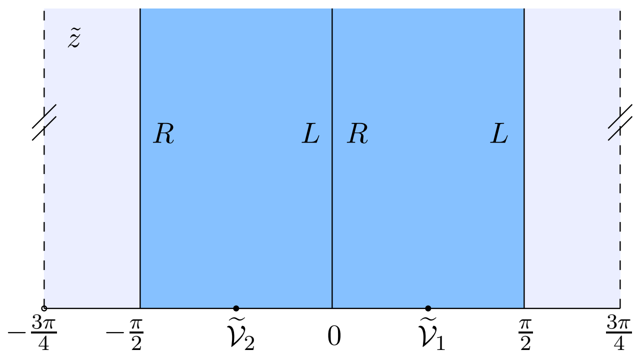

Figure 2: A pictorial description of the star product. Given two states $|\tilde{\mathcal{V}}_1\rangle$ and $|\tilde{\mathcal{V}}_2\rangle$ generated by inserting vertex operators $\tilde{\mathcal{V}}_1$ and $\tilde{\mathcal{V}}_2$ in the $\tilde{z}$ coordinate, the star product, $|\tilde{\mathcal{V}}_1\rangle * |\tilde{\mathcal{V}}_2\rangle$, is computed by gluing the right side of the $|\tilde{\mathcal{V}}_1\rangle$ state to the left side of the $|\tilde{\mathcal{V}}_2\rangle$ state. This gives a cylinder of width $3\pi/2$.

$z \to f(z)$, the upper half plane is mapped to an infinitely tall cylinder as illustrated in figure 1. In this frame, the star product can be described purely geometrically; one simply glues the strips of world-sheet together that correspond to the two string states. This is illustrated in figure 2.

Multiplying n strips of width $\pi/2$ will produce a strip of width $n\pi/2$ and it is useful to consider the class of all such states. When there are no operator insertions, a state described by a strip of width $n\pi/2$ is called a wedge state and is denoted $|n+1\rangle$. These states were first introduced in [23], and obey the algebra,

$$|n\rangle * |m\rangle = |m+n-1\rangle. \tag{2.8}$$

The state $|2\rangle$ is just the original strip of width $\pi/2$ with no vertex operator inserted at the origin and is, thus, the $SL(2,\mathbb{R})$ invariant vacuum $|0\rangle$.

It turns out that taking the limit as the width of the strip tends to infinity leads to a finite state; $|\infty\rangle = \lim_{n\to\infty} |n\rangle$. This state is known as the sliver [23] and as is a projector under star multiplication;

$$|\infty\rangle * |\infty\rangle = |\infty\rangle. \tag{2.9}$$

Notice that multiplying a state Λ by the wedge state of zero width, $|1\rangle$, leaves Λ invariant. Hence, $\mathcal{I} = |1\rangle$ is an identity of the star algebra;

$$\Lambda * \mathcal{I} = \mathcal{I} * \Lambda = \Lambda. \tag{2.10}$$

A useful property of $\mathcal{I}$ is that, at least formally, for any operator $\mathcal{O}$ it obeys [24, 25]

$$\mathcal{O}|\mathcal{I}\rangle = \mathcal{O}^\star|\mathcal{I}\rangle = \tfrac{1}{2}(\mathcal{O}+\mathcal{O}^\star)|\mathcal{I}\rangle, \tag{2.11}$$

where, in the notation of [26], $\mathcal{O}^\star$ denotes BPZ conjugation; $\mathcal{O}^\star = I \circ \mathcal{O}$, where $I(z) = -1/z$.

2.2 Some important operators

In general, each of the familiar operators in the $\tilde{z}$ coordinate can be pulled back into the z coordinate using $f^{-1}(\tilde{z}) = \tan(\tilde{z})$. We will occasionally denote such an operator using a tilde; e.g. $\tilde{c}(\tilde{z}) = f^{-1} \circ c(z)$. It is also useful to make the following definitions:

$$\mathcal{L}_0 = f^{-1} \circ L_0, \qquad \mathcal{B}_0 = f^{-1} \circ b_0, \qquad K_1 = f^{-1} \circ L_{-1}, \qquad B_1 = f^{-1} \circ b_{-1}. \tag{2.12}$$

Just as L_0 gave the mass level of fields in the z coordinate, $\mathcal{L}_0$ is the analogous level in the $\tilde{z}$ coordinates. Similarly, while the standard gauge fixing condition in the z-coordinate was $b_0\Phi = 0$, in the $\tilde{z}$-coordinate, one uses $\mathcal{B}_0\Phi = 0$.

Explicit mode expansions of $\mathcal{L}_0$ and $\mathcal{B}_0$ are given by

$$\mathcal{L}_0 = L_0 + \tfrac{2}{3}L_2 - \tfrac{2}{15}L_4 + \cdots\,, \tag{2.13}$$

$$\mathcal{B}_0 = b_0 + \tfrac{2}{3}b_2 - \tfrac{2}{15}b_4 + \cdots\,. \tag{2.14}$$

Note that while L_0 and b_0 are BPZ dual to themselves, their script cousins are not and we also have operators $\mathcal{L}_0^\star$ and $\mathcal{B}_0^\star$, which are given by $L_n^\star = (-1)^n L_{-n}$ and $b_n^\star = (-1)^n b_{-n}$. These obey the commutation relations[2],

$$[\mathcal{L}_0, \mathcal{L}_0^\star] = \mathcal{L}_0 + \mathcal{L}_0^\star, \tag{2.15}$$

as well as

$$[\mathcal{L}_0, \mathcal{B}_0] = [\mathcal{L}_0^\star, \mathcal{B}_0^\star] = 0, \qquad [\mathcal{L}_0^\star, \mathcal{B}_0] = -\mathcal{B}_0 - \mathcal{B}_0^\star, \qquad [\mathcal{L}_0, \mathcal{B}_0^\star] = \mathcal{B}_0 + \mathcal{B}_0^\star. \tag{2.16}$$

An important property of $\mathcal{L}_0$ is that the wedge states can be represented in the form [28, 29, 23, 30]

$$|r\rangle = U_r^\star |0\rangle, \tag{2.17}$$

where $U_r = (2/r)^{\mathcal{L}_0}$. The operators U_r and $U_s^\star$ obey the important relation [30],

$$U_r U_s^\star = U^\star_{2+\frac{2}{r}(s-2)} U_{2+\frac{2}{s}(r-2)}, \tag{2.18}$$

which can be used to derive (2.15).

The operators K_1 and B_1 take a very simply form,

$$K_1 = L_1 + L_{-1}, \qquad B_1 = b_1 + b_{-1}. \tag{2.19}$$

[2] These commutation relations are an important property of the conformal frame of the sliver. Recently [26], it has been shown that the conformal frames of other projectors, known as special projectors, lead to similar algebras; $[\mathcal{L}_0, \mathcal{L}_0^\star] = s(\mathcal{L}_0 + \mathcal{L}_0^\star)$. These special projectors have many similarities with the sliver and can be used to solve the ghostnumber zero equations of motion [27, 26].

These operators were first studied in [23], where it was shown that they are derivations of the star algebra;

$$K_1(\Phi_1 * \Phi_2) = (K_1\Phi_1) * \Phi_2 + \Phi_1 * (K_1\Phi_2). \tag{2.20}$$

$$B_1(\Phi_1 * \Phi_2) = (B_1\Phi_1) * \Phi_2 + (-1)^{\mathrm{gh}(\Phi_1)}\Phi_1 * (B_1\Phi_2). \tag{2.21}$$

They also annihilate the wedge states;

$$K_1|r\rangle = B_1|r\rangle = 0. \tag{2.22}$$

In the $\tilde{z}$ coordinates they take the form,

$$K_1 = \oint d\tilde{z}\, T(\tilde{z}), \qquad B_1 = \oint d\tilde{z}\, b(\tilde{z}). \tag{2.23}$$

It is also useful to define the "left" and "right" parts of these operators, which are given by taking only the left or right parts – as viewed from infinity – of the contour integral;

$$K_1^{L,R} = \oint_{\gamma^{L,R}} d\tilde{z}\ T(\tilde{z}), \qquad B_1^{L,R} = \oint_{\gamma^{L,R}} d\tilde{z}\ b(\tilde{z}). \tag{2.24}$$

In the $\tilde{z}$ coordinates, the contours, $\gamma^{L,R}$, are given by the vertical lines on the right and left of the strip. Note that, $K_1^L + K_1^R = K_1$ and $B_1^L + B_1^R = B_1$. Also,

$$K_1^L(\Phi_1 * \Phi_2) = (K_1^L\Phi_1) * \Phi_2, \qquad B_1^L(\Phi_1 * \Phi_2) = (B_1^L\Phi_1) * \Phi_2, \tag{2.25}$$

$$K_1^R(\Phi_1 * \Phi_2) = \Phi_1 * (K_1^R\Phi_2), \qquad B_1^R(\Phi_1 * \Phi_2) = (-1)^{\mathrm{gh}(\Phi_1)}\Phi_1 * (B_1^R\Phi_2). \tag{2.26}$$

An important property of the operators $K_1^{L,R}$ is that they act as a derivative with respect to the width of the state. This follows from (2.24). Since the $K^{L,R}$ are just integrals of T in the $\tilde{z}$ coordinate and $\int T(\tilde{z})$ is the world-sheet Hamiltonian, $\epsilon K^{R,L}$ can be thought of as adding/subtracting an infinitesimal strip of with ϵ from the right/left of the world-sheet. This gives the useful identity,

$$\partial_n|n\rangle = \pm\tfrac{\pi}{2}K_1^{R,L}|n\rangle, \tag{2.27}$$

which can be integrated to give

$$|n\rangle = e^{\pm\frac{\pi}{2}(n-2)K_1^{R,L}}|0\rangle. \tag{2.28}$$

The operators $K_1^{L,R}$, $\mathcal{L}_0$ and $\mathcal{L}_0^\star$, as well as $B_1^{L,R}$, $\mathcal{B}_0$ and $\mathcal{B}_0^\star$ are related through the identities,

$$K_1^L - K_1^R = \tfrac{2}{\pi}(\mathcal{L}_0 + \mathcal{L}_0^\star), \qquad B_1^L - B_1^R = \tfrac{2}{\pi}(\mathcal{B}_0 + \mathcal{B}_0^\star), \tag{2.29}$$

which follow from the definitions of these operators. Using (2.29), we can rewrite (2.28) as

$$|n\rangle = e^{\frac{(2-n)}{2}(\mathcal{L}_0+\mathcal{L}_0^\star)}|0\rangle. \tag{2.30}$$

This expression can be related to (2.17) using the identity,

$$e^{\frac{(2-n)}{2}(\mathcal{L}_0+\mathcal{L}_0^\star)} = U_n^\star U_n. \tag{2.31}$$

A more general collection of such identities can be found in [30, 31, 32, 15].

3 The exact tachyon vacuum solution

In this section, we review the exact tachyon vacuum state found in [15]. Define

$$\psi_n = \tfrac{2}{\pi} c_1|0\rangle * B_1^L|n\rangle * c_1|0\rangle. \tag{3.32}$$

Then the tachyon vacuum is given by[3]

$$\Psi = \lim_{N\to\infty}\left(\psi_N - \sum_{n=0}^{N}\partial_n\psi_n\right). \tag{3.33}$$

Formally, the ψ_N piece vanishes in level truncation as $N \to \infty$, but it gives finite contributions to the energy and is required for Ψ to satisfy the equations of motion when contracted with itself [16, 17];

$$\langle\Psi|Q_B\Psi\rangle + \langle\Psi|\Psi * \Psi\rangle = 0. \tag{3.34}$$

We will see that this term is also required to give a complete proof that the cohomology of Q_Ψ vanishes.

The solution satisfies the gauge fixing condition,

$$\mathcal{B}_0\Psi = 0, \tag{3.35}$$

which as alluded to earlier, is the analogue of Feynman-Siegel gauge fixing in the $\tilde{z}$-coordinate.

The states $-\partial_n\psi_n$ can be written using (2.27) as

$$-\partial_n\psi_n = c_1|0\rangle * B_1^L K_1^L|n\rangle * c_1|0\rangle. \tag{3.36}$$

These states take a simple form in the $\tilde{z}$ coordinate, as illustrated in figure 3.

4 Proof that Q_Ψ has no cohomology

Having defined Ψ, we can now turn to the main aim of this paper, to prove that Q_Ψ has vanishing cohomology so that there are no on-shell perturbative states around the tachyon vacuum. As discussed in the introduction, we can do this using a trick, which we state as a simple lemma:

Lemma: The cohomology of a BRST operator Q_Ψ vanishes if and only if there exists a string field A such that $Q_\Psi A = \mathcal{I}$.

Proof: First, suppose that Q_Ψ has no cohomology. Consider $Q_\Psi\mathcal{I} = Q_B\mathcal{I} + \Psi*\mathcal{I} - \mathcal{I}*\Psi = Q_B\mathcal{I}$. Since, as was first shown in [24, 25], $Q_B\mathcal{I} = 0$, it follows that $Q_\Psi\mathcal{I} = 0$. Since $\mathcal{I}$ is Q_Ψ-closed and Q_Ψ has no cohomology, there must exist some A such that $\mathcal{I} = Q_\Psi A$.

[3] The term $-\partial_n\psi_n$ for $n = 0$ can be defined by carefully taking the limit. Explicitly, one finds $Q_B B_1^L c_1|0\rangle$.

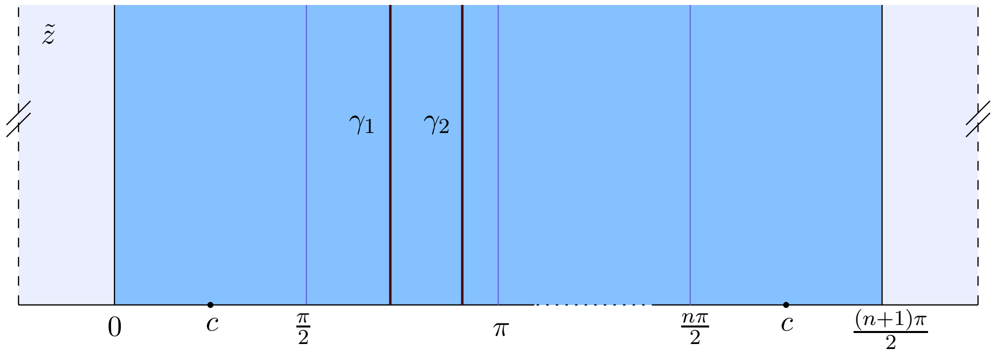

Figure 3: The state $-\partial_n \psi_n$ is given by a strip of width $\frac{\pi}{2}(n+1)$ with two insertions of $c(\tilde{z})$ as well as two contour integrals of $T(\tilde{z})$ and $b(\tilde{z})$ along the curves γ_1 and γ_2.

Now suppose, instead, that we have a state A such that $Q_\Psi A = \mathcal{I}$. Suppose we also have some Q_Ψ-closed state Λ such that $Q_\Psi \Lambda = 0$. Then

$$Q_\Psi(A * \Lambda) = (Q_\Psi A) * \Lambda = \mathcal{I} * \Lambda = \Lambda, \tag{4.37}$$

so that Λ is Q_Ψ-exact. Since any Q_Ψ-closed state is also Q_Ψ-exact, it follows that Q_Ψ has no cohomology.

Such an operator A is known in the math literature as a homotopy operator. Note that the existence of A proves that the cohomology of Q_Ψ vanishes at all ghost numbers, not just ghost number zero as required by Sen's conjectures.[4]

4.1 Finding the state A

We now describe how to find an A satisfying,

$$Q_\Psi A = Q_B A + \Psi * A + A * \Psi = \mathcal{I}. \tag{4.38}$$

Although (4.38) is a linear equation for A, a blind search for a solution could be very difficult. Fortunately, for the Feynman Siegel gauge solution, (4.38) was solved numerically in [14] and we can use the results found there to guess a solution.

Surprisingly, it was found in [14] that, in Feynman-Siegel gauge, A takes the approximate form,

$$A_{\rm FS} \sim \frac{1}{L_0} b_0 \mathcal{I}. \tag{4.39}$$

Curiously, this form of $A_{\rm FS}$ is the state one would write down if one was trying to show that, in the *perturbative* vacuum, Q_B had vanishing cohomology. Indeed one has

$$Q_B A_{\rm FS} = \mathcal{I} - |0\rangle, \tag{4.40}$$

[4] This seems to contradict the numerical results of [13]. Nonzero cohomology at other ghost numbers has also been found for the so-called universal solution [33] in [34].

so that one finds the identity state minus the one piece of the identity that is in the cohomology of Q_B.

A natural guess for the $\mathcal{B}_0$-gauge solution is to take the same form for A, but with b_0 and L_0 replaced by their counterparts in the $\tilde{z}$ coordinate, $\mathcal{B}_0$ and $\mathcal{L}_0$;

$$A = \frac{1}{\mathcal{L}_0}\mathcal{B}_0\mathcal{I}. \tag{4.41}$$

It turns out that this A can be written in a nicer form, as an integral over wedge states with insertions. Using (2.11), we have

$$A = \frac{1}{2\mathcal{L}_0}(\mathcal{B}_0 + \mathcal{B}_0^\star)\mathcal{I}. \tag{4.42}$$

Since $(\mathcal{B}_0 + \mathcal{B}_0^\star)$ raises the $\mathcal{L}_0$-level by one, we may rewrite (4.42) as

$$A = \tfrac{1}{2}(\mathcal{B}_0 + \mathcal{B}_0^\star)\frac{1}{\mathcal{L}_0 + 1}\mathcal{I}. \tag{4.43}$$

This can be further simplified by writing

$$\frac{1}{\mathcal{L}_0 + 1} = \int_0^1 z^{\mathcal{L}_0} dz = \int_0^1 dz\, U_{2/z}. \tag{4.44}$$

Using (2.18), we have

$$U_{2/z}\mathcal{I} = U_{2/z}U_1^\star|0\rangle = U_{2-z}^\star|0\rangle = |2-z\rangle, \tag{4.45}$$

which yields[5]

$$A = \tfrac{1}{2}(\mathcal{B}_0 + \mathcal{B}_0^\star)\int_0^1 dz\,|2-z\rangle = \tfrac{1}{2}(\mathcal{B}_0 + \mathcal{B}_0^\star)\int_1^2 dr\,|r\rangle. \tag{4.47}$$

Using (2.29) and (2.22) this becomes

$$A = \tfrac{\pi}{2}B_1^L\int_1^2 dr\,|r\rangle. \tag{4.48}$$

This state has a simple geometric interpretation, as shown in figure 4.

[5]This result can also be found directly in the $\mathcal{L}_0$-level expansion;

$$\begin{aligned} A &= \frac{1}{2\mathcal{L}_0}(\mathcal{B}_0 + \mathcal{B}_0^\star)\sum_{n=0}^\infty \frac{1}{2^n n!}(\mathcal{L}_0 + \mathcal{L}_0^\star)^n|0\rangle = \frac{1}{2}\sum_{n=0}^\infty(\mathcal{B}_0 + \mathcal{B}_0^\star)\frac{1}{2^n(n+1)!}(\mathcal{L}_0 + \mathcal{L}_0^\star)^n|0\rangle \\ &= \tfrac{1}{2}(\mathcal{B}_0 + \mathcal{B}_0^\star)\frac{e^{\frac{1}{2}(\mathcal{L}_0 + \mathcal{L}_0^\star)} - 1}{(\mathcal{L}_0 + \mathcal{L}_0^\star)/2}|0\rangle = \tfrac{1}{2}(\mathcal{B}_0 + \mathcal{B}_0^\star)\int_1^2 dr\, e^{\frac{2-r}{2}(\mathcal{L}_0 + \mathcal{L}_0^\star)}|0\rangle = \tfrac{1}{2}(\mathcal{B}_0 + \mathcal{B}_0^\star)\int_1^2 dr\,|r\rangle. \end{aligned} \tag{4.46}$$

$$A = \tfrac{\pi}{2}\int_1^2 dr$$

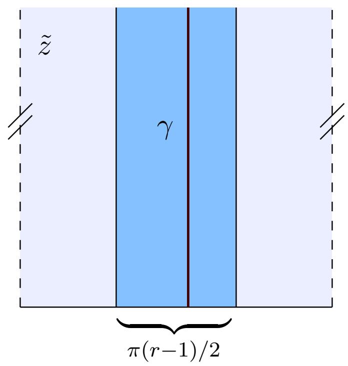

Figure 4: The state A can be represented as a sum over wedge states $|r\rangle$. The only operator insertion is a single contour integral of $b(\tilde{z})$ along the curve γ.

4.2 Computation of $Q_\Psi A$

The first term in $Q_\Psi A$ is just $Q_B A$. This is given by

$$Q_B A = \tfrac{\pi}{2} K_1^L \int_1^2 |r\rangle = -\int_1^2 dr\, \partial_r |r\rangle = \mathcal{I} - |0\rangle, \tag{4.49}$$

which reproduces the Feynman-Siegel gauge result, (4.40).

Next we must compute the star-products $\Psi * A$ and $A * \Psi$. Because the tachyon vacuum solution is twist invariant, these two computations are related to each other by a twist. Hence, we need to compute just one of them, $\Psi * A$.

Since $\Psi = \psi_N - \sum_{m=0}^{n} \partial_n \psi_n$, we begin by evaluating $\psi_n * A$. Using (3.36), we have

$$\psi_n * A = \int_1^2 dr\, c_1|0\rangle * B_1^L|n\rangle * c_1|0\rangle * B_1^L|r\rangle, \tag{4.50}$$

which we can rewrite using (2.25) as

$$\int_1^2 dr\, c_1|0\rangle * B_1^L B_1^R (|n\rangle * c_1|0\rangle * |r\rangle). \tag{4.51}$$

Now, using $B_1^L B_1^R = B_1^L(B_1 - B_1^L) = B_1^L B_1$ this becomes

$$\int_1^2 dr\, c_1|0\rangle * B_1^L B_1(|n\rangle * c_1|0\rangle * |r\rangle) = \int_1^2 dr\, c_1|0\rangle * B_1^L(|n\rangle * |0\rangle * |r\rangle), \tag{4.52}$$

where we have used the derivation property (2.20) of B_1 as well as (2.22). It follows that

$$\psi_n * A = \int_1^2 dr\, B_1^R(c_1|0\rangle * |n+r\rangle). \tag{4.53}$$

Similarly, one can compute $A * \psi_n$ either by repeating the above computation or by exploiting twist symmetry. Either way, one finds

$$A * \psi_n = \int_1^2 dr\, B_1^L(|r+n\rangle * c_1|0\rangle). \tag{4.54}$$

Now consider $-\sum_{n=0}^N \partial_n \psi_n * A$. Since n and r appear only in the combination $n+r$, we can replace the derivative ∂_n with ∂_r. This gives

$$-\sum_{n=0}^N \partial_n \psi_n * A = -\sum_{n=0}^N \int_1^2 dr\, B_1^R(c_1|0\rangle * \partial_r|n+r\rangle) = \sum_{n=0}^N B_1^R(c_1|0\rangle * \{|n+1\rangle - |n+2\rangle\}). \tag{4.55}$$

Notice that the sum can now be trivially performed since

$$\sum_{n=0}^N |n+1\rangle - |n+2\rangle = \mathcal{I} - |N+2\rangle. \tag{4.56}$$

Hence, we find

$$-\sum_{n=0}^N \partial_n \psi_n * A = B_1^R c_1|0\rangle - B_1^R(c_1|0\rangle * |N+2\rangle). \tag{4.57}$$

Similarly, one can compute

$$-A * \sum_{n=0}^N \partial_n \psi_n = B_1^L c_1|0\rangle - B_1^L(|N+2\rangle * c_1|0\rangle). \tag{4.58}$$

Using (4.53), (4.54), (4.57) and (4.58), we find, in total, that

$$\Psi * A + A * \Psi = |0\rangle - \Sigma, \tag{4.59}$$

where the state Σ is given by

$$\Sigma = B_1^R\left\{c_1|0\rangle * \left(|N+2\rangle - \int_1^2 dr\,|N+r\rangle\right)\right\} + B_1^L\left\{\left(|N+2\rangle - \int_1^2 dr\,|N+r\rangle\right) * c_1|0\rangle\right\}. \tag{4.60}$$

Now, as $N \to \infty$, the state $|N\rangle$ limits to the sliver so that $|N+2\rangle - \int_1^2 dr\,|N+r\rangle \to 0$ as $N \to \infty$. In fact, it is straightforward to check that, in the level expansion, it goes to zero as $\mathcal{O}(N^{-3})$. Hence, when we remove the regulator we find

$$\Psi * A + A * \Psi = |0\rangle. \tag{4.61}$$

Note that it was important to include the ψ_N piece in Ψ to cancel out the surface terms in the sums (4.57) and (4.58). Combining (4.61) with (4.49), we find the desired result;

$$Q_\Psi A = Q_B A + \Psi * A + A * \Psi = \mathcal{I}. \tag{4.62}$$

This proves that the cohomology of Q_Ψ is empty.

4.3 Comparison with vacuum string field theory

It is interesting to compare our results with the results of vacuum string field theory (VSFT) [35, 36]. In VSFT, the BRST operator around the tachyon vacuum is taken, by ansatz, to be a simple pure ghost operator. For example, one of the early choices was the zero mode of the c-ghost, c_0. To show that c_0 has empty cohomology, one notes that $\{c_0, b_0\} = 1$, so that b_0 plays the role of our string field, A.

This analogy can be made a little closer. Just as $b_0^2 = 0$, it happens that $A * A = 0$. This property is easy to see from the geometric form of A in figure 4. Moreover, just as b_0 is a Hermitian operator, one can also construct a Hermitian operator $\hat{A}$ defined by

$$\hat{A}\Phi = A * \Phi + (-1)^{\mathrm{gh}(\Phi)}\Phi * A, \tag{4.63}$$

which satisfies $\hat{A}^2 = 0$ and $\{Q_\Psi, \hat{A}\} = 1$, as well as the Hermiticity property, $\langle\Phi_1|\hat{A}\Phi_2\rangle = \langle\hat{A}\Phi_1|\Phi_2\rangle$. Since VSFT is thought to be a singular limit of ordinary OSFT, in which the BRST operator becomes a c-ghost operator inserted at the midpoint [37], it would be interesting to see whether $\hat{A}$ becomes a simple operator formed out of just the b-ghost in this limit.

4.4 Brane decay in the presence of other branes

In this subsection, we show that one can extend our cohomology arguments to the case where we include other branes that have not decayed. Consider OSFT around a 2 brane background, which we describe by adding Chan-Paton indices to our string fields;

$$\phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix}, \tag{4.64}$$

where $\phi^\dagger = \phi$. To decay one of the branes, we may turn on

$$\psi = \begin{pmatrix} \Psi & 0 \\ 0 & 0 \end{pmatrix}. \tag{4.65}$$

The BRST operator Q_ψ acts as

$$Q_B\phi + [\psi, \phi] = \begin{pmatrix} Q_B\Phi_{11} + [\Psi, \Phi_{11}] & Q_B\Phi_{12} + \Psi * \Phi_{12} \\ Q_B\Phi_{21} - (-1)^{\mathrm{gh}(\Phi_{21})}\Phi_{21} * \Psi & Q_B\Phi_{22} \end{pmatrix}. \tag{4.66}$$

Since we have decayed the first brane, we expect that there are no on-shell 11, 12 or 21 strings. This implies that the three BRST-operators,

$$Q_{11}\Phi = Q_B\Phi + [\Psi, \Phi], \qquad Q_{12}\Phi = Q_B\Phi + \Psi * \Phi, \qquad \text{and} \qquad Q_{21}\Phi = Q_B\Phi - (-1)^{\mathrm{gh}(\Phi)}\Phi * \Psi, \tag{4.67}$$

should all have vanishing cohomology. Since $Q_{11} = Q_\Psi$, there is nothing new to show. For Q_{12} and Q_{21}, our old argument still works as long as we are careful about left multiplication versus right multiplication. Suppose that $Q_{12}\Phi = 0$. Then it is easy to check that

$$Q_{12}(A * \Phi) = (Q_\Psi A) * \Phi = \Phi. \tag{4.68}$$

Thus, as we expect, every closed state is exact. Similarly, if $Q_{21}\Phi = 0$, we have

$$Q_{21}(-\Phi * A) = \Phi * (Q_\Psi A) = \Phi. \tag{4.69}$$

Putting the A on the left of Φ would not work. Hence, we have shown that the only open strings that remain in the spectrum are those that live on the undecayed brane. This argument generalizes to the case of n decayed branes and m undecayed branes in the expected way.

5 Pure-gauge-like form

One of the curious features of the analytic tachyon vacuum is that it is very close to being pure gauge. Indeed, it was found by Okawa [16] that if one ignores the ψ_N term – which one can in the L_0 basis[6] – the full solution can be written as the limit, $\lambda \to 1$, of the state,

$$\Psi_\lambda = U_\lambda * Q_B V_\lambda, \tag{5.1}$$

where[7]

$$U_\lambda = 1 - \lambda\Phi, \qquad V_\lambda = \frac{1}{1 - \lambda\Phi} \tag{5.3}$$

and

$$\Phi = B_1^L c_1 |0\rangle. \tag{5.4}$$

When $\lambda < 1$ the states U_λ and V_λ are well defined in the level-expansion and the state Ψ_λ is a true pure-gauge solution with zero energy.

Obviously, the tachyon solution itself, cannot be a pure-gauge solution related by a continuous deformation to the vacuum for two reasons. First, the energy of such a solution would have to be zero in contradiction with the now proven Sen's first conjecture. Second, it would imply that the cohomology of the kinetic operator at the true vacuum would be isomorphic to the cohomology of Q_B in contradiction with Sen's third conjecture. It is therefore interesting to understand how the solution ceases to be a pure gauge at $\lambda = 1$ and how Sen's conjectures are rescued.

[6] Interestingly, in the $\mathcal{L}_0$ level truncation we find $\Psi_\lambda = \frac{\lambda}{1-\lambda} Q\Phi + \cdots$, where the dots stand for terms of $\mathcal{L}_0$-level higher than 0. Hence, the $\lambda \to 1$ limit does not exist in this basis.

[7] V_λ is defined by the Taylor series,

$$\frac{1}{1-\lambda\Phi} = \sum_{n=0}^{\infty} \lambda^n \Phi^n; \qquad \Phi^n = \underbrace{\Phi * \Phi * \ldots * \Phi}_{n}. \tag{5.2}$$

The basic property of the pure-gauge solutions is that

$$U_\lambda * V_\lambda = V_\lambda * U_\lambda = \mathcal{I}. \tag{5.5}$$

This allows one to define an isomorphism between the states in the perturbative vacuum and their corresponding states in the pure-gauge vacuum;

$$\phi \to \mathcal{F}_\lambda[\phi] = U_\lambda * \phi * V_\lambda, \tag{5.6}$$

which has inverse, $\mathcal{F}_\lambda^{-1}[\phi] = V_\lambda * \phi * U_\lambda$.

This isomorphism relates the original BRST operator, Q_B, to the new BRST operator, Q_λ, around the pure-gauge vacuum;

$$Q_\lambda(\mathcal{F}_\lambda[\phi]) = \mathcal{F}_\lambda[Q_B\phi]. \tag{5.7}$$

It follows that the two operators have identical cohomology.

We can now ask how (5.5)-(5.7) break down when $\lambda \to 1$. Clearly, since the right hand side of (5.5) is independent of λ, we will find $\lim_{\lambda\to 1} U_\lambda * V_\lambda = \lim_{\lambda\to 1} V_\lambda * U_\lambda = \mathcal{I}$. However, the state V_λ by itself diverges in the $\mathcal{L}_0$ level-expansion, although it appears to remain finite in the L_0 expansion.

Similar divergences occur when we consider $\mathcal{F}_\lambda(\phi)$ and its inverse. For concreteness, take $\phi = c\mathcal{O}|0\rangle$, where $\mathcal{O}$ is a matter operator that satisfies

$$[\mathcal{L}_0, \mathcal{O}] = h\mathcal{O}. \tag{5.8}$$

Following the rules of [15] we find

$$\begin{aligned}\mathcal{F}_\lambda[\phi] = c\mathcal{O}(0)|0\rangle + \sum_{m=1}^{\infty} \lambda^m U^\star_{m+2} U_{m+2} \Big\{ &\frac{1}{2}\tilde{\mathcal{O}}(x)(\tilde{c}(x) + \tilde{c}(-x)) + \frac{1}{2}\tilde{\mathcal{O}}(y)(\tilde{c}(x) - \tilde{c}(y)) \\ &- \frac{1}{\pi}(\mathcal{B}_0 + \mathcal{B}^\star)\Big(\tilde{\mathcal{O}}(x)\tilde{c}(x)\tilde{c}(-x) + \tilde{\mathcal{O}}(y)(\tilde{c}(x) - \tilde{c}(y))\tilde{c}(-x)\Big)\Big\}|0\rangle,\end{aligned} \tag{5.9}$$

where, for brevity, we have introduced $x = \frac{\pi}{4}m$ and $y = \frac{\pi}{4}(m-2)$. Using this form, it is straightforward to work out the coefficients in the $\mathcal{L}_0$ basis

$$\begin{aligned}\mathcal{F}_\lambda[\phi] =& \frac{1}{1-\lambda} c\mathcal{O}(0)|0\rangle + \\ &+ \frac{\lambda}{(1-\lambda)^2}\left[-\frac{1}{2}(\mathcal{L}_0 + \mathcal{L}_0^\star)\,\tilde{c}\tilde{\mathcal{O}}(0) + (\mathcal{B}_0 + \mathcal{B}_0^\star)\,\tilde{c}\tilde{\partial}\tilde{c}\tilde{\mathcal{O}}(0) + \frac{\pi}{4}\big((1-\lambda)\tilde{\partial}\tilde{c}\tilde{\mathcal{O}}(0) + \tilde{c}\tilde{\partial}\tilde{\mathcal{O}}(0)\big)\right]|0\rangle + \\ &+ \cdots,\end{aligned} \tag{5.10}$$

where the dots stand for terms of higher $\mathcal{L}_0$-level. We see that, due to the presence of poles at $\lambda = 1$, the state $\mathcal{F}_{\lambda=1}[\phi]$ does not make sense in this basis. Note that one cannot rescale ϕ_λ

by a positive power of $1-\lambda$ to get a finite representative of the cohomology, since the maximal order of the poles grows with level.

We find similar behavior when we compute $\mathcal{F}[\phi]$ in the ordinary L_0 level truncation. Since such computations are more difficult, we have restricted ourselves to the case where $\mathcal{O}$ is a weight one primary. This case is of particular interest, as any cohomology class of Q_B has a representative of this form.

Computing the coefficient of $\mathcal{F}_\lambda[\phi]$ in front of $c\mathcal{O}(0)|0\rangle$, we find

$$1+\sum_{m=1}^{\infty}\lambda^m\frac{m+2}{2}\left[\frac{1}{2}+\frac{\alpha}{\pi}\left(1-\left(\frac{\sin\alpha}{\sin 2\alpha}\right)^2\right)-\frac{1}{\pi}\sin 2\alpha+\frac{1}{2\pi}\left(\frac{\sin\alpha}{\sin 2\alpha}\right)^2(\sin 4\alpha-\sin 2\alpha)\right], \tag{5.11}$$

where $\alpha=\pi/(m+2)$. Since the summand behaves as

$$\lambda^m\left[\frac{m+2}{4}-\frac{1}{2}+\frac{\pi^2}{12}\frac{1}{(m+2)^2}+\cdots\right],$$

we see that, apart from the mild polylogarithmic singularities at $\lambda=1$, which are present also for the solution Ψ_λ itself, $\mathcal{F}_\lambda[\phi]$ contains double and single poles and therefore the limit $\lim_{\lambda\to 1}\mathcal{F}_\lambda[\phi]$ does not exist.

So far in this discussion we have tried to show that elements of the cohomology of Q_B are not mapped via $\mathcal{F}$ to elements of the cohomology of Q_Ψ. However, it is also interesting to ask why A cannot be pulled back to the perturbative vacuum to show that Q_B has no cohomology. Hence, we compute

$$\begin{aligned}\mathcal{F}^{-1}(A)=\frac{1}{1-\Phi}*A*(1-\Phi)&=\tfrac{\pi}{2}\int_1^2 dr\,(1+\sum_{n=1}^{\infty}B_1^L|n\rangle*c_1|0\rangle)*B_1^L|r\rangle*(1-B_1^Lc_1|0\rangle)\\ &=\tfrac{\pi}{2}\sum_{n=1}^{\infty}\int_1^2 dr\,B_1^L|n+r-1\rangle.\end{aligned} \tag{5.12}$$

This simplifies to

$$\mathcal{F}^{-1}(A)=\tfrac{\pi}{2}\int_1^{\infty}dr\,B_1^L|r\rangle, \tag{5.13}$$

which should be thought of as the "A" of the perturbative vacuum. We can now act on this state with Q_B;

$$Q_B(\mathcal{F}_{-1}(A))=-\int_1^{\infty}dr\,\partial_r|r\rangle=\mathcal{I}-|\infty\rangle. \tag{5.14}$$

Happily, we do not find just the identity on the right hand side, so the cohomology of Q_B need not vanish.[8] Equation (5.14) has a nice interpretation in terms of half strings. Consider a state

[8] Formally one could write $Q_B\mathcal{F}^{-1}(A)=\mathcal{F}^{-1}(Q_\Psi A)=\mathcal{F}^{-1}(\mathcal{I})=V_{\lambda=1}*U_{\lambda=1}$. Using (5.14), this would imply $V*U=\mathcal{I}-|\infty\rangle$ suggesting that V and U are a nontrivial pair of partial isometries as first proposed in [38]. On the other hand a direct computation seems to yield $V*U=\mathcal{I}$ in the strict $\lambda\to 1$ limit, in both L_0 and $\mathcal{L}_0$ level truncation. It would be nice to understand this anomaly more deeply.

ϕ which is Q_B-closed, but whose left half has no overlap with the right half of $|\infty\rangle$. In other words, $|\infty\rangle * \phi = 0$. It follows that ϕ is Q_B-exact. To see this, consider

$$Q_B(\mathcal{F}^{-1}(A) * \phi) = (\mathcal{I} - |\infty\rangle) * \phi = \phi. \tag{5.15}$$

A similar result holds for states whose right half has no overlap with the left half of $|\infty\rangle$. This implies that the entire cohomology of Q_B should be found on states whose left and right halves are given by the left and right halves of $|\infty\rangle$. Such a set of states is easy to find. For example, at ghost number 0, the cohomology of Q_B is represented by just $|\infty\rangle$ itself. At ghost number 1, which is the interesting case, the cohomology of Q_B has representatives given by weight $(0,0)$ primaries of the form cJ, where J is a weight one matter primary. Inserting these operators at the midpoint of $|\infty\rangle$ gives a set of ghost number 1 states in the cohomology of Q_B with left and right halves given by the left and right halves of $|\infty\rangle$.

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