Robust Principal Component Analysis: Exact Recovery of Corrupted Low-Rank Matrices via Convex Optimization

Correction

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The supplementary material to the NIPS version of this paper [4] contains a critical error, which was discovered several days before the conference. Unfortunately, it was too late to withdraw the paper from the proceedings. Fortunately, since that time, a correct analysis of the proposed convex programming relaxation has been developed by Emmanuel Candes of Stanford University. That analysis is reported in a joint paper, *Robust Principal Component Analysis?* by Emmanuel Candes, Xiaodong Li, Yi Ma and John Wright, `http://arxiv.org/abs/0912.3599`. That work not only removes the error in our NIPS submission, but yields significantly stronger results than those claimed in the NIPS conference version. Below, we first briefly describe the error, and then describe how this affects the claims of the paper.

Description of the error. In Equation (63) of the supplementary material, we state a tail bound for a function of the "sign matrix" $\tilde{U}\tilde{V}^*$. This bound is obtained by introducing a random rotation $[\,^{I}\ _{R}]$. However, the tail bound for the quantity in (63) is not correct, since the expectation $\mathbb{E}_R[\cdot]$ of this quantity with respect to the auxiliary rotation R is not necessarily zero. This error was discovered by Emmanuel Candes.

Implication on the results. Theorem 2, regarding the completion of random matrices with uniformly distributed singular vectors, depends strongly on the above incorrect argument, and hence should be regarded as a conjecture.

Theorem 1, the main result of the paper, has been significantly strengthened. The paper asserted that the convex programming heuristic successfully recovers certain random matrices of rank $r < Cm/\log m$ from errors affecting ρm^2 of the m^2 entries. New results in [1] remove the necessity to assume any random model on the matrix to be recovered. All that needs to be assumed is that the singular vectors are *incoherent* with the standard basis, in the sense of [2, 3]. The correct result states that rank-r matrices with incoherence parameter μ can be recovered from ρm^2 errors, as long as $r < Cm/\mu \log^2 m$.

Please see [1] for more discussion of this result. [1] also generalizes the result to the non-square case, and to matrix completion with corrupted entries, and further gives a faster-converging algorithm than the proximal gradient approach discussed in the NIPS version.

References

[1] E. Candes, X. Li, Y. Ma, and J. Wright. Robust principal component analysis? *http://arxiv.org/abs/0912.3599*, 2009.

[2] E. Candes and B. Recht. Exact matrix completion via convex optimzation. *Foundatoins of Computational Mathematics*, 9:717–772, 2008.

[3] E. Candes and T. Tao. The power of convex relaxation: Near-optimal matrix completion. *IEEE Transactions on Information Theory, to appear*, 2009.

[4] J. Wright, A. Ganesh, S. Rao, Y. Peng, and Y. Ma. Robust principal component analysis: Exact recovery of corrupted low-rank matrices via convex optimization. In Y. Bengio, D. Schuurmans, J. Lafferty, and C. Williams, editors, *Advances in Neural Information Processing Systems 22*, 2009.