Relational Contracts in Competitive Labor Markets*

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Abstract

This paper characterizes the distribution of jobs in a relational contracting model where both employed and unemployed workers compete for jobs. In equilibrium, identical firms offer a continuous distribution of contracts, with some firms offering high-wage, high-productivity contracts and others offering low-wage, low-productivity contracts. An increase in on-the-job-search affects equilibrium contracts in two ways. First, by decreasing retention rates it leads to a deterioration in the quality of jobs. Second, by reducing the opportunity cost of employment it allows firms to enter at the bottom of the market. If employed workers receive better offers than the unemployed then free entry leads to full employment, and wage dispersion rather than unemployment incentivizes workers.

1 Introduction

Employment contracts are typically incomplete, lacking explicit incentives for important aspects of a worker's behavior, such as teamwork, initiative and judgment. Firms thus motivate workers through the promise of future wages inherent in long-term employment relationships. The effectiveness of such incentives increases with the longevity of the relationship, which in turn depends on the job offers of competing firms. The goal of this paper is to study the intra-firm contracting problem in the context of a competitive labor market, and characterize the equilibrium distribution of jobs. In doing so the paper furthers our understanding of the impact of turnover on firm productivity and the role of unemployment in incentivizing workers.

We show that labor market competition generates endogenous wage and productivity dispersion as identical firms offer a continuous distribution of equally profitable contracts. Workers in low-wage,

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low-productivity jobs are motivated by the prospects of climbing the job ladder, while workers in high-wage, high productivity jobs are motivated by the fear of losing their job and having to restart their career from scratch. When employed workers are sufficiently competitive in the labor market, wage dispersion by itself generates sufficient incentives that free entry leads to full employment.

Our model has important practical implications. It shows how relational contracting and on-the-job search amplify exogenous productivity differences as more efficient firms provide higher-powered incentives through higher wages. This predicts that productivity dispersion is higher in countries with limited contract enforcement or in professions with severe moral hazard. Moreover, it suggests controlling for wages and turnover when estimating differences in firms' fundamental productivities. The model can be used to evaluate the impact of technological changes, such as social networks that facilitate on-the-job search. It can also be used to assess policy issues: For example, prioritizing unemployed workers via a back-to-work program raises the quality of jobs offered to unemployed workers but may increase unemployment, as low-wage temp-firms exit the market.

In the model, measure 1 of identical workers and measure $n \leq 1$ of identical firms play an infinitely repeated game. Each firm has one job. Each period $t \in \{1, 2, \ldots\}$ has three stages. First, firm i pays its worker wage w_t. Second, the worker chooses his effort η_t and output $\phi(\eta_t)$ is publicly revealed. Third, both firm and worker can terminate the relationship unilaterally; separation also occurs with exogenous probability $1 - \alpha$.

A relational contract between a firm and a worker consists of a sequence of wages and effort requirements that may depend on the worker's tenure. Theorem 1 shows that there exists a firm-optimal self-enforcing contract with stationary wages and effort levels $\langle w, \eta \rangle$. Ideally, the firm would like to backload wages to extract the worker's rents, but such contracts are not self-enforcing since the firm would deviate by firing expensive old workers and replacing them with cheap new workers. This stationarity result allows us to analyze the continuum of contracting problems as industry equilibria of a static game where each firm simultaneously chooses a contract $\langle w, \eta \rangle$ to maximize profits $\pi = \phi(\eta) - w$ subject to its worker's incentive compatibility constraint.

At the end of each period some jobs are vacant because of exogenous turnover. We assume that labor market search is frictionless in the sense that all vacancies, including those that arise in the matching process, are offered to workers and are filled subject to workers' participation constraints. In contrast to the labor search literature with exogenous, technological frictions (e.g. Pissarides (2000)), the frictions in our model arise endogenously from the moral hazard problem. In equilibrium attractive jobs are in short supply, so workers need to be rationed with some workers receiving only low-quality job offers or no job offers at all. To operationalize this rationing, we assume that workers draw priorities that depend on their employment status but not on their history with current or previous employers; vacancies are then offered to workers in order of their priorities. This formulation nests many interesting specifications of labor market competition with different degrees of on-the-job search. Under *Shapiro-Stiglitz matching*, only unemployed workers receive job offers; under *anonymous matching*, employed and unemployed workers receive the same job offers; under *intern matching* employed workers receive offers first and only left-over entry-level jobs are offered to the unemployed. One theme of the paper is to characterize how the equilibrium contract distribution depends on the degree of on-the-job search.

Our main contribution is to integrate the single-firm analysis with the matching process and solve for industry equilibrium as a fixed point of stationary firm-optimal contracts. Theorem 2 shows that industry equilibrium exists and is unique. With Shapiro-Stiglitz matching, all firms offer the same job, and workers never quit. In contrast, with on-the-job search, equilibrium contracts are distributed continuously. The best of these jobs commands the same effort as the unique job in Shapiro-Stiglitz but all other jobs command strictly less effort and have higher turnover. Thus, the introduction of on-the-job search crowds out high-quality, permanent jobs with low-quality, temporary jobs.

To understand how this job deterioration comes about, consider a single firm's problem. At the optimal contract, the firm equates the marginal product of effort with its marginal cost, consisting of the direct compensation for the worker's effort and an incentive premium. Crucially, the marginal cost of effort decreases in the firm's retention rate since a wage raise incentivizes effort today only to the degree that the worker is still employed to collect the raise tomorrow. With Shapiro-Stiglitz matching the retention rate is constant, so all firms offer the same job. With on-the-job search this degenerate job distribution is not an equilibrium: on-the-job search acts like a non-pecuniary job benefit, so a single firm can increase its profits by offering an inferior contract with lower wage and effort.

More generally, with on-the-job search, there can be no atoms in the distribution of contracts. Counterfactually, assume that an atom of firms offers job $\langle w^*, \eta^* \rangle$. Then, turnover is discretely higher for jobs below $\langle w^*, \eta^* \rangle$ than for jobs above $\langle w^*, \eta^* \rangle$, causing the marginal cost of effort to decrease discontinuously at $\langle w^*, \eta^* \rangle$. Since the marginal benefit of effort is continuous, this implies that either an upward or a downward deviation must be profitable (or both). Figure 1 illustrates this for the case where all firms offer $\langle w^*, \eta^* \rangle$.

Next, we analyze how on-the-job search affects wages, profits and unemployment in equilibrium. To do so we need to take a stance on firm entry; we first consider the conceptually and analytically simpler case with a fixed number of firms. Theorem 3 shows that an increase in on-the-job search decreases effort and wages but increases profits. Effort decreases because on-the-job search increases turnover and thereby the marginal cost of incentive provision. The deterioration in job quality in turn reduces the prospects of unemployed workers and increases the relative value of a job with given wage and effort. The worker's incentive constraint thus becomes slack, allowing all firms to cut wages. Hence wages decrease - both through the deterioration of job quality and through the ensuing wage cuts - while profits increase.

With free entry, the slack in workers' incentive constraints is met by additional firms entering at the bottom of the market. The loss of high quality jobs is thus compensated by a reduction in unemployment and results in full employment if the employed receive better offers than the unemployed (Theorems 5 and 6). Effort in good jobs is then incentivized by the risk of underemployment rather than unemployment while effort in bad jobs is incentivized by the prospects of climbing the job ladder.

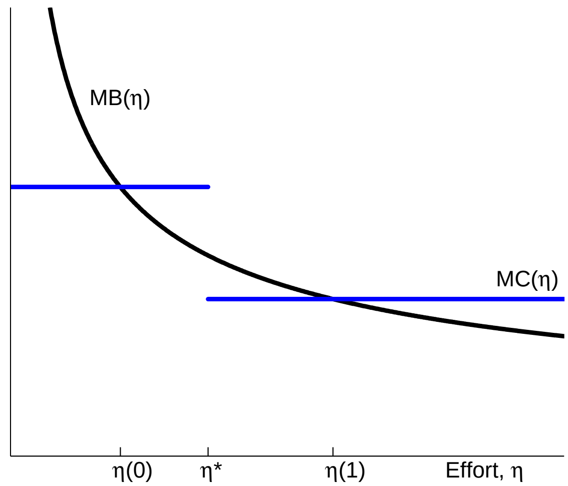

Figure 1: **Impossibility of Single-Wage Equilibria with On-the-Job Search.** This figure illustrates the marginal benefit and cost of effort for a single firm when all other firms offer the same contract $\langle w^*, \eta^* \rangle$. Turnover and the marginal cost of effort are discontinuous at η^* while the marginal benefit of effort is continuous. Thus, MB<MC just below η^* yielding a profitable downward deviation; similarly, MB>MC just above η^* yielding a profitable upward deviation. More generally, one or other of these deviations is profitable for any value of η^*.

For expositional simplicity, most of the paper assumes that unemployed workers receive better job offers than employed workers. In Section 7 we characterize equilibrium for arbitrary matching functions. In the corner case of intern matching, the deterioration of jobs is so extreme that an atom of firms offer internships paying zero wages and negative utility. These firms exploit their role as gatekeepers, offering the opportunity of a job with a better firm, but extracting all the future rents through their high effort demands.

To highlight the flexibility of the framework we extend the model to allow for firm and worker heterogeneity in Section 8. When firms differ in intrinsic productivity, high productivity firms strictly prefer contracts with high wages and high effort, purifying the mixed strategy equilibrium with identical firms. This extension allows us to examine how one firm's profit is affected by changes in competitors' productivities, which has implications for firm location decisions. Second, we consider heterogeneous workers and show that low-wage firms refuse to hire high-type workers because they view them as 'overqualified'.

Empirical Evidence. The model is based on the twin premises that higher wages raise the value of a job and encourage effort, and that workers search on-the-job. The first assumption is consistent with evidence that high-wage plants have fewer disciplinary actions (Raff and Summers (1987), Cappelli and Chauvin (1991)), that wages are positively correlated with a worker's reported commitment and effort (Levine (1993)), and that firms give bonuses and refuse to cut pay in order to motivate workers (Bewley (1999, Chap. 11)). These correlations could be the result of reciprocal preferences as found in the experiments of Fehr and Falk (1999).[1] Alternatively, they could result from repeated interactions, consistent with the findings that unemployment increases effort (Bewley (1999, Table 8.3)), reductions in employment protection increase effort (Jacob (2010)), and that effort declines at the end of a relationship (Hansen (2009)).

The assumption that employed workers engage in on-the-job search is directly supported by Nagypal's (2008) finding that job-to-job transitions account for 75% of all job transitions. Hornstein, Krusell, and Violante (2011) argue that short unemployment durations together with large wage differentials are also indicative of on-the-job search. The underlying idea - that on-the-job reduces the opportunity cost of taking up low-wage jobs - is the driving force behind quality deterioration and wage dispersion in our model.

Our results are consistent with the large unexplained inter-firm productivity differences (Syverson (2011)) and wage differentials (Krueger and Summers (1988), Mortensen (2003)). The model yields job ladders where a large fraction of wage growth occurs at job transitions and jumps are more frequent and larger at the start of a worker's career (Topel and Ward (1992)). It also predicts that "job ladders appear in parts of the economy where individual performance is difficult to disentangle"(Bulow and Summers (1986, p. 378)). A case in point are professional occupations where effort is very subjective (Milgrom and Roberts (1992, p. 258)) and the rate of job-to-job transitions is high (Nagypal (2008)). Indeed, such jobs are characterized by greater residual wage inequality (Lemieux (2006)) with workers more productive at more desirable firms (e.g. Oyer (2006)).

Related Literature. The paper builds on the relational contracting models of Shapiro and Stiglitz (1984) and MacLeod and Malcomson (1998). These models assume that firms only recruit from the pool of unemployed. They show how high wages can motivate workers, and argue that unemployment is a necessary component of any equilibrium.[2] Intuitively, workers only exert effort if shirking is punished by a spell of unemployment. These models have given rise to the view that "If all firms were identical, one would not expect to see different firms paying different wages even if efficiency wages were important," Krueger and Summers (1988, p. 261). Our paper shows that when the employed also compete for positions, firms offer different quality jobs. As a result, equilibrium unemployment falls, with incentives in a good job maintained by fear of being fired and having to start over in a bad job.

On-the-job search also gives rise to wage dispersion in the seminal model of Burdett and Mortensen (1998).[3] In their paper, on-the-job search matters because a single firm, facing a degenerate contract distribution, can poach workers from its competitors; so a single firm reacts to on-the-job search by raising wages. This motive is absent in our model as a firm always fill its single job. Rather,

[1] While we write the model in terms of relational contracting, one can interpret the results in terms of a model where workers have reciprocal preferences and exert high effort at firms that offer high wages and good job opportunities, as captured by the workers' value functions.

[2] MacLeod and Malcomson (1998) exhibits two types of equilibria. In the first type, some workers are unemployed; in the second type, some firms are 'unemployed'.

[3] Wage dispersion is also seen in Albrecht and Vroman (1992), Peters (2010), and Galenianos and Kircher (2009). As in Burdett and Mortensen (1998), these papers are market-based with high wages helping a firm hire more or better workers.

on-the-job search matters because it allows the firm's employee to search for better jobs at other firms in the future; so the firm regards on-the-job search as a non-pecuniary job benefit to its worker and cuts wages. Thus, in contrast to Burdett and Mortensen (1998), our firm would never want to ban its employee from searching and does not care to match outside offers. Beside these different predictions, our model addresses issues that are absent in Burdett–Mortensen, such as the effect of moral hazard and turnover on productivity and the effectiveness of incentive-pay. In practice, both inter- and intra-firm forces are important: Bewley's survey (1999, Tables 7.1, 11.2) reveals that employers' wage setting is influenced by turnover (58% of firms), the ability to hire (57%) and morale (32%), with most firms refusing to cut pay during a recession because it would hurt morale and demotivate workers (69%).

Overview. Section 2 introduces the bare-bones model. The subsequent sections analyze the infinite game between the continua of firms and workers in several steps. Section 3 analyzes the interaction between a single firm and its sequence of employees. Section 4 describes the matching process. Section 5 characterizes the industry equilibrium as the unique fixed point of the continuum of firm-optimal stationary contracts assuming the number of firms is fixed, while Section 6 solves for equilibrium with entry. We extend the model to arbitrary matching functions in Section 7, and to heterogeneous firms and workers in Section 8.

2 Model

Basics. The economy consists of measure 1 of identical workers and measure $n \leq 1$ of identical firms. Each firm has one job. Time is discrete and infinite, $t \in \{1, 2, \ldots\}$. The timing of each period t is summarized in Figure 2. Both firms and workers discount the future at rate $\delta \in (0, 1)$.

Job Market Matching. At the start of each period some firms have vacancies. These vacancies are offered to workers, as are all subsequent vacancies that arise in the matching process. Since jobs may differ, offers are rationed via priorities that depend on workers' employment status, but not on their history with current or previous employers. We elaborate on the matching process in Section 4.

Jobs. In each period, first the firm pays the worker wage $w \in [0, \infty)$.[4] Second, the worker chooses observable but non-contractible effort $\eta \in [0, \infty)$ and produces output $\phi(\eta)$ for a surplus of $\phi(\eta) - \eta$. Third, at the end of the period, either party can terminate the relationship; separation also occurs with exogenous probability $1 - \alpha$. The firm's profit and worker's utility (or rent) are given by

$$\pi := \phi(\eta) - w \qquad \text{and} \qquad u := w - \eta.$$

[4] In Section 7 we need to consider the possibility of negative wages. We can incorporate this into our model by giving the worker the possibility of rejecting such a negative wage.

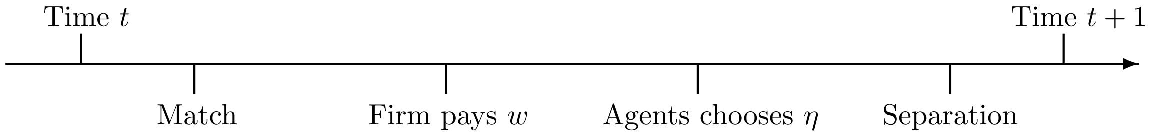

Figure 2: **Timeline of Period t.**

For convenience, we assume that the production function $\phi(\cdot)$ is continuously differentiable and strictly concave with $\phi(0) = 0, \phi'(0) = \infty, \lim_{\eta \to \infty} \phi'(\eta) = 0$.

3 Single Firm's Problem

In this section we focus on the interaction between a single firm and its sequence of employees; the interaction with other firms is captured through abstract outside options. We first argue that we can restrict attention to stationary contracts and then analyze the first-order condition for the firm-optimal contract.

During the matching stage, at the beginning of period t, let W be the continuation value of the best outside job offer to the firm's worker. W is distributed according to cdf F^e with support in $\varnothing \cup [0, \infty)$, where $\varnothing$ represents the possibility of obtaining no job offer in period t. If the firm loses its worker, either at the end of period $t-1$ or in the matching stage of period t, it posts a vacancy. The fill probability $p(\cdot)$ depends on the initial continuation value of the firm's job and equals one (resp. zero) if this value is strictly greater than (resp. strictly less than) some reservation utility.

The rest of the period t transpires as above: first the firm pays its worker wage w, then the worker exerts effort η, and finally the worker, firm or nature can terminate the relationship. In case of termination, the worker starts the next period with continuation value $V^{\varnothing}$.

All parameters $F^e(\cdot), p(\cdot)$ and $V^{\varnothing}$ are stationary and independent of history.

3.1 Stationary Contracts

We focus on *contract-specific* strategies in which the actions of the firm and workers do not depend on the identity of the worker, calendar time, or history outside the current relationship. Thus, wage w_t in period t of a contract only depends on $(w_1, \eta_1, \cdots, w_{t-1}, \eta_{t-1})$, effort η_t additionally depends on w_t, and separation decisions additionally depend on η_t.

We define a *self-enforcing contract* as a subgame-perfect equilibrium in pure contract-specific strategies. A self-enforcing contract remains self-enforcing if deviations off the equilibrium path are punished in the harshest possible way. For our simple game, this means that a worker who receives an off-equilibrium wage w_t shirks and quits, and the firm fires a worker after a deviation from equilibrium effort η_t. This allows us to write a self-enforcing contract as a sequence of wages and effort levels $\langle w_t, \eta_t \rangle$. Let V_t be the worker's continuation value, and Π_t the firm's continuation profit at the beginning of period t of the contract, and let $\Pi_{\varnothing}$ be the continuation profit of the firm with a vacancy. We call a contract *stationary* if $w_1 = w_2 = \cdots = w$ and $\eta_1 = \eta_2 = \cdots = \eta$.

Theorem 1 (Stationary Contracts) *Assume stationary conditions $F^e(\cdot), p(\cdot), V^{\varnothing}$. For any self-enforcing contract $\langle w_t, \eta_t \rangle$ with profits $\Pi_{\varnothing}$, there is a stationary self-enforcing contract $\langle w^*, \eta^* \rangle$ that yields weakly higher profits, $\Pi^*_{\varnothing} \geq \Pi_{\varnothing}$.*

Proof. See Appendix A. □

The idea of the proof is to define the (weakly) more profitable stationary contract $\langle w^*, \eta^* \rangle$ by setting effort to maximise surplus $\phi(\eta^*) - \eta^* = \sup_t \phi(\eta_t) - \eta_t$, and wages w^* such that $V^* = \sup_t V_t$. This contract is incentive compatible for the worker because it requires no more effort than the incentive compatible contract $\langle w_t, \eta_t \rangle$ and provides a higher continuation value. To see why $\langle w^*, \eta^* \rangle$ is weakly more profitable than $\langle w_t, \eta_t \rangle$, let τ be the period when the latter attains its maximal continuation value $V_\tau = V^*$. Assuming the contract is accepted with probability $p = 1$, profits $\Pi^*_{\varnothing}$ of $\langle w^*, \eta^* \rangle$ are higher than continuation profits Π_τ because surplus is higher by choice of η^* and the worker's share is the same by choice of w^*. Continuation profits Π_τ in turn must exceed initial profits $\Pi_{\varnothing}$ by the firm's incentive constraint not to fire and replace its worker. Intuitively, firms cannot backload wages in equilibrium since this would create incentives to fire old, expensive workers and replace them with new, cheap workers.[5]

The assumption of contract-specific strategies engenders two restrictions. First, it implies that no party conditions their behavior on information outside their current relationship. Such information is payoff-irrelevant and so it is optimal to ignore such information if everybody else ignores it. However, there may be other equilibria that do condition on payoff-irrelevant information, and these equilibria may yield higher profits. For example, if all workers ostracized firm i for falsely firing worker j, then firm i could commit to backload wages. We rule out such equilibria because they demand a high level of coordination and information (worker k must know whether worker j shirked at firm i). Second, contract-specific strategies do not condition on calendar time. If such conditioning were possible, firms and workers could agree on wage cuts in the very first period of the game and backload wages without consequences for incentives. While interesting, such initial backloading is transitory and should have little effect in the long run.[6]

From now on, we restrict attention to stationary firm-optimal contracts that maximize firm profit subject to the worker's incentive and participation constraints. Additionally, we assume that these contracts are accepted with probability $p = 1$. This assumption is justified by the proof of Theorem

[5] In contrast, equilibrium back-loading is possible in Levin (2003)'s single-firm, single-worker setting where back-loading is realized by bonus payments. The key difference between the models is that the firm's outside option when firing its worker is fixed exogenously in Levin (2003), so the firm can only steal the backloaded wages/bonus once. In our setting, the outside option is to hire a new worker with the same contract, so the firm could steal the back-loaded wages over and over again.

MacLeod and Malcomson (1998) find equilibrium bonus contracts in a multi-firm, multi-worker setting. In their equilibrium, firms refrain from firing workers and stealing the promised bonus because they can't replace workers instantaneously and forgo profits while their job is vacant. Theorem 1 implies that a firm cannot strictly increase its profits through such backloading.

[6] We also focus on contracts that maximize an individual firm's profits, taking the other firms' contracts as given. It would be jointly profitable for firms to collude on self-enforcing contracts with lower wages, depressing workers' rents and thus their outside options $V^{\varnothing}$. Such contracts can be sustained as equilibrium by fiat: If the parties punish each other for unexpected levels of wage or effort by termination, then no party can profitably deviate. However, such 'collusive' equilibria are not optimal for a single firm and are thus susceptible to renegotiation.

1 which shows that a strictly profitable, self-enforcing stationary contract $\langle w, \eta \rangle$ that is accepted with probability $p < 1$ cannot be firm-optimal since the firm can raise the wage a little.[7]

3.2 Analyzing the First-Order Condition

We now analyze the first-order condition of the firm-optimal contract. To simplify notation we write a stationary contract $\langle u, \eta \rangle$ as function of utility $u = w - \eta$ rather than the wage w. We also write outside options and their distribution $F^e(\cdot)$ in terms of per-period utility u rather than the continuation value W. Finally, we write $V(u)$ for the continuation value of an employed worker with contract $\langle u, \eta \rangle$ at the beginning of the period. The Bellman equations for employed and unemployed workers are given by

$$V(u) = \int \max\{u + \alpha\delta V(u); \hat{u} + \alpha\delta V(\hat{u})\} dF^e(\hat{u}) + \delta(1-\alpha)V^{\varnothing} \tag{3.1}$$

$$V^{\varnothing} = \int (\hat{u} + \alpha\delta V(\hat{u}))\, dF^{\varnothing}(\hat{u}) + \delta(1 - \alpha\theta) V^{\varnothing} \tag{3.2}$$

where $F^{\varnothing} : \varnothing \cup [0, \infty) \to [0, 1]$ is the distribution function of job offers to unemployed workers and $\theta := 1 - F^{\varnothing}(\varnothing)$ is the probability an unemployed worker receives an offer. We endogenize these in Section 4.

The firm-optimal contract $\langle u, \eta \rangle$ maximizes firm profits

$$\pi = \phi(\eta) - \eta - u$$

subject to incentivizing effort and participation

$$w - \eta + \alpha\delta V(u) + (1-\alpha)\delta V^{\varnothing} \geq w + \delta V^{\varnothing} \tag{IC}$$

$$w - \eta + \alpha\delta V(u) + (1-\alpha)\delta V^{\varnothing} \geq \delta V^{\varnothing} \tag{IR}$$

As long as wages are positive IR is implied by IC, so we ignore the IR constraint until Section 7.[8] At the firm-optimal contract, IC binds and we define $u_*(\eta)$, the utility to incentivize effort η, by[9]

$$\eta = \alpha\delta(V(u_*(\eta)) - V^{\varnothing}).$$

The firm's problem is then to choose η to maximize $\pi_*(\eta) = \phi(\eta) - \eta - u_*(\eta)$.

Assume for a moment that the distribution $F^e(\cdot)$ is continuous. The marginal continuation value

[7] This assumption rules out spurious equilibrium multiplicity when we allow for free entry and profits are zero. Without the assumption a pool of firms could enter at the bottom of the market, attracting workers with probability $p < 1$ or even $p = 0$.

[8] If $n = 1$, the firm also needs to beat the other firms in order to obtain a worker, i.e. $u \geq \sup\{u' : F^{\varnothing}(u') = 0\}$. By Theorem 4, this is not an issue until we consider general matching functions with entry in Section 7.

[9] If employed workers receive no job offer with positive probability, $F^e(\varnothing) > 0$, such a $u_*(\eta)$ exists and is unique, because $V(u)$ is strictly increasing, continuous with $\lim_{u \to -\infty} V(u) = -\infty$ and $\lim_{u \to \infty} V(u) = \infty$. If all employed workers receive job offers, $F^e(\varnothing) = 0$, then $V(u)$ is bounded below and we set $u_*(\eta) := -\infty$ if $\eta < \alpha\delta\left(V(u) - V^{\varnothing}\right)$ for all $u \in \mathbb{R}$.

of current utility is then given by

$$V'(u) = (1 + \alpha\delta V'(u))F^e(u) = \frac{F^e(u)}{1 - \alpha\delta F^e(u)}.$$

When utility u increases, the incremental contract value equals the perpetuity value of the incremental utility at the contract-specific discount rate $\alpha\delta F^e(u)$, discounted by the probability $F^e(u)$ that the worker is poached before receiving the incremental utility for the first time. Holding tight the incentive constraint, this implies that the marginal cost of effort to the firm

$$\frac{d}{d\eta}(\eta + u_*(\eta)) = 1 + \frac{1}{\alpha\delta V'(u_*(\eta))} = \frac{1}{\alpha\delta F^e(u_*(\eta))} \tag{3.3}$$

decreases in the retention rate $F^e(u_*(\eta))$ but always exceeds one, so the firm always chooses η inefficiently low. Intuitively, tomorrow's wages incentivize today's effort, so the incentive premium equals one period's interest on the cost of effort and the effective interest rate increases in turnover. The first-order condition for the optimality of contract $\langle u, \eta \rangle$ is thus

$$\phi'(\eta) = \frac{1}{\alpha\delta F^e(u_*(\eta))}. \tag{3.4}$$

This equation shows that the solution to the firm's problem need not be unique because both marginal benefits and marginal costs decrease in effort. Indeed, in Section 5 we show that with on-the-job search firms are indifferent between a continuum of contracts in equilibrium.

We complete the single-firm analysis by showing that the distribution $F^e(\cdot)$ is indeed continuous. Assume otherwise. Then the formula for $V'(u)$ and (3.3) remains valid for the right-handed derivative while we need to replace $F^e(u)$ by its left-sided limit $\lim_{\epsilon\to 0} F^e(u-\epsilon)$ for the left-handed derivative. The necessary left- and right-handed first-order conditions for the optimality $\langle u, \eta \rangle$ are thus $u = u_*(\eta)$ and

$$\lim_{\epsilon\to 0} \frac{1}{\alpha\delta F^e(u_*(\eta - \epsilon))} \leq \phi'(\eta) \leq \frac{1}{\alpha\delta F^e(u_*(\eta))}. \tag{3.5}$$

As the LHS exceeds the RHS, (3.5) implies that $F^e(\cdot)$ is continuous at any optimal contract (see Figure 1).

4 Job Market Matching

In this section we analyze the transitions between unemployment and jobs of heterogeneous quality u. We assume that job offers depend on workers' employment status, but not on their history with current or previous employers. The transitions can thus be described by the distribution of an employed worker's best offer $F^e(u)$, and that of an unemployed worker $F^{\varnothing}(u)$.

Absent voluntary terminations, there are αn filled jobs, $(1-\alpha)n$ vacancies, and $1-\alpha n$ unemployed workers at the beginning of each period. The distribution of jobs, both filled and vacant, is given by

$F(u)$ and will be endogenized in Section 5. Market clearing for jobs above u requires

$$\underbrace{(1-\alpha n)(1-F^{\varnothing}(u))}_{\text{unemployed}}+\underbrace{\alpha nF(u)(1-F^{e}(u))}_{\text{employed below } u}=\underbrace{(1-\alpha)n(1-F(u))}_{\text{vacancies above } u} \tag{4.1}$$

The probability that an unemployed worker receives some job offer equals the labor market tightness $\theta := 1 - F^{\varnothing}(\varnothing) = (1-\alpha)\, n/\,(1-\alpha n)$. As there is more competition for good jobs, the job offers to unemployed workers are a negative selection of the job distribution, that is $1 - F^{\varnothing}(u) \leq \theta\,(1 - F(u))$.

To determine the two distributions of job offers, $F^{\varnothing}(u)$ and $F^{e}(u)$, from the market clearing condition (4.1) we need to specify the comparative advantage of employed and unemployed workers in the labor market. We do so by supposing that all workers draw independent 'matching priorities' $z \in [0,1]$; we normalize the distribution for employed workers to be uniform but do not restrict the cdf ψ for unemployed workers. Workers then get to pick vacancies in order of their priorities, with highest priorities first. The employed worker with quantile z then ties with the unemployed worker with quantile $\psi(z)$, so $F^{\varnothing}(u)$ and $F^{e}(u)$ in (4.1) are connected by $F^{\varnothing}(u) = \psi(F^{e}(u))$, whenever such a $F^{e}(u)$ exists. Existence may fail if $\psi(\cdot)$ is discontinuous, in which case one employed worker ties with an atom of unemployed workers who need to be rationed to satisfy (4.1). Formally, we define $F^{\varnothing}(\cdot), F^{e}(\cdot)$ as the unique right-continuous functions that satisfy (4.1) together with $F^{\varnothing}(u) \in [\lim_{\epsilon\to 0}\psi(F^{e}(u)-\epsilon), \psi(F^{e}(u))]$.[10]

Equation (4.1) defines the retention rate $F^{e}(u)$ at job u solely based on the quantile $q = F(u)$ of the job, so we disentangle the effect of matching technology ψ and job distribution $F(u)$ by writing the retention rate as a function of the job's quantile $F^{e}(u) = \beta(F(u))$. It will be convenient to extend this definition of $\beta(\cdot)$ from the range of F to the entire unit interval $q \in [0,1]$ via

$$(1-\alpha n)(1-\psi(\beta(q))) + \alpha n q(1-\beta(q)) = (1-\alpha)n(1-q), \tag{4.2}$$

whenever such a $\beta(q)$ exists.[11]

We say that a matching technology $\psi(\cdot)$ satisfies *on-the-job search* (OJS) if it is continuous on $[0,1]$. This implies that $F^{e}(\cdot)$ and $\beta(\cdot)$ are strictly increasing on their supports and captures the idea that there is no interval of jobs $[u,u']$ that is offered exclusively to unemployed workers, unless all employed workers get job offers above u'. Also we say that there is *more on-the-job search* under $\widetilde{\psi}$ than under ψ, if $\widetilde{\psi}(z) \geq \psi(z)$ for all $z \in [0,1]$. This translates to lower retention rates in the sense that $\widetilde{\beta}(q) \leq \beta(q)$ for all $q \in [0,1]$.

We illustrate the formalism with three examples:

[10] This formulation is more flexible than that used in typical search models where employed and unemployed workers may differ in the probability of getting any job offer but, conditional on getting a job offer, share the same distribution. This usual approach corresponds to linear $\psi(\cdot)$ and, for example, excludes matching systems where the unemployed have a comparative advantage at obtaining low quality jobs, such as intern matching.

[11] If $\psi(\cdot)$ is discontinuous, we proceed as above and define $\beta(q)$ as the unique solution of

$$(1-\alpha n)(1-x) + \alpha n q(1-\beta(q)) = (1-\alpha)n(1-q)$$

where $x \in [\lim_{\epsilon\to 0}\psi(\beta(q)-\epsilon), \psi(\beta(q))]$.

Shapiro-Stiglitz matching: Employed workers never receive any job offers, so $\psi_{\mathrm{S}} \equiv 0$ on $[0,1)$. The retention rate is then given by $\beta_{\mathrm{S}}(\cdot) \equiv 1$. This process does not satisfy on-the-job search.

Anonymous matching: Employed and unemployed workers receive job offers from the same distribution, so $\psi_{\mathrm{A}}(z) = z$. This process satisfies on-the-job search. To be explicit, equation (4.2) implies that retention rates are given by

$$\beta_{\mathrm{A}}(q) = \big(1 - n(1-q)\big)/\big(1 - \alpha n\,(1-q)\,\big). \tag{4.3}$$

The probability $\beta'_{\mathrm{A}}(q)dq$ of receiving a job offer in $[q, q+dq]$ is decreasing in q. This is because the pool of underemployed workers interested in jobs $[q, q+dq]$ is increasing in q, and the supply of such jobs is decreasing in q as fewer of them open up during the matching process.

Intern matching: All jobs are first offered to employed workers, so $\psi_{\mathrm{I}}(\cdot) \equiv 1$. In this case, we get $\beta_{\mathrm{I}}(q) = 0$ for $q \in [0, 1-\alpha]$ and

$$\beta_{\mathrm{I}}(q) = \left(q - (1-\alpha)\right)/\alpha q$$

for $q \in [1-\alpha, 1]$. However, we show in Section 7 that the retention rate is strictly positive at all jobs as more than $1-\alpha$ firms offer identical jobs at the bottom of the job distribution, so that the quantile for all these jobs is greater than $1-\alpha$.

For ease of exposition we assume that job offers to unemployed workers first-order-stochastic-dominate job offers to employed workers, $F^{\varnothing}(u) \leq F^{e}(u)$, by imposing $\psi(z) \leq z$. We relax this assumption in Section 7.

5 Equilibrium

In this section we integrate the analysis of a single firm's problem in Section 3 with the market analysis in Section 4. It will prove useful to label the identical firms by $x \in [0,1]$ and assume without loss that firms with higher labels offer contracts with higher utilities. Formally, let $\langle u, \eta \rangle : [0,1] \to \mathbb{R} \times \mathbb{R}^{+}$ be a pair of functions where $u(\cdot)$ is weakly increasing. The utility mapping $u(\cdot)$ is essentially the inverse of the distribution function $F(\cdot)$. More precisely, we have $F(u(x)) = \sup\{x' : u(x') \leq u(x)\}$ and the quantile of a firm's job $F(u(x))$ equals its label x unless the job distribution $F(\cdot)$ has an atom at $u(x)$.

Definition: An *industry equilibrium* consists of a distribution of contracts $\langle u(x), \eta(x) \rangle$ such that

(a) Every contract $\langle u(x), \eta(x) \rangle$ is firm-optimal; and

(b) Given the matching technology $\psi(\cdot)$ and distribution of utilities $F(\cdot)$, the distributions of job offers $F^{e}(\cdot), F^{\varnothing}(\cdot)$ are derived from market clearing (4.1) and the value of unemployment $V^{\varnothing}$ is given by equation (3.2).

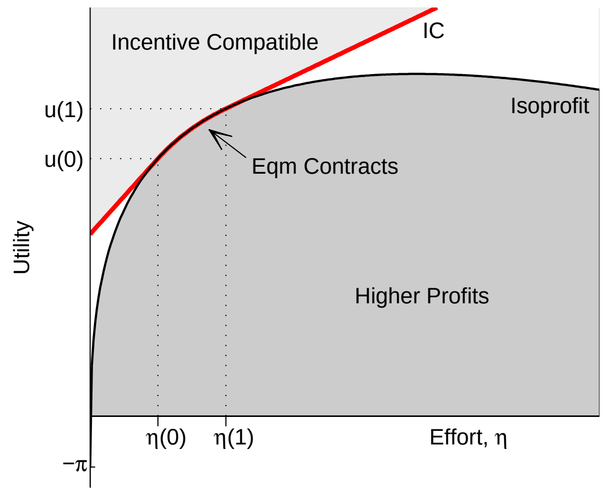

Figure 3: **The Set of Equilibrium Contracts.** This figure shows the firm's isoprofit curve $u = \phi(\eta) - \eta - \pi$ and the worker's incentive compatibility constraint $u_*(\eta)$ in equilibrium. Contracts under the isoprofit line are more profitable for the firm; contracts above the IC constraint are incentive compatible. When solving for equilibrium, the firm's first-order condition says the isoprofit curve must be tangent to the IC constraint; profits are determined so these curves coincide over the range of equilibrium contracts.

Firm-optimality implies that all firms maximize profit subject to incentive compatibility. We can then solve for equilibrium in three steps.

1. Effort $\eta(x)$ is uniquely determined by the firm's first-order condition. If firms $[\underline{x}, \overline{x}] \ni x$ offer the same job, then the necessary left- and right-handed first-order conditions (3.5) become

$$\frac{1}{\alpha\delta\beta(\underline{x})} \leq \phi'(\eta(x)) \leq \frac{1}{\alpha\delta\beta(\overline{x})}.$$

   Since $\beta(\cdot)$ is weakly increasing, these inequalities imply that $\beta(\cdot)$ is flat on $[\underline{x}, \overline{x}]$ and equilibrium effort $\eta(x)$ satisfies

$$\phi'(\eta(x)) = \frac{1}{\alpha\delta\beta(x)} \tag{5.1}$$

   for all $x \in [0, 1]$ irrespective of the job distribution $F(\cdot)$. This first-order condition says that the firm's isoprofit curve is tangent to the worker's incentive constraint (see Figure 3). Hence it can be interpreted as a marginal IC constraint in the following sense: If contract $\langle u(1), \eta(1)\rangle$ is firm-optimal, and (5.1) holds for all contracts $\langle u(x), \eta(x)\rangle$, then all these contracts are firm-optimal.

2. Utility $u(x)$ is determined up to a single parameter by the fact all firms make equal profits $\pi \geq 0$.[12] That is,

$$u(x) = \phi(\eta(x)) - \eta(x) - \pi. \tag{5.2}$$

[12]If profits were negative, some firms would exit the market and the analysis of Section 6 applies.

3. Profits π are uniquely determined by the worker's IC constraint for firm $x = 1$ (or any other firm),

$$\eta(x) = \alpha\delta(V(u(x)) - V^{\varnothing}). \tag{5.3}$$

Uniqueness follows because an increase in profits π comes at the expense of workers' utility $u(1)$ and continuation value $V(u(1))$. The value of unemployment $V^{\varnothing}$ also decreases in π, but less so because unemployed workers experience the decrease in rents with a delay. Thus $V(u(1)) - V^{\varnothing}$ is decreasing in π, and there is a unique value of π for which the IC constraint of firm $x = 1$ is met with equality.

The firms' first-order condition (5.1) has several immediate consequences for the job distribution. Effort in the best and worst jobs are given by

$$\eta(1) = (\phi')^{-1}(1/\alpha\delta) \quad \text{and} \quad \eta(0) = (\phi')^{-1}(1/\alpha\delta\beta(0)).$$

The best job has retention rate $\beta(1) = 1$, so the highest effort level $\eta(1)$ does not depend on the matching technology and is the same as in Shapiro-Stiglitz. The lowest effort level $\eta(0)$ does depend on the matching technology, and is strictly positive since $\beta(0) > 0$. Between the extremes, effort $\eta(\cdot)$ is continuous and, if the matching technology satisfies OJS, strictly increasing. In other words the contract distribution $F(\cdot)$ has no gaps, and with OJS, no atoms. If an atom of firms $[\underline{x}, \overline{x}]$ offer the same job $\langle u, \eta \rangle$ then the marginal cost of effort falls at η from $1/\alpha\delta\beta(\underline{x})$ to $1/\alpha\delta\beta(\overline{x})$. If no employed workers are offered jobs $x \in [\underline{x}, \overline{x}]$, then $\beta(\underline{x}) = \beta(\overline{x})$ and this is consistent with equilibrium. However if OJS is satisfied, then $\beta(\underline{x}) < \beta(\overline{x})$ implying the marginal cost of effort jumps down, so that either an upward or downward deviation is profitable (see Figure 1). If there is a gap in the contract distribution, the marginal benefit of effort decreases over this gap but the marginal cost of effort does not, which again is incompatible with local optimality. To summarize:

Theorem 2 (Equilibrium Characterization) *Industry equilibrium exists and is unique. Effort $\eta(x)$ is determined by the first-order condition (5.1). The effort function $\eta(\cdot)$ is continuous, increasing, and strictly increasing if $\psi(\cdot)$ satisfies OJS.*

Proof. *Existence.* We discuss above how to construct an equilibrium and complete this construction by formalizing step three, that for a fixed distribution of effort levels the difference in value between an employed and an unemployed worker $V(u(x)) - V^{\varnothing}$ is decreasing linearly in π. Equation (5.2) implies that $du(x)/d\pi = -1$ so the employed worker's Bellman equation (3.1) implies that $dV(u(x))/d\pi$ is independent of x. Thus,

$$\frac{d}{d\pi}\left(V(u(x)) - V^{\varnothing}\right) = -(1-\theta) + \alpha\delta(1-\theta)\frac{d}{d\pi}\left(V(u(x)) - V^{\varnothing}\right) = -\frac{1-\theta}{1-\alpha\delta(1-\theta)} < 0. \tag{5.4}$$

By the intermediate value theorem there exists a π such that IC binds.

Figure 3 shows that all contracts $\langle u(x), \eta(x) \rangle$ are indeed firm-optimal. More formally, all contracts $\langle u(x), \eta(x) \rangle$ generate the same profit π, and no contract $\langle u, \eta(x) \rangle$ with $u < u(x)$ satisfies the worker's IC constraint. To see that no contract $\langle u, \eta_- \rangle$ with $\eta_- < \eta(0)$ is more profitable, we extend the definition of $u_*(\eta)$ to all $\eta \in \mathbb{R}^+$ using (5.3), so $\langle u_*(\eta_-), \eta_- \rangle$ is the most profitable incentive compatible contract with effort η_-. As $\langle u(0), \eta(0) \rangle$ is the lowest contract offered by other firms, the marginal cost of effort on $[\eta_-, \eta(0)]$ equals $1/\alpha\delta\beta(0)$ while the marginal benefit $\phi'(\eta)$ is greater than $\phi'(\eta(0)) = 1/\alpha\delta\beta(0)$. With marginal benefits exceeding marginal costs, $\langle u_*(\eta_-), \eta_- \rangle$ is less profitable than $\langle u(0), \eta(0) \rangle$. Similarly, if $\eta_+ > \eta(1)$ then marginal costs of effort equal $1/\alpha\delta = \phi'(\eta(1))$ on $[\eta(1), \eta_+]$ and exceed marginal benefits $\phi'(\eta)$, so $\langle u_*(\eta_+), \eta_+ \rangle$ is less profitable than $\langle u(1), \eta(1) \rangle$. This is illustrated in Figure 3.

Uniqueness. By construction, $\langle u(x), \eta(x) \rangle$ are unique up to profit π. Uniqueness of the π follows from (5.4).

Contract distribution. The inverse marginal productivity $(\phi')^{-1}$ is strictly decreasing and continuous. As $\beta(\cdot)$ is weakly increasing and continuous, so is $\eta(x) = (\phi')^{-1}(1/\alpha\delta\beta(x))$, and if $\beta(\cdot)$ is strictly increasing, then so is $\eta(\cdot)$. □

We now elaborate on the equilibrium characterization by considering two specific matching technologies, illustrated in Figure 4. The top panels show the Shapiro-Stiglitz equilibrium. The middle panels show how the introduction of on-the-job search enables firms to increase their profits by offering an inferior contract with lower wage and effort. Intuitively, on-the-job search acts like a non-pecuniary job benefit and increases the worker's value for such inferior contracts. Thus the deviating firm can offer lower wages in the inferior contracts while maintaining its worker's incentives, making the downward-deviation strictly profitable. The bottom panels show the equilibrium with anonymous matching.

Shapiro-Stiglitz matching. Suppose that only unemployed workers receive job offers, so $\psi_{\mathrm{S}}(\cdot) \equiv 0$ on $[0,1)$, yielding a retention rate $\beta_{\mathrm{S}}(\cdot) \equiv 1$. The marginal cost of effort to the firm then equals $1/\alpha\delta$ by (3.3). The intercept of the firm's profit function $\pi_*(0) = \phi(0) - 0 - u_*(0)$ is negative, equal to the worker's opportunity cost of forgoing one period of job offers, $(1-\delta)V^{\varnothing}$. The firm's profit function is thus given by

$$\pi_*(\eta) = \phi(\eta) - \frac{1}{\alpha\delta}\eta - (1-\delta)V^{\varnothing}. \tag{5.5}$$

In accordance with Theorem 2, profit is maximized at $\eta_{\mathrm{S}}(x) = (\phi')^{-1}(1/\alpha\delta) =: \overline{\eta}$ for each firm x.

To compute profits without reference to the endogenous value $V^{\varnothing}$ note that the worker's IC constraint binds; in particular an employed worker is indifferent between (1) working today and shirking tomorrow, and (2) shirking today, searching for a job tomorrow and then shirking again. Plan (2) saves η in effort costs today but the probability of forfeiting wage w tomorrow increases from $(1-\alpha)(1-\theta)$ in plan (1) to $1-\theta$ in plan (2); so indifference implies $\eta = \alpha\delta(1-\theta)w$. As firms optimally choose effort $\overline{\eta}$ firm profits are given by

$$\pi_{\mathrm{S}} = \phi(\overline{\eta}) - \frac{1}{\alpha\delta(1-\theta)}\overline{\eta}. \tag{5.6}$$

If there are too many firms and the labor market is very tight with $\theta \approx 1$, profits in equation (5.6) are negative, so firms exit until $\pi_{\mathrm{S}} = 0$ (see Section 6).

Anonymous matching. Suppose that $\psi_{\mathrm{A}}(z) = z$, so that employed and unemployed workers receive job offers from the same distribution. The retention rate is then strictly increasing from $\beta_{\mathrm{A}}(0) = 1 - \theta$, the probability that an unemployed worker stays unemployed, to $\beta_{\mathrm{A}}(1) = 1$.

With anonymous matching the intercept of the profit function is zero, because the worker does not bear any opportunity cost for forgoing search when he takes up job $\langle 0, 0 \rangle$, which is identical to unemployment. For the lowest firm, the marginal cost of effort is constant equal to $1/\alpha\delta(1-\theta)$ for all $\eta \in [0, \eta(0)]$. Thus, on $[0, \eta(0)]$, the firm's profit function is given by

$$\pi_*(\eta) = \phi(\eta) - \frac{1}{\alpha\delta(1-\theta)}\eta. \tag{5.7}$$

In accordance with Theorem 2, profit is maximized at $\eta_{\mathrm{A}}(0) = (\phi')^{-1}(1/\alpha\delta(1-\theta)) =: \underline{\eta}$. As profits are constant for all firms we get

$$\pi_{\mathrm{A}} = \phi(\underline{\eta}) - \frac{1}{\alpha\delta(1-\theta)}\underline{\eta}. \tag{5.8}$$

Profits are higher under anonymous matching (5.8) than under Shapiro-Stiglitz matching (5.6).

Surprisingly, profits under anonymous matching coincide with profits under Shapiro-Stiglitz matching if all firms could collude on contracts since such firms would jointly choose effort $\underline{\eta}$. This is no coincidence. Under anonymous matching, profits at the lowest contract (5.7) do not depend on the contracts offered by other firms. Thus for firm $x = 0$, the effect of reducing effort in its own contract equals the effect of reducing effort in all contracts, which is exactly the consideration of the colluding firms under Shapiro-Stiglitz matching.

Returning to general matching technologies ψ, we now study how the industry equilibrium depends on the degree of on-the-job search. For matching technologies $\psi, \widetilde{\psi}$ we denote equilibrium effort by $\eta(x), \widetilde{\eta}(x)$, equilibrium utility by $u(x), \widetilde{u}(x)$, and so on.

Theorem 3 (Comparative Statics of OJS) *When on-the-job search increases, $\psi(z) \leq \widetilde{\psi}(z)$ for all $z \in [0,1]$, then:*

(a) Turnover increases for all jobs: $1 - \beta(x) \leq 1 - \widetilde{\beta}(x)$ for all x.

(b) Effort and surplus decrease for all jobs: $\eta(x) \geq \widetilde{\eta}(x)$ and $\phi(\eta(x)) - \eta(x) \geq \phi(\widetilde{\eta}(x)) - \widetilde{\eta}(x)$ for all x.

(c) Profits increase for all firms: $\pi \leq \widetilde{\pi}$.

(d) Utilities and continuation values decrease for all employed and unemployed workers: $u(x) \geq \widetilde{u}(x)$, $V(u(x)) \geq V(\widetilde{u}(x))$ and $V^{\varnothing} \geq \widetilde{V}^{\varnothing}$.

Proof. (a) This follows directly from market clearing (4.2).

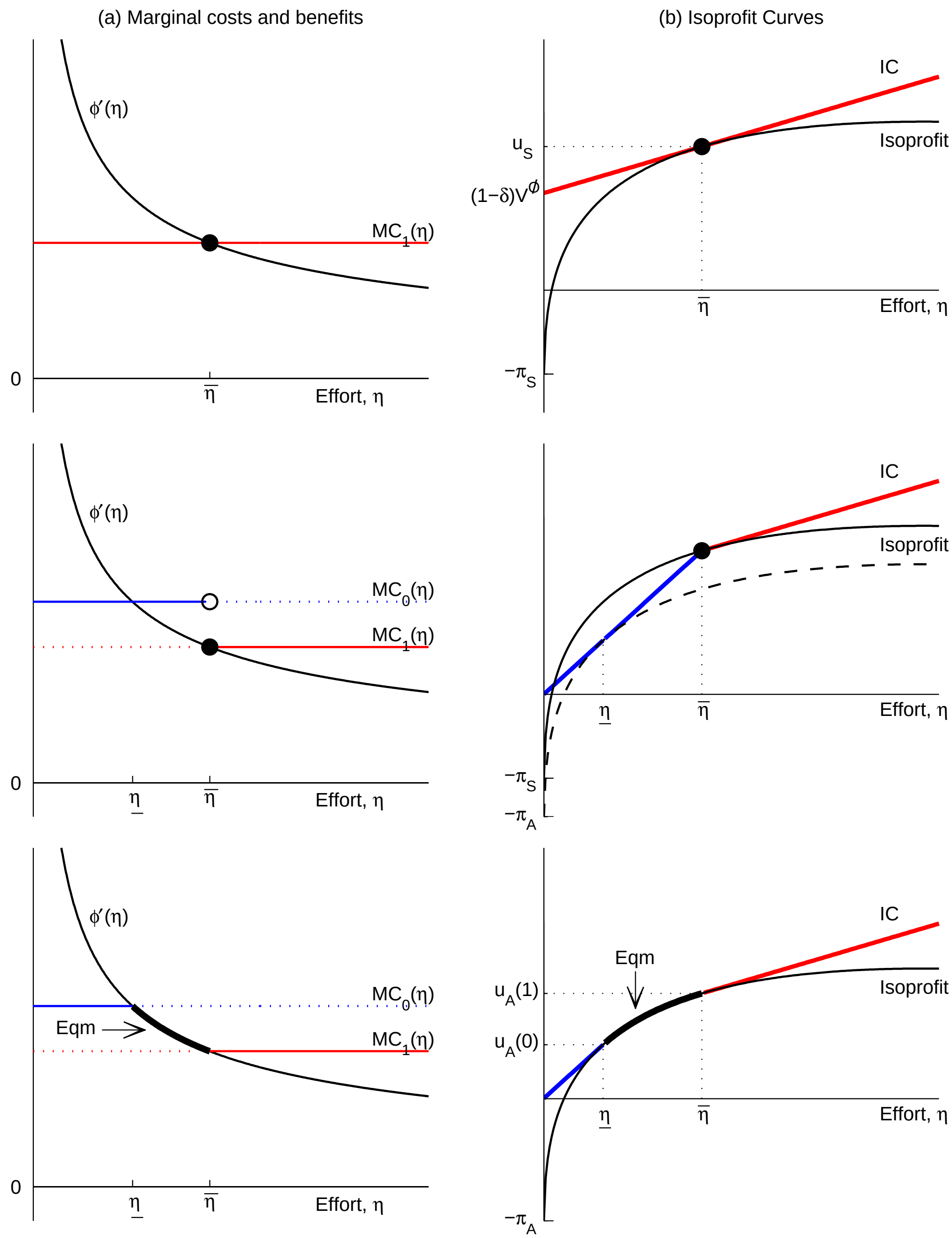

Figure 4: **The Impact of On-the-Job Search.** The **top left panel** illustrates the marginal benefit and marginal cost of effort $\mathrm{MC}_1(\eta)$ under Shapiro-Stiglitz matching when other firms offer the Shapiro-Stiglitz equilibrium contract with effort $\overline{\eta}$. The **top right panel** shows the firm's isoprofit function and the worker's IC constraint. The **middle panels** introduce anonymous matching and suppose other firms continue to offer the Shapiro-Stiglitz contract. When the firm offers a lower wage, it faces a lower retention rate and higher marginal cost of effort $\mathrm{MC}_0(\eta)$, but does not need to compensate its worker for the opportunity cost of forgone searching, $(1-\delta)V^{\varnothing}$. This yields a profitable deviation with effort $\underline{\eta}$. The **bottom panels** show the equilibrium under anonymous matching. Here, $\mathrm{MC}_1(\eta)$ and $\mathrm{MC}_0(\eta)$ are marginal costs for highest and lowest firms (i.e. quantiles 1 and 0). The distribution of contracts is determined so that each firm equates the marginal cost and benefit of effort, and achieves the same profit.

(b) By the first-order condition (5.1) turnover increases the marginal cost of effort to firm x, so effort at firm x decreases: $\eta(x) \geq \widetilde{\eta}(x)$. As effort is always below its first-best level $(\phi')^{-1}(1)$ and surplus $\phi(\eta) - \eta$ is concave, a reduction in effort reduces surplus.

(c) We first fix profits π, and show that with increased OJS $\widetilde{\psi}$ and corresponding utility levels $\widehat{u}(x) = \phi(\widetilde{\eta}(x)) - \widetilde{\eta}(x) - \pi$, the worker's IC constraint (5.3) is slack for the top firm. Since $V(u(1)) - V^{\varnothing}$ is decreasing in π by equation (5.4), firm profits must then increase in order for IC to bind, so $\pi \leq \widetilde{\pi}$.

To show that the IC constraint is slack we first show that, fixing profits, the value of an unemployed worker $V^{\varnothing} = V^{\varnothing}(\psi, u(\cdot))$, as defined by (3.2), decreases with an increase on OJS. First fix utility $u(x)$, but suppose ψ increases to $\widetilde{\psi}$. By definition, this decreases job offers to unemployed workers $F^{\varnothing} = \psi(F^e(\cdot))$, so $V^{\varnothing}(\widetilde{\psi}, u(\cdot)) \leq V^{\varnothing}(\psi, u(\cdot))$. Second, keep profits π and the matching function $\widetilde{\psi}$ fixed, but consider utility levels $\widehat{u}(x) = \phi(\widetilde{\eta}(x)) - \widetilde{\eta}(x) - \pi$ where $\widetilde{\eta}(x)$ is equilibrium effort under $\widetilde{\psi}$, and π is equilibrium profit under ψ. Part (b) implies that $\widehat{u}(x) \leq u(x)$, so the value of unemployment decreases further, $V^{\varnothing}(\widetilde{\psi}, \widehat{u}(\cdot)) \leq V^{\varnothing}(\widetilde{\psi}, u(\cdot))$. Thus, $V^{\varnothing}(\widetilde{\psi}, \widehat{u}(\cdot)) \leq V^{\varnothing}(\psi, u(\cdot))$.

Now, for the top job $\langle u(1), \eta(1) \rangle$ in the original equilibrium the Bellman equation is given by $V(u(1)) = u(1) + \delta\left(\alpha V(u(1)) + (1-\alpha) V^{\varnothing}\right)$. Because the effort of the top firm equals $\eta(1)$ and profits are fixed, the transition from $(\psi, u(\cdot))$ to $(\widetilde{\psi}, \widehat{u}(\cdot))$ does not alter $u(1) = \widehat{u}(1)$ but decreases $V^{\varnothing}$. Therefore the value of the top job $V(u(1))$ decreases only through the decrease of $V^{\varnothing}$ and, by discounting, the increase in on-the-job search increases $V(u(1)) - V^{\varnothing}$, so the IC constraint is slack.

(d) Parts (b) and (c) show that surplus is lower and profits are higher for all jobs, so workers' utility is also lower. As in (c), the higher profits, poorer selection of jobs and lower effort also lower the value of unemployment, $V^{\varnothing} \geq \widetilde{V}^{\varnothing}$. Workers' continuation values $V(u(x))$ also decrease because the IC constraint implies $V(u(x)) = V^{\varnothing} + \eta(x)/\alpha\delta$. □

The proof of Theorem 3 shows that an increase in on-the-job search increases turnover, and thereby the marginal cost of effort. When some firms accordingly substitute good stable jobs with bad temp jobs, the prospects of unemployed workers get worse. This introduces slack into workers' incentive constraints and in a second step allows firms to cut wages. Hence, workers lose twice: First when firms substitute some stable jobs with temp jobs, and then again when all firms cut wages. Firms win because one firm's lower wage reduces the value of unemployment and exerts a positive externality on other firms who can cut wages and increase profits accordingly. On-the-job search allows firms to reap this positive externality in equilibrium, essentially inducing endogenous backloading over a worker's career.

6 Entry

So far we have treated the number of firms in the economy as exogenous. We now endogenize entry to integrate a further important aspect of inter-firm competition with the intra-firm contracting analysis. Entry allows us to assess the effects of on-the-job search on the level of unemployment and leads to a more nuanced welfare analysis. In particular, the deterioration in job quality of existing jobs analyzed in Section 5 is now counteracted by the creation of additional jobs at the bottom of the job distribution.

Definition: An *industry equilibrium* consists of a mass $n \leq 1$ of firms and distribution of contracts $\langle u(x), \eta(x) \rangle$ such that

(a) Every contract $\langle u(x), \eta(x) \rangle$ is firm-optimal, generates profits equal to $\pi = 0$ and attracts a worker with probability 1;[13] and

(b) Given the matching technology $\psi(\cdot)$ and distribution of utilities $F(\cdot)$, the distributions of outside offers $F^e(\cdot), F^{\varnothing}(\cdot)$ are derived from market clearing (4.1), and the value of unemployment $V^{\varnothing}$ is given by equation (3.2).

Industry equilibrium is characterized by the same three conditions as in Section 5. Before, they determined the contract distribution and profit level; here they determine the contract distribution and the number of jobs.

1. The first-order condition (or marginal IC constraint) for contract $\langle u(x), \eta(x) \rangle$:

$$\phi'(\eta(x)) = \frac{1}{\alpha\delta\beta_n(x)}, \tag{6.1}$$

where the retention rate $\beta_n(x)$ at firm x now depends on the number of firms in the market via market clearing (4.2). This uniquely determines effort $\eta(x)$ up to the constant n.

2. Zero profits:

$$u(x) = \phi(\eta(x)) - \eta(x). \tag{6.2}$$

This uniquely defines utility $u(x)$ up to the constant n.

3. The IC constraint at the top:

$$\eta(1) = \alpha\delta \left(V(u(1)) - V^{\varnothing} \right) \tag{6.3}$$

where $\eta(1) = (\phi')^{-1}(1/\alpha\delta)$. This uniquely defines the number of firms, n, because an increase in the number of firms benefits unemployed workers more immediately than employed workers, and tightens the IC constraint of the highest firm (6.3).

Theorem 4 (Equilibrium Characterization) *Industry equilibrium with entry exists and is unique. With anonymous matching, there is full employment: $n = 1$. With less on-the-job search, some unemployment remains: $n < 1$.*

Proof. As in Theorem 2, equations (6.1), (6.2) and (6.3) are necessary and sufficient for equilibrium and the first two uniquely determine $\langle u(x), \eta(x) \rangle$ up to n. Thus, we need to show that there exists

[13] The restriction to equilibria with $n \leq 1$ firms where each firm attracts a worker rules out spurious equilibrium multiplicity that arises when additional firms enter that either (a) never attract a worker, or (b) tie with other firms for the last available worker and make zero profits.

a unique number of firms $n \in (0,1]$ such that incentive compatibility (6.3) is satisfied. To do so, we show that the RHS of (6.3) is (1) continuous and strictly decreasing in $n \in (0,1]$, (2) strictly greater than the LHS as $n \to 0$, and (3) weakly smaller than the LHS for $n = 1$.

For (1), the first-order condition (6.1) yields an important independence property: As long as (6.1) holds at contract $\langle u, \eta \rangle$, the probability of receiving a superior job offer is independent of n. In particular, it equals $1 - F^e(u) = 1 - 1/\alpha\delta\phi'(\eta)$ for the employed and $1 - F^{\varnothing}(u) = 1 - \psi(1/\alpha\delta\phi'(\eta))$ for the unemployed.

Thus, when $\langle u(0), \eta(0) \rangle$ is the lowest job with n firms, then with $n' > n$ firms an unemployed worker faces a higher probability of getting any job, but the same probability of finding a job above $u(0)$, and therefore a higher probability of finding a job at or below $u(0)$. Denote the average value of these additional job offers at the bottom of the job distribution by W. For any $u \geq u(0)$, the transition probability $F^e(u)$ is unaffected by the increase in n, so $V(u)$ only changes via the value of unemployment $V^{\varnothing}$.

Denote by $\Delta V^{\varnothing}$ (resp. $\Delta V = \Delta V(u(1))$) the difference in the value of unemployment (resp. value of employment in the highest job, or any other) when n increases to n' and θ to θ'. The Bellman equations for employed workers (3.1) and unemployed workers (3.2) imply

$$\begin{aligned} \Delta V^{\varnothing} &= (\theta' - \theta)W + \delta((1 - \alpha\theta')V^{\varnothing} + \theta\alpha\Delta V) \\ \Delta V &= \delta(\alpha\Delta V + (1 - \alpha)\Delta V^{\varnothing}) \end{aligned} \tag{6.4}$$

Together, these equations imply that $(\theta' - \theta)W/(1-\delta) > \Delta V^{\varnothing} > \Delta V > 0$. Hence $V^{\varnothing}$ is continuously and strictly increasing in n, and $V(u(1)) - V^{\varnothing}$ is continuously and strictly decreasing in n.

For (2), when n approaches 0, unemployed workers never find a job and $V^{\varnothing}$ vanishes. Thus $V(u(1))$ approaches $u(1)/(1 - \alpha\delta)$ and the IC constraint is slack because

$$\lim_{n \to 0} \alpha\delta(V(u(1)) - V^{\varnothing}) = \frac{\alpha\delta}{1 - \alpha\delta} u(1) = \frac{\alpha\delta}{1 - \alpha\delta}(\phi(\eta(1)) - \eta(1)) > \eta(1).$$

The last inequality owes to $\frac{\alpha\delta}{1-\alpha\delta}(\phi(\eta) - \eta) - \eta \propto \phi(\eta) - \eta/\alpha\delta$ being maximized at $\eta(1) = (\phi'^{-1}(1/\alpha\delta)$.

For (3), when $n = 1$, the unemployed receive better job offers than the employed and get some job for sure, so $V^{\varnothing} \geq V(u(0))$. With anonymous matching $n = 1$ is an equilibrium: the lowest job has retention $\beta(0) = 0$, effort $\eta(0) = 0$, utility $u(0) = 0$ and is thus equivalent to unemployment both in terms of current utility and in terms of future matching opportunities. The IC constraint holds since $\eta(0) = 0 = \alpha\delta \left(V(u(0)) - V^{\varnothing} \right)$.

For other matching functions, where $\psi(z) < z$ for some $z \in [0,1]$, the IC constraint is violated when $n = 1$. If $F^e(u(0)) = 0$ then there exists $u \in [u(0), u(1)]$ with $F^e(u) = z$, because $F^e(u(1)) = 1$ and $F^e(\cdot)$ is continuous. Hence $F^{\varnothing}(u) \leq \psi(F^e(u)) < F^e(u)$, so unemployed workers receive strictly better job offers and $V^{\varnothing} > V(u(0))$, violating IC for the bottom job. If $F^e(u(0)) > 0$ then the first-order condition, $\phi'(\eta) = 1/\alpha\delta F^e(u)$, implies that $\eta(0) > 0$ and the violation follows directly from $V^{\varnothing} \geq V(u(0))$. $\square$

In the classical model of Shapiro-Stiglitz where only unemployed workers receive job offers, unemployment is necessary to maintain workers' incentives. Theorem 4 shows that, in the presence of on-the-job search, workers are also motivated by the threat of underemployment. To understand this result, remember that when a worker takes a job (in particular, the lowest job), he must be compensated for the opportunity cost of foregoing search. This creates a fixed cost for a new firm to enter the industry and means that when the unemployed are better searchers than the employed, there will be less than full employment. In the case of anonymous matching, there is no opportunity cost to take a job, no fixed cost to be paid, and therefore we obtain full employment. Anonymous matching should be viewed as a dividing line rather than a corner case: In Section 7 we show that equilibrium exhibits full employment whenever the employed receive better offers than the unemployed.[14]

We now return to the question how on-the-job search affects the job distribution.

Theorem 5 (Comparative Statics of OJS) *When on-the-job search increases, $\psi(z) \leq \widetilde{\psi}(z)$ for all $z \in [0,1]$, then:*

(a) Overall employment increases: $n \leq \widetilde{n}$.

(b) The number of good jobs decreases: $n(1-F(u)) \geq \widetilde{n}(1-\widetilde{F}(u))$ for all $u \in [u(0), u(1)]$.

More specifically, when on-the-job search increases from $\psi = \psi_S$ to $\widetilde{\psi} = \psi_A$, then:

(c) Aggregate worker utility, and thus welfare decrease: $n_S \int u_S(q)dq \geq n_A \int u_A(q)dq$.

Proof. (a) As in the proof of Theorem 3, if we fix the number of firms, an increase in on-the-job search increases $V(u(1)) - V^{\varnothing}$ and introduces slack into the IC constraint (6.3). Since this difference is decreasing in n, as shown in Theorem 4, the number of firms must increase in order for IC to bind.

(b) For $u \in [u(0), u(1)]$ the number of jobs above u is given by $n(1-F(u))$. Rearranging, market clearing (4.1) yields

$$n(1-F(u)) = \frac{(1-\alpha n)(1-F^{\varnothing}(u)) + \alpha n(1-F^e(u))}{1-\alpha F^e(u)}. \tag{6.5}$$

When we increase OJS from $\psi(\cdot)$ to $\widetilde{\psi}(\cdot)$, $F^e(u)$ remains unaffected since it is determined by the marginal IC constraint $\phi'(\eta) = 1/\alpha\delta F^e(u)$ independently of ψ and n. Hence the increase in OJS increases $F^{\varnothing}(u) = \psi(F^e(u))$ and, using part (a), increases n. This decreases the RHS of (6.5).

(c) To compare welfare we first show that the value functions $V^{\varnothing}$ and $V(u)$ are independent of on-the-job search. Utility and effort in the top contract $\langle \eta(1), u(1) \rangle$ are given by the first-order condition (6.1) and zero profits (6.2) and are independent of the matching technology. The IC constraint at the top (6.3) and the Bellman equation for the top job (3.1) thus pin down $V(u(1))$ and $V^{\varnothing}$ independent

[14] The full employment result depends on our parametric assumption on ϕ. Unemployment remains if there is a fixed cost of job creation, $\phi(0) < 0$. Under anonymous matching, unemployment also remains if marginal productivity is bounded, $\phi'(0) < \infty$, although full employment is attained if the employed are strictly better searchers, $\psi(z) > z$.

of ψ. For $u < u(1)$, the marginal value $V'(u)$ is also independent of ψ,

$$V'(u(q)) = \frac{\beta(q)}{1-\alpha\delta\beta(q)} = \frac{1}{\alpha\delta\left(\phi'(\eta(q))-1\right)}$$

using the first-order condition (6.1) to substitute for $\beta(q)$.

Now, label workers by $y \in [0,1]$ and assume that higher workers get higher jobs, so that (post matching) worker $y \in [0, 1-n]$ is unemployed and worker $y \in [1-n, 1]$ occupies job $q(y) = (y-(1-n))/n$. Denote the post-match value of worker y by

$$W(y) := \begin{cases} \delta V^{\varnothing} & \text{if } y < 1-n \\ u(q(y)) + \delta\left(\alpha V(u(q(y))) + (1-\alpha)V^{\varnothing}\right) & \text{if } y > 1-n \end{cases}$$

Hence $W(0) = \delta V^{\varnothing}$ and $W(1) = V(u(1))$ are independent of the matching process. For Shapiro-Stiglitz and anonymous matching, we then have the following single-crossing property,

$$W_{\mathrm{S}}(y) = \begin{cases} W_{\mathrm{S}}(0) = W_{\mathrm{A}}(0) < W_{\mathrm{A}}(y) & \text{if } y \in (0, 1-n_{\mathrm{S}}) \\ W_{\mathrm{S}}(1) = W_{\mathrm{A}}(1) > W_{\mathrm{A}}(y) & \text{if } y \in (1-n_{\mathrm{S}}, 1) \end{cases} \tag{6.6}$$

Let $G^{\varnothing}(y)$ be the probability than an unemployed worker receives a job with utility $u(q(y))$ or less in the matching process. Substituting for $q(y)$ in $F^{\varnothing}$,

$$G^{\varnothing}_{\mathrm{A}}(y) = \frac{y}{1-\alpha(1-y)} \quad \text{and} \quad G^{\varnothing}_{\mathrm{S}}(y) = \begin{cases} 1-\theta_{\mathrm{S}} & \text{if } y < 1-n_{\mathrm{S}} \\ \theta_{\mathrm{S}} & \text{if } y \geq 1-n_{\mathrm{S}} \end{cases}$$

where $\theta_{\mathrm{S}} = (1-\alpha)n_{\mathrm{S}}/(1-\alpha n_{\mathrm{S}})$. The difference in welfare is then given by

$$\begin{aligned} \int_0^1 W_{\mathrm{A}}(y)dy - \int_0^1 W_{\mathrm{S}}(y)dy &< \int_0^1 \left(W_{\mathrm{A}}(y) - W_{\mathrm{S}}(y)\right) dG^{\varnothing}_{\mathrm{A}}(y) \\ &= V^{\varnothing} - G^{\varnothing}_{\mathrm{A}}(1-n_{\mathrm{S}})\, W_{\mathrm{S}}(0) - \left(1 - G^{\varnothing}_{\mathrm{A}}(1-n_{\mathrm{S}})\right) W_{\mathrm{S}}(1) \\ &= V^{\varnothing} - (1-\theta_{\mathrm{S}})W_{\mathrm{S}}(0) - \theta_{\mathrm{S}} W_{\mathrm{S}}(1) \\ &= 0 \end{aligned}$$

The first line follows from the single crossing condition (6.6) and the concavity of G_{A}. Economically, this says that the planner evaluates the effect of on-the job search less favourably than an unemployed worker. The second line follows from $\int_0^1 W_{\mathrm{A}}(y)dG^{\varnothing}_{\mathrm{A}}(y) = V^{\varnothing}$ and the fact that $W_{\mathrm{S}}(\cdot)$ is a step-function. The final line uses the definition of $V^{\varnothing}$ under Shapiro-Stiglitz matching. □

Both with a fixed number of firms and with entry, increasing on-the-job search raises turnover and causes firms to replace stable, high-quality jobs with temporary, low-quality jobs. The key difference is in the knock-on effect of this deterioration. With a fixed number of firms, firms cut wages, reinforcing the detrimental effects on worker welfare and leading to the stark results in Theorem 3. With free entry, on-the-job search incentivizes entry and the loss of high-quality jobs is balanced by an influx of jobs at the bottom of the market and a reduction in unemployment. However, the rise in the number of firms leads to an additional deterioration in the quality of jobs offered: the first-order condition (6.1) implies that turnover in any job $\langle u, \eta \rangle$ must remain constant, so the increase in n replaces unemployed workers (who are good searchers) with employed workers (who are worse searchers) and necessitates a reduction in the number of jobs above $\langle u, \eta \rangle$.

On-the-job search thus leads to a deterioration in job quality but an increase in the number of jobs, so the overall welfare effect is hard to sign. However, Theorem 5(c) shows that, even with entry, an increase in on-the-job search from Shapiro-Stiglitz matching to anonymous matching lowers welfare. The key step in the proof is that the creation of jobs exactly cancels the loss of job quality from the perspective of an unemployed worker, whose equilibrium value is independent of the matching process. Compared to the unemployed worker, the social planner is less concerned with the new jobs at the lower end of the job distribution and more concerned with the job deterioration at the top end, so welfare decreases.

7 Intern Matching

So far we have assumed that unemployed workers receive better job offers than employed workers. This entailed an opportunity cost of employment, so workers' utility u in contract $\langle u, \eta \rangle$ must both incentivize effort η and overcome the opportunity cost. This simplified our analysis by ensuring positive utility u and wages w, and allowing us to ignore workers' participation constraint.

We now relax this restriction and allow for arbitrary matching technologies $\psi : [0,1] \to [0,1]$. In particular, we allow for matching functions where employed workers receive better job offers than unemployed workers, $\psi(z) \geq z$, perhaps because they have superior contacts. We show that the equilibrium analysis in Sections 5 and 6 broadly extends but needs adjustment at the bottom of the job distribution. More precisely, when the workers' participation constraint is binding, the job distribution is truncated and an atom of firms $x \in [0, \tilde{x}]$ offers the same entry-level job $\langle u(0), \eta(0) \rangle$.

First we argue that in equilibrium, the IC constraint binds and wages are non-negative. For general matching technologies this is not obvious because a worker would accept employment at a negative wage if job offers to employed workers $F^e(\cdot)$ are sufficiently better than offers to unemployed workers $F^{\varnothing}(\cdot)$. To prove our claim, assume to the contrary that IC is slack in an equilibrium contract $\langle u, \eta \rangle$. Then the contract $\langle u, \eta + \epsilon \rangle$ is also self-enforcing and $\langle u, \eta \rangle$ can only be optimal if effort is first-best, $\phi'(\eta) = 1$. If IR binds in $\langle u, \eta \rangle$, then this contract generates maximal surplus and allocates all of it to the firm. This is impossible in equilibrium as workers' utility would be negative in all contracts if all firms can achieve this profit level. If IR is slack in $\langle u, \eta \rangle$, then the contract $\langle u - \epsilon, \eta \rangle$ is self-enforcing and more profitable. It follows that IC binds which, together with IR, implies that wages are non-negative.

The firm's problem is thus to choose $\langle u, \eta \rangle$ to maximise profits $\pi = \phi(\eta) - \eta - u$ subject to IC and wages being nonnegative, $u + \eta \geq 0$. Since the IC constraint binds at any firm-optimal contract, this is equivalent to choosing $\eta \geq \tilde{\eta}$ to maximize $\pi_*(\eta) = \phi(\eta) - \eta - u_*(\eta)$ where $u_*(\eta)$ is defined by IC and $\tilde{\eta} := \inf \{\eta \geq 0 | u_*(\eta) + \eta \geq 0\}$.

We first consider the case with a fixed number of firms, $n < 1$. In equilibrium, effort $\eta(x)$ at firms $[\tilde{x}, 1]$ satisfies the first-order condition (5.1)

$$\phi'(\eta(x)) = \frac{1}{\alpha\delta\beta\,(x)},$$

while firms $x \in [0, \tilde{x}]$ command effort $\tilde{\eta} = \eta\,(\tilde{x})$. Utilities are given by $u\,(x) = \phi\,(x) - \eta(x) - \pi$, and profits π are uniquely determined by the IC constraint at the top (5.3)

$$\eta(1) = \alpha\delta\left(V(u(1)) - V^{\varnothing}\right),$$

where $\eta(1) = (\phi')^{-1}(1/\alpha\delta) =: \overline{\eta}$. The proof of equilibrium existence and uniqueness (Theorem 2) extends easily to general matching technologies. The only step that changes is that $du(1)/d\pi = -1$ for $x \in (\tilde{x}, 1]$, while $du(x)/d\pi = 0$ for $x \in [0, \tilde{x})$. Since some jobs have to offer strictly positive utility, we have $\tilde{x} < 1$, $du(1)/d\pi = -1$ and $V(u(1)) - V^{\varnothing}$ strictly decreases in π.

Figure 5 illustrates the equilibrium contracts. Firms $x \in [0, \tilde{x}]$ exploit the non-pecuniary benefit of on-the-job search by offering entry level jobs that command positive effort $\tilde{\eta}$ and pay zero wages. Workers are willing to work for free and incur negative utility in exchange for the opportunity of a job with a better firm.

The possibility of such an atom is in contrast to the smooth job distribution in Section 5, where firms can profitably undercut a contract offered by a positive mass of competitors. Undercutting is not feasible here because the workers' participation constraint is binding. Of course, the atom may be empty, e.g. if the unemployed are better searchers. However, in the special case of intern matching we show below that the lowest job $\langle u(0), \eta(0)\rangle$, the *internship*, is offered by more than αn firms and is the only job ever offered to the unemployed.

Next, suppose there is free entry of firms into the market. In Section 6 effort and utility are determined by the first-order condition (6.1) and the zero-profit condition (6.2), while employment is determined by the IC constraint of the top firm (6.3). The proof of equilibrium existence and uniqueness (Theorem 4) extends without modification to general matching technologies as long as the IC constraint of the top firm is violated when $n = 1$. Otherwise, firms $x \in [0, \tilde{x}]$ all offer contract $\langle u(\tilde{x}), \eta(\tilde{x})\rangle$, while the first-order condition (6.1) determines contracts of firms $x \in [\tilde{x}, 1]$. The cutoff $\tilde{x}$ is uniquely determined so the IC constraint at the top (6.3) binds: The values $V^{\varnothing}$ and $V(u(1))$ increase in $\tilde{x}$, but the difference $V(u(1)) - V^{\varnothing}$ decreases because employees in the top job only benefit from the improvement at the bottom of the distribution after they become unemployed.[15]

The following result extends Theorem 4, showing that there is full employment if employed workers are better searchers than the unemployed. Moreover, there is a nonempty atom at the bottom of the distribution paying strictly positive wages if employed workers are strictly better

[15] As with a fixed number of firms, a firm that undercuts the atom fails to obtain a worker. However, since $n = 1$, the relevant IR constraint is not that the firm's job has to be better than unemployment, but that the offer cannot be strictly worse than mass 1 of other offers (see footnote 8).

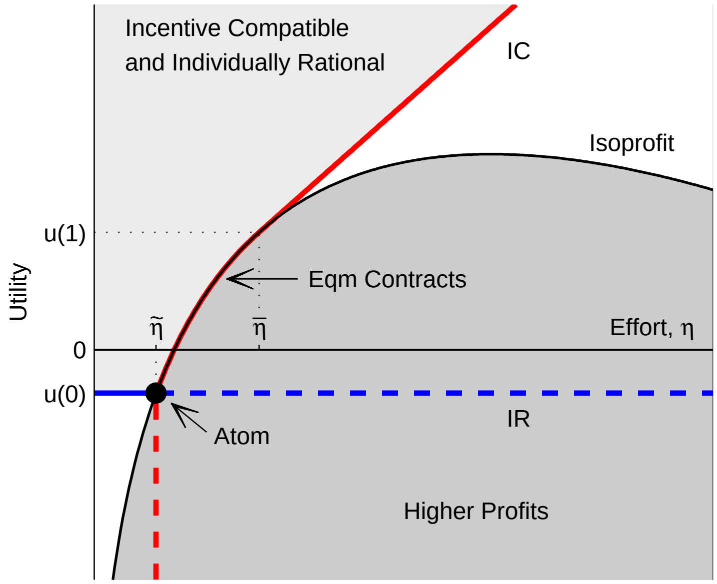

Figure 5: **The Set of Optimal Contracts with Intern Matching.** This figure shows the firm's isoprofit curve $u = \phi(\eta) - \eta - \pi$, the worker's IC constraint $u_*(\eta)$, and the IR constraint for the equilibrium contracts. Firms $x \geq \tilde{x}$ have effort satisfying the first-order condition; firms $x \leq \tilde{x}$ offer the contract $\langle u(0), \eta(0)\rangle = \langle -\tilde{\eta}, \tilde{\eta}\rangle$ which pays wage $w(0) = 0$.

searchers.

Theorem 6 (Equilibrium Characterization) *Suppose there is free-entry. If $\psi(z) \geq z$ then industry equilibrium has full employment $n = 1$. If additionally $\psi(z) > z$ for some z then there exists $\tilde{x} > 0$ such that all firms $x \in [0, \tilde{x}]$ offer the same contract $\langle \eta(0), u(0)\rangle$ with strictly positive wage $w(0) > 0$.*

Proof. First, note that with $\psi(z) \geq z$ employed workers get weakly better job offers than unemployed workers because $F^{\varnothing}(u) \geq \lim_{\epsilon\to 0} \psi(F^e(u-\epsilon)) \geq \lim_{\epsilon\to 0} F^e(u-\epsilon) = F^e(u)$. Thus, 'employed unemployment' $\langle u, \eta\rangle = \langle 0, 0\rangle$ satisfies IC because $V(0) \geq V^{\varnothing}$.

Now suppose $n < 1$ is an equilibrium. Then all contracts are characterized by the first-order condition (6.1) and the zero-profit condition. We consider separately the cases $\beta(0) > 0$ and $\beta(0) = 0$. If $\beta(0) > 0$ then $\eta(0) > 0$. As $\langle u, \eta\rangle = \langle 0, 0\rangle$ satisfies IC, so does the strictly profitable contract $\langle u, \eta\rangle = \langle \epsilon/\alpha\delta\beta(0), \epsilon\rangle$ for any $\epsilon \leq \eta(0)$, contradicting the optimality of the original contract. If $\beta(0) = 0$, then IC is slack in contract $\langle u, \eta\rangle = \langle 0, 0\rangle$ because employed workers are better searchers and obtain jobs with utility $u > 0$ in the following period while unemployed workers remain unemployed with positive probability. This contradiction implies that equilibrium must exhibit full employment $n = 1$.

Finally, let $\psi(z) > z$ for some z and assume to the contrary that there is no atom at the bottom of the job distribution and so $\langle u(0), \eta(0)\rangle = \langle 0, 0\rangle$. Then, the function $F^e(u(\cdot))$ has full range, so there exist equilibrium contracts $\langle u', \eta'\rangle, \langle u'', \eta''\rangle$ with $u'' > u'$ and $\psi(F^e(u')) > F^e(u'') > F^e(u')$. This implies that employed workers receive strictly better job offers than unemployed workers, $F^{\varnothing}(u'') \geq \lim_{\epsilon \to 0} \psi(F^e(u'' - \epsilon)) \geq \psi(F^e(u')) > F^e(u'')$, implying that IC is slack in $\langle u(0), \eta(0) \rangle = \langle 0, 0 \rangle$. This contradiction implies that there must be an atom of entry-level jobs at the bottom of the job distribution. By the same argument it must be that $\eta(0) > 0$ and therefore $w(0) > 0$. □

Intern matching. We now illustrate the above results for the special case of intern matching. Suppose that $\psi_{\mathrm{I}} \equiv 1$, so employed workers get first pick at jobs. More specifically, employed workers compete for the top α quantiles of the job distribution, while unemployed workers compete for the bottom $1 - \alpha$ quantiles.

First, consider an exogenous number of firms $n < 1$. In the lowest job $\langle u(0), \eta(0) \rangle$, the participation constraint binds. If not, then contract $\langle u(0) - \varepsilon, \eta(0) \rangle$, which has value $V(u(0) - \varepsilon) = V(u(0))$, would be IR, IC and strictly more profitable. The internship $x \in [0, \tilde{x}]$ thus has zero wages. Since $V(u(0)) > V^{\varnothing}$ the internship also has strictly positive effort $\tilde{\eta} > 0$ and thus strictly negative utility. Above the internship, $x \geq \tilde{x}$, effort $\eta(x)$ is determined by its first-order condition (5.1).

As the internship has non-zero retention rate $1/\alpha\delta\phi'(\tilde{\eta})$, some interns are only offered other internships. Therefore unemployed workers, who are only offered jobs that employed workers have turned down, are only offered internships. When offered an internship, an unemployed worker is indifferent between accepting it and turning it down, implying that $V^{\varnothing} = 0$. All the attractive jobs offered in equilibrium are worth nothing to an unemployed worker because the firms offering internships exploit their gatekeeper role and dissipate the rents from these attractive prospects.

As workers' outside option $V^{\varnothing}$ equals 0, firms effectively do not compete for workers, in the sense that a firm that offers the highest contract $\langle u(1), \eta(1) \rangle$ with $\eta(1) = \overline{\eta}$ needs to pay its worker the same wage as if it were the only firm in the market. Utility $u(1)$ thus equals the interest on effort $\overline{\eta}$ and the wage equals $w(1) = \overline{\eta}/\alpha\delta$, yielding monopoly profits[16]

$$\pi_{\mathrm{I}} = \phi(\overline{\eta}) - \frac{1}{\alpha\delta}\overline{\eta}.$$

As internships are equally profitable and pay zero wages, the effort level in the internship is pinned down by $\phi(\eta(0)) = \phi(\overline{\eta}) - \overline{\eta}/\alpha\delta$.

When the number of firms $n < 1$ increases, retention rates $\beta(x)$, and thus equilibrium effort $\eta(x)$, are unaffected because employed and unemployed workers do not compete for the same jobs. Hence the increase in the number of firms just scales up the contract distribution without affecting turnover, surplus, profits or utility at firm x. Free-entry thus leads to full employment $n = 1$, with identical effort levels $\eta(x)$ and wages higher by π_{I}.[17]

[16] These "monopoly" profits exceed the "collusive" profits that firms attain in equilibrium with anonymous matching. There, the *lowest* firm has no effective competition and maximizes profits with marginal cost of effort $1/\alpha\delta(1-\theta)$, resulting in $\pi_{\mathrm{A}} = \phi(\underline{\eta}) - \underline{\eta}/\alpha\delta(1-\theta)$. Here the *highest* firm has no effective competition and maximizes profits with marginal cost of effort $1/\alpha\delta$, resulting in $\pi_{\mathrm{I}} = \phi(\overline{\eta}) - \overline{\eta}/\alpha\delta$.

[17] With a fixed number of firms $n = 1$, equilibrium is indeterminate. Equilibrium effort is given by $\eta(x) = \eta(0)$ for $x \in [0, \widetilde{x}]$ and by (5.1) for $x \in [\widetilde{x}, 1]$, and any profit level $\pi \in [0, \pi_{\mathrm{I}}]$ is compatible with equilibrium. The reason underlying this indeterminacy is that the IC constraint is independent of a constant wage shift when $n = 1$. This is in contrast to the case $n < 1$ where an increase in wages benefits the employed more than the unemployed and there

8 Heterogeneity

In this section we analyze the impact of firm and worker heterogeneity on the distribution of profits and wages. These extensions demonstrate the flexibility of the model, and allow us to analyze how exogenous heterogeneity is amplified endogenously by on-the-job search.

8.1 Heterogeneous Firms

First we consider heterogeneous firms and study how the profits of a firm depend on the productivity of its competitors; this comparative static may be relevant for firms' location decisions. Standard intuition suggests that an increase in competitors' productivities increases labor market competition and decreases profits. We corroborate this intuition for Shapiro-Stiglitz matching and anonymous matching, but show that it fails for intern matching.

Productivity p is distributed with a continuous distribution $G(\cdot)$ on $[\underline{p},\overline{p}]$ and the production function $\phi(\eta,p)$ is strictly supermodular; profits are given by

$$\pi = \phi(\eta,p) - \eta - u. \tag{8.1}$$

Firm p chooses a contract $\langle u(p),\eta(p)\rangle$ to maximize profit (8.1) subject to incentive compatibility,

$$\eta(p) = \alpha\delta\left(V(u(p)) - V^{\varnothing}\right). \tag{8.2}$$

which defines $u = u_*(\eta)$. The resulting profit function, $\pi_*(\eta,p) = \phi(\eta,p) - \eta - u_*(\eta)$, is strictly supermodular in (η,p), so any optimal selection $\eta(p)$ is increasing.

Following Section 5, first suppose there are $n < 1$ firms and $\psi(z) \leq z$. We determine equilibrium in three steps.

1. Effort $\eta(p)$ is uniquely determined by the firm's first-order condition

$$\frac{\partial}{\partial\eta}\phi(\eta(p),p) = \frac{1}{\alpha\delta\beta(G(p))}. \tag{8.3}$$

2. Utility is given by $u(p) = \phi(\eta(p),p) - \eta(p) - \pi_*(\eta(p),p)$ where firms' profits are determined up to a constant by applying the envelope theorem to $\pi_*(\eta(p),p)$,

$$\pi(p) = \pi(\overline{p}) - \int_p^{\overline{p}} \frac{\partial}{\partial p}\phi(\eta(p),\hat{p})d\hat{p} \tag{8.4}$$

   and $\pi(p) := \pi_*(\eta(p),p)$.

3. The constant $\pi(\overline{p})$ is determined by the incentive constraint (8.2) for firm $\overline{p}$.

exists a unique level of π where the IC constraint binds.

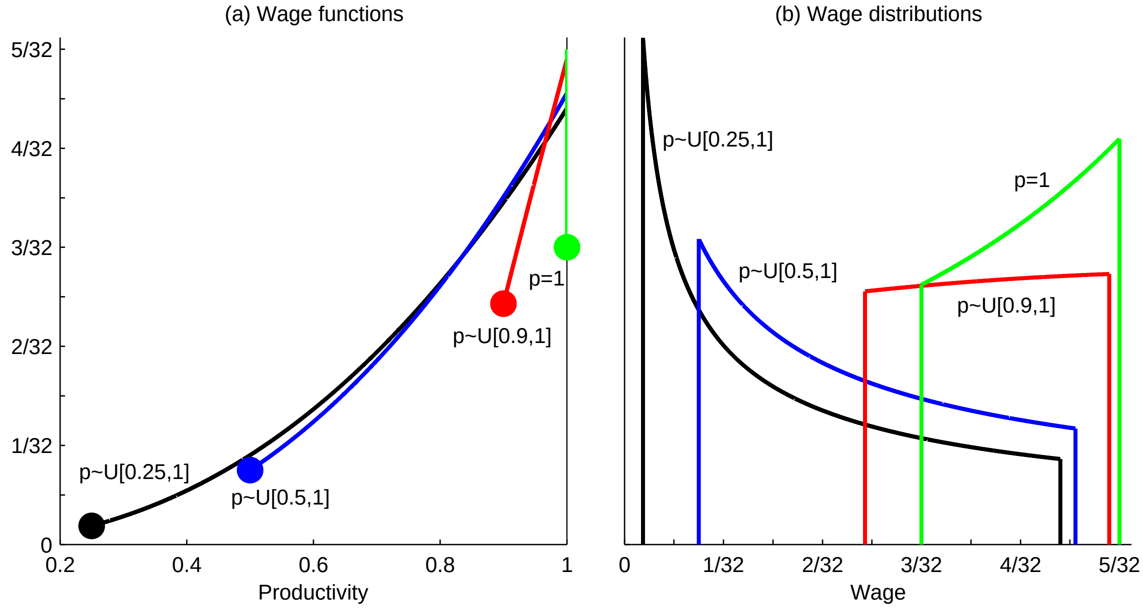

Figure 6: **Wages with Heterogeneous Firms.** This figure shows wages for $p \sim [0.25, 1]$, $p \sim [0.5, 1]$, $p \sim [0.9, 1]$ and $p = 1$. Panel (a) shows the wage function $w(p)$, while panel (b) shows the resulting densities of wages. This figure assumes output is $\phi(\eta, p) = p\sqrt{2\eta}$, the mass of firms is $n = 2/5$, the breakup rate is $1 - \alpha = 1/2$, and the discount rate is $\delta = 1/2$.

With intern matching, low-productivity firms $p \in [\underline{p}, \widetilde{p}]$ offer an internship contract $\langle u(\widetilde{p}), \eta(\widetilde{p})\rangle$ with wage $w(\widetilde{p}) = 0$ while high-productivity firms $p \in [\widetilde{p}, \overline{p}]$ offer contracts $\langle u(p), \eta(p)\rangle$ where effort is given by (8.3) and utilities are determined as above.

Figure 6 shows how exogenous and endogenous heterogeneity aggregate into job dispersion. It illustrates the wage function $w(p)$ and resulting wage density for different distributions of productivity under anonymous matching. An increase in the lowest productivity, $\underline{p}$, has two effects. First, the average wage level rises, increasing the value of unemployment and forcing any higher firm to increase wages. For this reason, the highest wage is increasing in $\underline{p}$. Second, the increase in competition lowers the productivity ranking of any given firm, lowering the retention rate and reducing effort. Consequently, firm p pays less when it is at the bottom of the distribution than when it is in the middle. Together, these two effects cause the wage function $w(p)$ to become steeper as $\underline{p}$ rises. In the limit, as $\underline{p} \to \overline{p}$, the distribution converges to that in Section 5.

We now examine how firm p's profit $\pi(p)$ depends on the productivity of its competitors. More productive competitors command higher effort and pay higher wages, which affects firm p through the incentive constraint of its worker $\eta(p) = \alpha\delta(V(u(p)) - V^{\varnothing})$. The critical question is thus whether an improvement in competitors' job offers is more valuable to an unemployed worker or to firm p's employee. For the sake of a clean analysis we restrict attention to our three example matching technologies.

With Shapiro-Stiglitz matching the standard intuition is correct and firm p's profits decrease as competitors get more productive. The reason is that an unemployed worker benefits from a better job distribution as soon as he is offered one of the improved jobs, while firm p's employee first has to lose his current job before receiving the improved job offers. Therefore, an increase in competitors' productivities decreases $V(u(p)) - V^{\varnothing}$, tightening the incentive constraint and decreasing firm p's profits.

With anonymous matching, the same argument applies when productivity increases for firms below p and inferior competitors catch up: Firm p's employee first has to lose his job before benefiting from the improvements below p, so the incentive constraint tightens and profits decrease. However, if productivity increases for firms above p, firm p's profit is not affected. By definition, firm p's employee receives the same job offers as unemployed workers, so $V(u(p))$ and $V^{\varnothing}$ increase in lock-step.

With intern matching the standard intuition fails and firms benefit from more productive competitors. The value of unemployment $V^{\varnothing}$ is constant equal to zero for any productivity distribution. The value of firm p's job $V(u(p))$ to the contrary increases in the productivity of competitors above p as firm p's worker is hoping for job offers from these competitors. This introduces slack into firm p's incentive constraint and allows it to increase profits. Intuitively, firm p exploits its gatekeeper role to cut wages. When productivity increases for competitors below p, both $V(u(p))$ and $V^{\varnothing}$ are unaffected, so profits $\pi(p)$ do not change.

The qualitative effect of competitors' productivities on firm p's profit is summarized by the following table: (see Appendix B for a proof)

	Below p	Above p
Shapiro-Stiglitz	$-$	$-$
Anonymous	$-$	0
Intern	0	$+$

With Shapiro-Stiglitz matching and anonymous matching, firms prefer to be in a market with unproductive competitors. Thus an American firm may move to India to decrease turnover and attract more committed, loyal workers. In contrast, with intern matching, firms prefer markets with productive competitors. This may lead to agglomeration, with small production studios moving to Los Angeles, obtaining a cheap workforce who are motivated by future job prospects at glitzier competitors.

8.2 Heterogeneous Workers

We now turn to the case of heterogeneous workers and use the model to show that some low-wage firms will refuse to hire workers they regard as "overqualified".

Suppose there are two publicly observed types of worker $\kappa \in \{L, H\}$, where worker κ has cost of effort η/κ and $H \geq L > 0$. There is mass m_κ of type κ, with $m_L + m_H = 1$. An equilibrium is described by the number of firms n_κ offering contracts to workers of type κ and two distributions of contracts $\langle \eta_\kappa(x), u_\kappa(x) \rangle$.

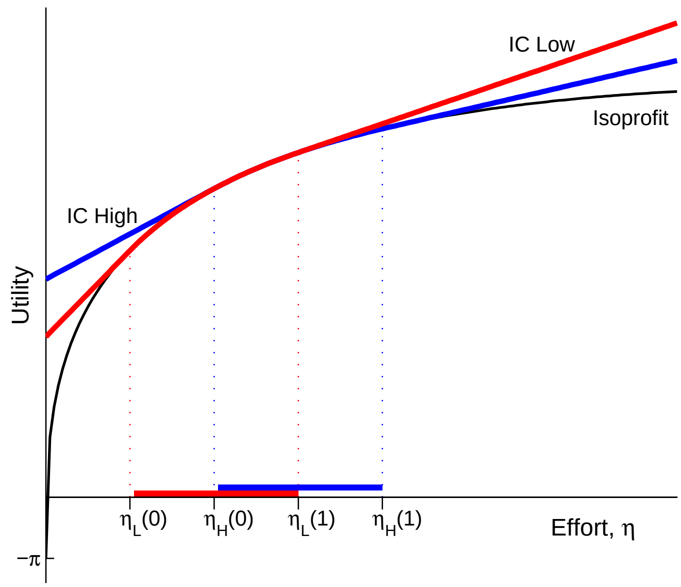

Figure 7: **Equilibrium Effort and Wages with Heterogeneous Workers.** This figure illustrates the equilibrium effort ranges for low-type and high-type workers. Moving along the isoprofit curve, worker κ's IC constraint is satisfied with equality for $\eta \in [\eta_\kappa(0), \eta_\kappa(1)]$ and violated for other effort levels.

Following Section 5, assume there are $n < 1$ firms and $\psi(z) \leq z$. Equilibrium can be characterized in the usual three steps.

1. The first order condition (or marginal IC constraint) for all contracts

$$\phi(\eta_\kappa(x)) = \frac{1}{\alpha\delta\kappa\beta_{n_\kappa}(x)}$$

   where the retention rate $\beta_{n_\kappa}(x)$ at firm x depends on the number of firms in market κ via equation (4.2). This uniquely determines effort $\eta_\kappa(x)$ up to the constants n_κ.

2. Constant profits: There is π such that for $\kappa \in \{L, H\}$

$$u_\kappa(x) = \phi(\eta_\kappa(x)) - \eta_\kappa(x) - \pi$$

   This uniquely determines utilities $u_\kappa(x)$ up to the constants, $\{n_L, n_H, \pi\}$.

3. The IC constraints at the top state that

$$\eta_\kappa(1) = \alpha\delta(V_\kappa(u_\kappa(1)) - V_\kappa^\varnothing).$$

   Together with $n_L + n_H = n$, these uniquely determine the constants, $\{n_L, n_H, \pi\}$.

One can think of $\{n_L, n_H, \pi\}$ being determined in two steps. First, for fixed $n_L \in [n - m_H, m_L]$ there is a unique equilibrium profit level $\pi_\kappa(n_L)$ in each market by Theorem 2. Second, n_L is determined to equate profits across the two markets, $\pi_L(n_L) = \pi_H(n - n_L)$. By the proof of Theorem 4, $\pi_L(n_L) - \pi_H(n - n_L)$ is continuous and strictly decreasing in n_L, is strictly positive if $n_L = n - m_H$ and strictly negative if $n_L = m_L$. Hence in any equilibrium, there are unemployed workers of each type.[18]

In equilibrium, there are three ranges of jobs, all with the same profit (see Figure 7). Low-wage jobs with $\eta \in [\eta_L(0), \eta_H(0))$ are only offered to low-type workers, even though unemployed high-type workers would willingly take them. High-type workers are rejected for such jobs because their value of unemployment $V_H^{\varnothing}$ is too high, so they would shirk after accepting the job. That is, such high-type workers are "overqualified". This is consistent with Bewley's (1999, Chap. 15.2) finding that 80% of firms are unwilling to hire overqualified workers because of the belief that such workers would be unhappy and less motivated (50%) and ultimately quit for higher paying jobs (78%). Intermediate jobs with $\eta \in [\eta_H(0), \eta_L(1)]$ are offered to both types of workers. The marginal costs of incentivizing effort is independent of the worker's type: A high type has lower cost of effort but requires a higher incentive premium due to the lower retention rate. Finally, jobs with $\eta \in (\eta_L(1), \eta_H(1)]$ are only offered to high-type workers. Low-type workers are rejected for such jobs because their cost of effort is too high, so they would shirk after accepting the job.

Finally, we can combine firm and worker heterogeneity in a single model. As in Section 8.1, the supermodularity of payoffs implies that high productivity firms offer high effort jobs. As in this section, each type of worker obtains jobs over some range $[\eta_\kappa(0), \eta_\kappa(1)]$ which may overlap for different types of workers. Hence worker-firm matching is partially assortative, with some high-productivity firms preferring low-type workers with high retention rates to high-type workers with low retention rates.

9 Conclusion

This paper analyzes relational firm-worker contracting in the context of a competitive labor market. Relational incentives at any one firm depend on the longevity of its relationships which in turn depends on the contracts offered by other firms. In equilibrium, identical firms offer a continuous distribution of contracts with some firms offering high-wage, high-productivity contracts and others offering low-wage, low-productivity contracts. On-the-job-search affects equilibrium contracts in two ways. First, by decreasing retention rates it leads to a deterioration in the quality of jobs. Second, by reducing the opportunity cost of employment it induces entry of temp firms and can generate full employment. We highlight the flexibility of our model by allowing for arbitrary prioritizations of employed and unemployed workers, and heterogeneous firms and workers.

We have focused on the firm's intensive margin of motivating its single worker while deliberately ignoring the extensive margin of attracting more workers, which is central in the search literature. It would therefore be interesting to integrate the analyses and study the interaction between incentive provision and direct competition. It is also important to think about the source of workers' priorities

[18] With intern matching, profits are discontinuous at $n_\kappa = m_\kappa$ (see footnote 17). Hence there will be m_H firms in more profitable high-type market, and $n - m_L$ firms in the low-type market.

in the labor market. In practice these depend on the accumulation of experience, the screening role of employment and the network structure of contacts.

Appendix

A Theorem 1: Single Firm's Problem

Before proving Theorem 1, we formally state value functions and the firm's problem when offering a contract $\langle w_t, \eta_t \rangle$.

The worker's pre-matching continuation value V_t and post-matching continuation value W_t are given by

$$V_t = \int \max\{W, W_t\} dF^e(W), \tag{A.1}$$
$$W_t = w_t - \eta_t + \delta \left(\alpha V_{t+1} + (1-\alpha) V^{\varnothing} \right).$$

Note that $V_t \leq V_{t'}$ iff $W_t \leq W_{t'}$. For the firm, the pre-matching continuation profits in period-t of a contract Π_t and the profits when having a vacancy $\Pi_{\varnothing}$ are given by

$$\Pi_t = F^e(W_t) \left[\phi(\eta_t) - w_t + \delta(\alpha \Pi_{t+1} + (1-\alpha)\Pi_{\varnothing}) \right] + (1 - F^e(W_t))\Pi_{\varnothing}$$
$$\Pi_{\varnothing} = p(W_1) \left[\phi(\eta_1) - w_1 + \delta(\alpha \Pi_2 + (1-\alpha)\Pi_{\varnothing}) \right] + (1 - p(W_1))\delta \Pi_{\varnothing}$$

where the term in brackets is the post-matching continuation profits. Letting $F^e(W_t)$ be the retention probability in the expression for Π_t implicitly assumes that employed workers reject outside offers when they are indifferent. Note also, that although V_t and Π_t do not correspond to pre-matching continuation values for $t = 1$ because the relationship is yet to be formed, the expressions are well-defined and we use them below.

A contract $\langle w_t, \eta_t \rangle$ is self-enforcing if it satisfies the individual rationality and incentive compatibility constraints for the firm and worker. The worker must accept the offer and exert the required effort

$$\eta_t \leq \alpha\delta \left(V_{t+1} - V^{\varnothing} \right) \qquad \text{for all } t \geq 1 \tag{WIC}$$
$$p(W_1) = \begin{cases} 0 & \text{if } W_1 < Z \\ 1 & \text{if } W_1 > Z \end{cases} \tag{WIR}$$

where Z is the worker's outside option. The firm must make positive profits and prefer to keep its worker,

$$\Pi_{t+1} \geq \Pi_{\varnothing} \qquad \text{for all } t \geq 1 \tag{FIC}$$
$$\Pi_{\varnothing} \geq 0. \tag{FIR}$$

The *firm's problem* is to choose $\langle w_t, \eta_t \rangle$ to maximize $\Pi_{\varnothing}$, subject to (WIC), (WIR), (FIC), and (FIR).

A.1 Proof of Theorem 1

Fix an 'original' self-enforcing contract $\langle w_t, \eta_t \rangle$ with profit $\Pi_\varnothing$. We first impose two simplifying assumptions: $p(W_1) = 1$, that is the firm immediately fills its vacancy when posting its original contract; and $F^e(W_t) > 0$ for some $t \geq 1$, that is the retention rate does not equal zero in all rounds. With these assumptions we then define a self-enforcing stationary contract $\langle w^*, \eta^* \rangle$ with weakly higher profit $\Pi^*_\varnothing \geq \Pi_\varnothing$. Finally, we drop these two assumptions and show how to adapt the arguments.

Define $\eta^* \leq (\phi')^{-1}(1)$ so that

$$\phi(\eta^*) - \eta^* = \sup_{t\geq 1}[\phi(\eta_t) - \eta_t].$$

η^* is well-defined, since $\phi(\eta) - \eta$ is continuous and strictly increasing on $[0, (\phi')^{-1}(1)]$. Next, let $V^* = \sup_{t\geq 1} V_t$ be the highest continuation value for the worker, and w^* be the stationary wage that generates this continuation value

$$V^* = \int \max\left\{W, w^* - \eta^* + \delta\left(\alpha V^* + (1-\alpha)V^{\varnothing}\right)\right\} dF^e(W).$$

Define the resulting pre-matching profits by $\Pi^*_\varnothing$ when the firm has a vacancy and Π^* when the firm has an employee.

Since $\langle w^*, \eta^* \rangle$ is stationary it satisfies (FIC). It also satisfies (WIC): for all $t \geq 1$, $\eta_t \leq \alpha\delta(V_{t+1} - V^{\varnothing})$, so taking the supremum of each side implies that $\eta^* \leq \alpha\delta(V^* - V^{\varnothing})$. It also delivers utility $V^* \geq V_1$, so the corresponding post-matching value is higher $W^* \geq W_1$, (WIR) holds and the firm fills its vacancy with probability $p(W^*) = 1$ when posting $\langle w^*, \eta^* \rangle$.

We now wish to show that the stationary contract delivers higher profits than the original contract. We do so by finding for any $\epsilon > 0$ a period $\tau \geq 1$ with pre-matching profits $\Pi^* \geq \Pi_\tau - \epsilon$. By (FIC) we have $\Pi_t \geq \Pi_\varnothing$ for all $t \geq 2$, and $\Pi_1 \geq \Pi_\varnothing$ follows from the definitions of these two terms. Since $p(W^*) = 1$, we thus have $\Pi^*_\varnothing = \Pi^* \geq \Pi_\varnothing$, as required.

To see $\Pi^* \geq \Pi_\tau - \epsilon$, note that there exists $\tau \geq 1$ with $V_\tau \geq V^* - \epsilon(1-\delta)F^e(W_\tau)$.[19] We can write continuation profits in the original contract as sum of period profits until separation at time T, plus continuation profits

$$\Pi_\tau = \mathbb{E}\left[\sum_{t=\tau}^{T-1} \delta^{t-\tau}\left(\phi\left(\eta_t\right) - w_t\right) + \delta^{T-\tau}\Pi_\varnothing\right] \tag{A.2}$$

where the stochastic separation time T is distributed according to $\Pr(T = \tau) = 1 - F^e(W_\tau)$, $\Pr(T = \tau + 1) = F^e(W_\tau)\left(1 - \alpha F^e\left(W_{\tau+1}\right)\right)$, and so on. In the stationary contract $\langle w^*, \eta^* \rangle$, firm

[19] Here we use the assumption $F^e(W_t) > 0$ for some t.

profits are constant and thus equal[20]

$$\Pi^* = \mathbb{E}\left[\sum_{t=\tau}^{T-1} \delta^{t-\tau}\left(\phi\left(\eta^*\right) - w^*\right) + \delta^{T-\tau}\Pi^*\right] \tag{A.3}$$

for any distribution of separation times T, so in particular for the distribution induced by the original contract. As effort η^* maximizes surplus $\phi(\eta) - \eta$ and continuation profits in the original contract do not fall below their initial level $\Pi_\tau \geq \Pi_\varnothing$, we subtract (A.2) from (A.3) to obtain

$$\Pi^* - \Pi_\tau \geq \mathbb{E}\left[\sum_{t=\tau}^{T-1} \delta^{t-\tau^*}\left(u_t - u^*\right) + \delta^{T-\tau}\left(\Pi^* - \Pi_\tau\right)\right]. \tag{A.4}$$

The worker's value is the discounted sum of his utilities,

$$V_\tau = \mathbb{E}\left[\sum_{t=\tau}^{T-1} \delta^{t-\tau} u_t + \delta^{T-\tau} X_T\right] \tag{A.5}$$

where X_T is the continuation value after separation at time T,

$$X_T = \begin{cases} \delta V^\varnothing & \text{in case of natural attrition,} \\ W & \text{in case of an outside offer } W \geq W_T. \end{cases}$$

Using (A.5) and the analogous expression for V^*,

$$\mathbb{E}\left[\sum_{t=\tau}^{T-1} \delta^{t-\tau}(u_t - u_t^*)\right] = [V_\tau - V^*] - \mathbb{E}\left[\delta^{T-\tau}(X_T - X^*)\right] \geq -\epsilon(1-\delta)F^e(W_\tau) \tag{A.6}$$

where the inequality uses $X^* \geq X_T$, which follows from the fact that $V^* \geq V_T$ and hence $W^* \geq W_T$. Inserting (A.6) back into (A.4) we get

$$\Pi^*_\varnothing - \Pi_\tau \geq -\epsilon \frac{(1-\delta)F^e(W_\tau)}{\mathbb{E}[1-\delta^{T-\tau}]} \geq -\epsilon \tag{A.7}$$

as required.

We now return to the knife-edge cases that we excluded so far and show case-by-case that we can find a stationary contract with higher profits than $\langle w_t, \eta_t \rangle$. First assume that $F^e(W_t) = 0$ for all $t \geq 1$, so the original contract terminates after the first round. Here, the contract $\langle \eta, w \rangle = \langle \eta_1, w_1 \rangle$ is stationary and trivially satisfies (FIC), (WIR) and (WIC). It also yields the same profit as the original contract, so is weakly better.

If $F^e(W_t) > 0$ for some t, but $p(W_1) \in (0,1)$ we need to consider three sub-cases. First, if $V_1 < V^*$ then the stationary contract $\langle \eta^*, w^* \rangle$ defined above has a fill-rate of $p(W^*) = 1$, so (A.3) holds and we can conclude $\Pi^*_\varnothing \geq \Pi_\varnothing$ as above.

[20] Here we use that $p(W^*) \geq p(W_1) = 1$

Second, suppose $V_1 = V^*$ and $\Pi_\varnothing = 0$. The stationary contract $\langle w^*, \eta^* \rangle$ is incentive compatible for the firm and worker, and is accepted with probability $p(W_1)$ since $V_1 = V^*$. Then $\Pi^* \geq \Pi_\varnothing = 0$, and

$$\Pi^*_\varnothing = p(W_1)\Pi^* + (1 - p(W_1))\delta\Pi^*_\varnothing = \frac{p(W_1)}{1 - \delta(1 - p(W_1))}\Pi^* \geq 0$$

To see this suppose, by contradiction, that $\Pi^*_\varnothing < 0$ so $\Pi^* < \Pi^*_\varnothing$, implying that the LHS of equation (A.3) is greater than the RHS. The above arguments then imply that $\Pi^* \geq \Pi_1 \geq \Pi_\varnothing = 0$. It thus follows that $\Pi^*_\varnothing \geq 0 = \Pi_\varnothing$, yielding a contradiction.

Finally, suppose $V_1 = V^*$ and $\Pi_\varnothing > 0$. Consider the stationary contract $\langle \eta^*, w^*_\epsilon \rangle$ where w^*_ϵ is chosen so that the corresponding value is $V^*_\epsilon = V^* + \epsilon(1-\delta)F^e(W_1)$ for $\epsilon > 0$. This contract is incentive compatible for the firm and worker, and is accepted with probability $p(W^*_\epsilon) = 1$ since $V^*_\epsilon > V_1$. Setting $\tau = 1$, equation (A.7) implies that, $\Pi^*_\epsilon \geq \Pi_1 - \epsilon$. Since $p(W^*_\epsilon) = 1$, we thus have $\Pi^*_{\varnothing,\epsilon} = \Pi^*_\epsilon \geq \Pi_1 - \epsilon > \Pi_\varnothing - \epsilon$, where the last inequality uses $p(W_1) < 1$, $F^e(W_1) > 0$ and $\Pi_\varnothing > 0$. Hence $\Pi^*_{\varnothing,\epsilon} > \Pi_\varnothing$ for sufficiently small ϵ.

B Derivation of Table in Section 8.1

Shapiro-Stiglitz Matching: The average Bellman equation for employed workers (3.1) is

$$(1 - \alpha\delta)\widehat{V} = \widehat{u} + \delta(1 - \alpha)V^\varnothing$$

where $\widehat{u} := \int u(p)dG(p)$, and so forth. Substituting this into unemployed Bellman equation (3.2),

$$(1 - \delta)V^\varnothing = \frac{\theta}{1 - \alpha\delta(1 - \theta)}\widehat{u}. \tag{B.1}$$

Using these two equations, the average IC constraint, $\widehat{\eta} = \alpha\delta(\widehat{V} - V^\varnothing)$ becomes

$$\widehat{u} = \frac{1 - (1 - \theta)\alpha\delta}{(1 - \theta)\alpha\delta}\widehat{\eta} \tag{B.2}$$

Finally, using (B.1) and (B.2) to substitute for $(1 - \delta)V^\varnothing$, the profit function (5.5) of one particular firm is given by

$$\pi_*(\eta) = \phi(\eta) - \frac{1}{\alpha\delta(1 - \theta)}[(1 - \theta)\eta + \theta\widehat{\eta}]$$

Hence an increase in competitors' productivity raises their effort $\widehat{\eta}$ via the first-order condition (8.3) and lowers p's profit.

Anonymous Matching: Using the envelope theorem, firm p's equilibrium profits are given by

$$\pi(p) = \pi(\underline{p}) + \int_{\underline{p}}^{p} \partial_p \phi(\eta(\hat{p}), \hat{p})\, d\hat{p}$$

where the profits of the lowest firm are given by

$$\pi(\underline{p}) = \phi(\eta(\underline{p}), \underline{p}) - \eta(\underline{p})/(\alpha\delta(1-\theta)).$$

When productivity increases below p, then any competitor with fixed productivity $\hat{p} < p$ is lower ranked, faces higher turnover and thus commands lower effort $\eta(\hat{p})$. Marginal revenue of productivity $\partial_p \phi(\eta(\hat{p}), \hat{p})$ thus decreases and so do firm p's profits. When productivity increases above p, firm p's profits are unaffected.[21]

Intern Matching: Firm p's equilibrium profits are given by

$$\pi(p) = \pi(\overline{p}) - \int_p^{\overline{p}} \partial_p \phi(\eta(\hat{p}), \hat{p})\, d\hat{p}$$

where the profits of the highest firm are given by

$$\pi(\overline{p}) = \phi(\eta(\overline{p}), \overline{p}) - \eta(\overline{p})/\alpha\delta.$$

When productivity increases above p, then any competitor with fixed productivity $\hat{p} > p$ is lower ranked, faces higher turnover and thus commands lower effort $\eta(\hat{p})$. Marginal revenue of productivity $\partial_p \phi(\eta(\hat{p}), \hat{p})$ thus decreases and firm p's profits increase. When productivity increases below p, firm p's profits are unaffected.

[21] This does not assume a fixed support $[\underline{p}, \overline{p}]$ because we do not require the density to be positive.

References

ALBRECHT, J. W., AND S. B. VROMAN (1992): “Non-Existence of Single-Wage Equilibria in Search Models with Adverse Selection,” *Review of Economic Studies*, 59(3), 617–624.

BEWLEY, T. F. (1999): *Why Wages Don’t Fall During a Recession*. Harvard University Press, Cambridge, MA.

BULOW, J. I., AND L. H. SUMMERS (1986): “A Theory of Dual Labor Markets with Application to Industrial Policy, Discrimination, and Keynesian Unemployment,” *Journal of Labor Economics*, 4(3), 376–414.

BURDETT, K., AND D. T. MORTENSEN (1998): “Wage Differentials, Employer Size, and Unemployment,” *International Economic Review*, 39(2), 257–273.

CAPPELLI, P., AND K. CHAUVIN (1991): “An Interplant Test of the Efficiency Wage Hypothesis,” *Quarterly Journal of Economics*, 106(3), 769–787.

FEHR, E., AND A. FALK (1999): “Wage Rigidity in a Competitive Incomplete Contract Market,” *Journal of Political Economy*, 107(1), 106–134.

GALENIANOS, M., AND P. KIRCHER (2009): “Directed Search with Multiple Job Applications,” *Journal of Economic Theory*, 114(2), 445–471.

HANSEN, M. (2009): “Career Concerns Incentives and Teacher Effort,” Working paper, University of Washington.

HORNSTEIN, A., P. KRUSELL, AND G. L. VIOLANTE (2011): “Frictional Wage Dispersion in Search Models: A Quantitative Assessment,” *American Economic Review*, forthcoming.

JACOB, B. A. (2010): “The Effect of Employment Protection on Worker Effort: Evidence from Public Schooling,” NBER Working paper no. 15655.

KRUEGER, A. B., AND L. H. SUMMERS (1988): “Efficiency Wages and the Inter–Industry Wage Structure,” *Econometrica*, 56(2), 259–293.

LEMIEUX, T. (2006): “Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?,” *American Economic Review*, 96(3), 461–498.

LEVIN, J. (2003): “Relational Incentive Contracts,” *American Economic Review*, 93(3), 835–857.

LEVINE, D. I. (1993): “What Do Wages Buy?,” *Administrative Science Quarterly*, 38(3), 462–483.

MACLEOD, W. B., AND J. M. MALCOMSON (1998): “Motivation and Markets,” *American Economic Review*, 88(3), 388–411.

MILGROM, P., AND J. ROBERTS (1992): *Economics, Organization and Management*. Prentice Hall.

MORTENSEN, D. (2003): *Wage Dispersion*. MIT Press.

NAGYPAL, E. (2008): “Worker Reallocation over the Business Cycle: The Importance of Job-to-Job Transitions,” Working paper, Northwestern University.

OYER, P. (2006): “Initial Labor Market Conditions and Long-Term Outcomes for Economists,” *Journal of Economic Perspectives*, 20(3), 143–160.

PETERS, M. (2010): “Noncontractible Hetergeneity in Directed Search,” Working paper, UBC.

PISSARIDES, C. A. (2000): *Equilibrium Unemployment Theory*. MIT Press, Cambridge, MA.

RAFF, D. M. G., AND L. H. SUMMERS (1987): "Did Henry Ford Pay Efficiency Wages?," *Journal of Labor Economics*, 5(4), S57–S86.

SHAPIRO, C., AND J. E. STIGLITZ (1984): "Equilibrium Unemployment as a Worker Discipline Device," *American Economic Review*, 74(3), 433–444.

SYVERSON, C. (2011): "What Determines Productivity?," *Journal of Economic Literature*, forthcoming.

TOPEL, R. H., AND M. P. WARD (1992): "Job Mobility and the Careers of Young Men," *Quarterly Journal of Economics*, 107(2), 439–479.