Complexity and Contract

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Abstract

It is well known that contract incompleteness can arise from the impossibility of planning for all future contingencies in a relationship (e.g. Williamson (1975)). In this paper it is shown that whether or not such incompleteness constrains the efficiency of the contract is very sensitive to assumptions concerning the timing of the resolution of uncertainty. It is shown that when agents must respond to an unforeseen contingency before being able to renegotiate the contract, then contract complexity is a binding constraint, a case that is called *ex post hold-up*. Secondly, it is suggested that the amount of multi-tasking can provide a measure of contract complexity. When complexity is low, contingent contracting is efficient, while subjective performance evaluation is more efficient when complexity is high. In this case the optimal contract for *ex post* hold-up is based upon the ability of humans to make subjective judgements that are in some cases more informative than explicit performance measures. Moreover, the efficiency of the contract is not sensitive to human error per se, but is an increasing function of the correlation in judgements between the contracting parties.

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"The time is not here yet, but I hope it is coming when judges realize that the people who draft...contracts cannot envisage all the things that the future will bring."[1]

1 Introduction

Building upon the work of Simon (1957), Williamson (1975) observes that a fundamental reason for transaction costs is the impossibility of planning for all future contingencies in a relationship.[2] The purpose of this paper is to explore the conditions under which such complexity can constrain the set of feasible contracts, and help us better understand the contracts observed in practice. Specifically, situations where agents are asked to make decisions when unforeseen events occur, but cannot renegotiate the contract is one I call *ex post* hold-up. In these cases, complexity can have an important impact upon the form of the optimal contract. The paper begins by comparing the structure of the *ex post* hold-up problem to other contracting problems in the literature and suggests that a key ingredient in understanding the form of the optimal contract is the timing of information and actions in a relationship. Secondly, a way to measure contract complexity is suggested that has empirical implications. Finally, the optimal governance of contracts facing *ex post* hold-up when complexity is high depends upon the degree of correlation in subjective beliefs between the contracting parties.

Beginning with Simon (1951), there is a large literature that takes as given contract incompleteness due to transaction costs and then explores its implications for efficient governance. Simon argues that giving one agent authority over another economizes on transaction costs by allowing one to delay decision making until after uncertainty has been resolved. In a similar vein, the recent property rights literature, beginning with Grossman and Hart (1986), argues that problems of contract incompleteness are resolved by an appropriate reallocation of bargaining power in a relationship through ownership rights. Agency theory, beginning with Ross (1973), Mirrlees (1999) and Holmström (1979), focuses upon how asymmetric information can explain observed contracting arrangements. Holmström and Milgrom (1991) show that in a multi-tasking context when signals concerning one task are not available, then the optimal contract may ignore information regarding performance on other tasks.

While contract incompleteness and asymmetric information are central theme in this literature, the role of human cognition is not. One reason, as observed by Oliver Hart (1990), is both agency theory and the property rights literature assume that agents select their actions immediately after the contract is signed. The contract is designed to provide the appropriate incentives for performance at this stage, and hence if *ex post* unanticipated events occur these cannot affect actions that are sunk, and therefore cannot affect the structure of the optimal contract. Agents may anticipate events that cannot be described *ex ante,* but this is a different problem, and one which Maskin and Tirole (1999b) demonstrate that under the appropriate conditions does not affect the ability of individuals to optimally regulate their relationship, leading Tirole (1999) to conclude that there does not exist a satisfactory foundation for the theory of incomplete contract theory.

[1]A. Denning, The Discipline of Law 56 (1979). As quoted in Farnsworth (1990), page 543.

[2]In particular the discussion in section 2.1 of Williamson (1975).

How then do we reconcile these results in contract theory demonstrating the irrelevance of human cognition for contract formation with Williamson (1985)'s view that bounded rationality is central to the theory of transaction costs?[3] My first point is that we can usefully categorize different contracting problems as a function of *when* information is revealed. In the next section the sequence of moves for the agency model, the hold-up model and Simon's authority model are reviewed. While these are important classes of problems that correspond to many interesting contracting situations, they are not exhaustive. In many principal-agent situations the agent is called upon to respond to unexpected events in a way that is personally costly, but for which there is not sufficient time to renegotiate the outstanding contract with the principal. I call this contracting hazard *ex post* hold-up, and show in section 3 that the nature of human cognition may play an important role in the optimal regulation of the relationship.

Many employment relationships have exactly this characteristic. For example, fireman may have to respond quickly to events while a building is burning, and cannot renegotiate the contract with the city in mid-blaze. Emergency room doctors must deal with a variety of unexpected events, some of which are dangerous to the physician, especially when the patient has a communicable disease. In these situations hold-up can take one of two forms. First the agent after taking an action may not receive the compensation that he or she feels is appropriate. Secondly, the principal may worry that the agent may not have the correct incentives to take the appropriate action *ex post.*

Section 3 continues with a discussion of why contracting in these situations is difficult. If each event that an agent faces could be described beforehand, along with the appropriate response, then *ex post* hold-up would be solved with a complete state-contingent contract. However when the services to be provided entail multi-tasking with random benefits and costs, the number of contract contingencies grows exponentially with the number of tasks. This implies that even with a moderate number of tasks, complete state-contingent contracting is impossible. It is worth emphasizing that contract incompleteness in this case is *not* exclusively due to the bounded cognitive abilities of the contracting parties: when complexity grows exponentially with a variable of interest, the problem quickly becomes intractable for any finite computation device for even modest values of this variable.[4] This is an empirically useful result because it suggests that the number of tasks in a relationship is a measure of transaction costs that is independent of individual characteristics.

Anderlini and Felli (1994) take a complementary approach to contract incompleteness. They use the notion of a *computable* contract, namely any complete contract must have the property that it is possible to determine the terms and conditions using a finite number of computations. They give examples of contracts that are not computable, and hence are incomplete. Though this condition is a *necessary* condition, it is not *sufficient* to ensure the existence of a complete contract. All the state contingent contracts considered in this paper satisfy Anderlini and Felli's necessary condition, however, like many problems in computer science, being solvable in finite time does not imply practical solution since the time needed to write a complete contract is an astronomically long period.[5] This approach is extended in Anderlini and Felli

[3] See chapter 1.

[4] A point that is well appreciated in the computer science literature. See for example Garey and Johnson (1979). Williamson (1975) makes a similar point on page 23 in reference to the game of chess.

[5] For example, decoding an encrypted message is a *computable* problem that it can be achieved in finite time. However, such

(1999) where they derive the optimal incomplete contract as a function of complexity costs.

In this paper a somewhat different approach is explored. Even if contingent contracting is impossible, the contract may still provide a mechanism to determine what constitutes appropriate performance *ex post*, and ensure that the agent is rewarded for taking the appropriate action. This issue is addressed in section 4, where it is shown that the problem of performance evaluation is formally a problem in pattern recognition where the goal is to characterize event-action pairs into the sets acceptable or not acceptable. In cognitive science it is widely recognized that while humans are quite poor at thinking logically, they have very powerful pattern recognition abilities.[6] For example, the reason that humans are good at chess is not because of their ability to reason about the game, a skill for which computers are far more skilled, but rather their ability to recognize board patterns that represent strong positions.[7] This ability is so difficult to program that only recently have computers been consistently better than humans at chess, and only with programs that are highly specialized. This implies that human judgment of performance is in many situations superior to any mechanical measuring system, and hence optimal contracts should be designed to incorporate this ability.

Incentives can be provided in these cases by observing that both the principal and agent have subjective evaluations of an agent's performance. As long as these evaluations are sufficiently correlated, then it is possible to construct a mechanism that ensures efficient performance. The optimal contract in this case takes the form of a bonus payment by the principal to the agent when the principal has judged performance to be acceptable. Given that third parties, such as the courts, are at a disadvantage in determining if performance is acceptable, the optimal contract must depend upon the agent's self assessment of performance. Should the principal not reward the agent when the agent believes he or she is deserving then the optimal contract requires the principal to pay a penalty to a third party.

The difficulty with such payments is that they are subject to the hazard of renegotiation. In the event of a disagreement, the principal and agent have a strong incentive to renegotiation to avoid paying the third party. Two well known solutions to this problem are discussed in section 5: enforcement with repeated interaction combined with the threat of termination and the use of rank order tournaments. This is a useful exercise because it answers an open question in the legal theory of relational contract raised by Goetz and Scott (1981). They observe that the right to unilateral termination, while part of many bilateral relational contracts, is not a usual condition for collective agreements, and hence they question the efficacy of such termination rights. The results here show that unilateral termination clauses may be a necessary condition for efficiency when bargaining is restricted to two individuals, and can only be modified when there are three or more individuals in a relationship.

2 Contracting Scenarios

Consider the following generic exchange problem between an agent (he) who produces a good or service for a principal (her) in exchange for compensation:

messages are believed to be secure because the time required is sufficiently long as to be impracticable.

[6] See Churchland and Sejnowski (1993) for an excellent introduction to these issues.

[7] This was shown in a wonderful paper by Newell, Shaw, and Simon (1963).

1. The agent is expected to choose an action $\mathbf{y}$ from a set of possible actions $\mathbf{Y}$ (in general multi-dimensional) at a cost $C(\mathbf{y}, \beta)$, where β is a random parameter chosen by Nature.

2. The benefit to the principal from this action is $qB(\mathbf{y}, \alpha)$, where α is random parameter chosen by Nature, and q is the quantity of trade, which is normalized to represent trade (1) or no-trade (0), or the probability of trade if $q \in (0, 1)$.

3. The principal and agent write a binding contract at the beginning of the relationship conditional upon their expectations and information available. I assume that the principal has all the bargaining power at each stage.[8] The payoffs to the principal and agent are respectively given by:

$$U_P = qB(\mathbf{y}, \alpha) - W, \tag{1}$$

$$U_A = W - C(\mathbf{y}, \beta). \tag{2}$$

The principal is assumed to offer a contract that maximizes her payoff subject to the agent receiving his reserve payoff from the relationship. The term "contract" is used in the economist's sense rather than in the more restrictive legal sense. That is the contract specifies a mechanism or game between the principal and agent, including expected actions and beliefs, even when these cannot be verified in court. In contrast the legal notion of contract refers to promises enforced by the threat of court-awarded damages in the case of default. In particular for the economist these damage awards are an explicit part of the agreement between the two parties, as are actions taken after events that only the contracting parties can observe. An important element of this broader notion of contract is the potential for one party (the principal) to reallocate bargaining power to the other party (the agent). This reallocation of bargaining power is central to the property rights literature beginning with Grossman and Hart (1986). The purpose of this section is to illustrate how the form of the optimal contract and the nature of property rights are sensitive to the *timing* of information revelation. I briefly outline the three important classes of contracting problems that have been considered in the literature, agency theory and the hold-up problem of Williamson (1975) and Grossman and Hart (1986), and Simon (1951)'s authority model, and discuss the relevance of theories of bounded rationality for each of these contracting problems. I then introduce the hazard of *ex post hold-up*, that is more appropriate for addressing the role of human cognition in contract formation.

2.1 Agency Theory

Agency theory, beginning with Ross (1973) and Holmström (1979), is the starting point for the modern theory of contract. It is always possible to view the economic theory of contract as an application of agency theory: namely observed contracts are the result of negotiations between a principal and an agent, who choose optimal contracts as a function of the available information. However, in this paper I follow Hart and Holmström (1987), and adopt a narrower definition of agency theory corresponding to the class of models that focus upon how to structure contracts as a function of mutually observed (and enforceable)

[8] For simplicity, I follow the recent literature (Hart and Moore (1999) and Maskin and Tirole (1999a)) and assume that the principal has all the bargaining power in any *ex post* negotiation. This assumption can be dropped, but at the cost of unnecessarily complicating the argument.

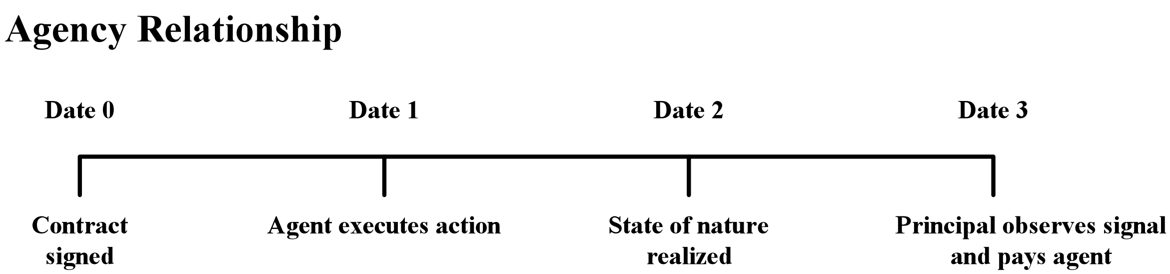

Figure 1: Time Line for Agency Relationship

signals of performance. In the context of our simple model let us fix β, and set $q = 1$. The timing of decisions are as illustrated in figure 1. At date 0 the contract is signed, then the agent chooses y, which is assumed to be a real number representing effort or some personally costly action: $\partial C/\partial y > 0$. The choice of effort affects the underlying distribution of α in such a way that more effort is beneficial to the principal: $\partial E\left(B\left(y,\alpha\right)\right)/\partial y > 0$ for all α. The principal then pays the agent a wage that is a function of the observed benefit, or $W = f\left(B\right)$.

In agency theory it is typically assumed that the agent is risk averse, and hence he would prefer a wage W that is independent of the random shock α. In this case the agent has no incentive to work and would select y to minimize the personal cost of effort. A major insight of this literature, as discussed in Hart and Holmström (1987), is in order to avoid this moral hazard problem the optimal contract should be a function of *any* signal that adds information regarding worker effort.

There is a great deal of evidence to suggest that the basic hypothesis of agency theory is correct, namely individuals do respond to incentives. Hence, if workers are paid a wage that is independent of income one expects to observe some shirking. Despite this fact, explicit pay-for-performance systems, while common, are far from being ubiquitous, leading many experts such as Gibbons (1997) and Prendergast (1999), to conclude that agency theory alone cannot explain all the variation observed in the data.

One solution, provided by Holmström and Milgrom (1991), begins with the observation that while effort is often multi-dimensional, performance measures may not be sufficiently rich to capture this variation. For example suppose that a home-owner is contracting for the services of a contractor who must allocate effort between speedy completion of the project and quality, whose actions are represented by vector $y = \{y_s, y_q\}$, where y_s represents speed and y_q represents quality. In the absence of explicit contract terms, the cost-minimizing effort is strictly positive: $\left\{y_s^o, y_q^o\right\} = \arg\min_{y_s,y_q\geq 0} C\left(y_s, y_q\right) > 0$. It is also reasonable to suppose that quality and speed are substitutes, and hence $C_{sq} > 0$.

In this simple example the benefit to the home-owner is assumed to have no uncertainty and is given by $B\left(y\right)$. Given that the payoff represents the subjective preferences of the home-owner, then one cannot write a contract conditional upon an explicit measure of B or for that matter quality y_q, also a subjective variable. Rather, the only contractible variable is y_s, speed. In this case, assuming that the problem is convex, it follows that under the optimal contract $\left\{y_s^*, y_q^*\right\}$ solves:

$$C_{y_q}\left(y_s^*, y_q^*\right) = 0, \tag{3}$$

$$B_{y_s}\left(y_s^*, y_q^*\right) = C_{y_s} + B_{y_q}\left(y_s^*, y_q^*\right)\left(\frac{C_{y_s y_q}}{C_{y_q y_q}}\right) \tag{4}$$

The first term is the consequence of the contractor minimizing costs in the quality dimension, while the second term is the first order condition for speed. Since speed and quality are substitutes ($C_{sq} > 0$) then it follows that y_s^* is less than the first best.[9] Under Holmström and Milgrom (1991)'s assumption, if the substitution effect is sufficiently strong, or C_{qq} sufficiently small, then $y_s^* < y_s^o$. In other words the optimal contract may entail providing either no incentive or negative incentive for speed.

Hence incomplete contracts in agency theory arise from a paucity of information regarding performance. Notice that the hypothesis of rational expectations is central to the theory. The principal structures the incentive contract as a function of her expectations regarding future performance by the agent. The introduction of bounded rationality regarding the formation of expectations would imply that we may sometimes observe incentive contracts with unintended consequences (a possibility that is often observed in practice, as the examples in Kerr (1975)'s seminal article demonstrate). However, aside from the potential for error, agency theory provides little guidance regarding the implications of bounded rationality for observed contract form.

Also, Holmström and Milgrom (1991)'s explanation for the lack of high power incentives for quality performance ignores the potential for incentives based upon non-contractible signals. In the case of the contractor, their model suggests that in a one period relationship the contractor would simply choose his most preferred quality. Yet, disputes over quality are quite common during construction. In many cases contracts are structured so that in areas that the quality is lacking, the builder may ask the contractor to take corrective actions, even though some aspects of quality were not explicitly contracted upon *ex ante*. This type of *ex post* renegotiation over non-contractibles is central to the hold-up model considered next.

2.2 Hold-up

Suppose now that the contractor is building a custom-designed house. Given that time of completion is contractible, we focus only upon the provision of non-contractible quality. The main difference with respect to the agency model is the existence of a physical asset whose ownership rights can be transferred. Uncertainty plays a role in that *ex post*, it may be more efficient to allocate the good to another buyer in the market. Suppose that the value of the house to the principal and the market are respectively given by $B\left(y_q, \omega\right)$ and $B^o\left(y_q, \omega\right)$, where it is assumed that $B\left(y_q, \omega\right) - B^o\left(y_q, \omega\right) = k\left(\omega\right)$, and $k\left(\omega\right)$ is an uncertain amount of relationship-specific rent that depends upon the state of nature ω. When this is negative, it is efficient to breach the contract, while performance is efficient when $k\left(\omega\right) > 0$. Let the expected value of the relationship given that there is efficient breach, be positive and given by $\bar{k} = E\left(\max\left\{0, k\left(\omega\right)\right\}\right) > 0$. The time line for the contract is illustrated in figure 2.

[9] A similar equation is derived by Baker (1992) who works out the optimal contract when the contractible variable is not perfectly aligned with benefits.

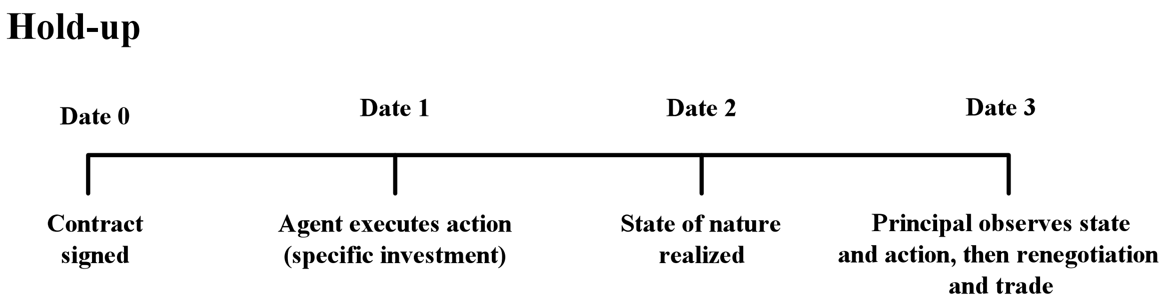

Figure 2: Time Line for Hold-Up Problem

The insight of the property rights literature, beginning with Williamson (1975) and Grossman and Hart (1986), is that the *ex post* distribution of bargaining power is an important determinant of the efficiency of the relationship, and that this bargaining power can be reallocated via ownership rights. Consider first the case in which the principal owns the house. Given that the principal has all the *ex post* bargaining power we obtain exactly the same solution as in the agency model above: the contractor selects his preferred quality, y_q^o, and agrees to a fixed price contract $p = C\left(y_q^o\right)$. In this case if *ex post* efficiency requires that the building be owned by another person, then the principal would simply sell the building to that person. Though this contract ensures *ex post* allocative efficiency, the lack of performance incentives implies that the contractor does not supply an efficient level of quality.
An alternative contract is for the principal to sell her right to the project to the contractor at price $p = \max_{y_q} E\left\{B^o\left(y_q,\omega\right)\right\} - C\left(y_q\right)$, with the provision that she must be given the chance to match any offer that the contractor might receive from the market. This is a contract that provides the principal with the *right of first refusal,* a contract that was common in Hollywood for some actors and producers.[10] Under this contract whenever $B^o\left(y_q,\omega\right) > B\left(y_q,\omega\right)$ the principal is unwilling to match the market price and the contractor receives $B^o\left(y_q,\omega\right)$. Whenever $B\left(y_q,\omega\right) > B^o\left(y_q,\omega\right)$, the principal simply matches the market offer, and again the contractor obtains $B^o\left(y_q,\omega\right)$. It is assumed that the marginal return from quality is the same in the market and for the principal, and hence this contract provides first best incentives for quality, while ensuring efficient matching. More formally the payoff of the contractor is:

$$\begin{aligned} U_A\left(y_q\right) &= E\left\{B^o\left(y_q,\omega\right)\right\} - C\left(y_q\right) - p, \\ &= E\left\{\max\left\{B^o\left(y_q,\omega\right), B\left(y_q,\omega\right)\right\}\right\} - \bar{k} - C\left(y_q\right) - p. \end{aligned} \tag{5}$$

This case is an example of general investment combined with turnover costs that are independent of investment. As in MacLeod and Malcomson (1993), it is also possible to obtain the first best in this case with appropriately chosen liquidation damages (proposition 5).

[10] In personal correspondence relating his discussions with Ben Klein and Earl Thompson, Alchian (1998) observes that many Hollywood contracts for shows were exactly of this form. An actor or producer on a long term contract could entertain outside offers. However, if the studio matched the offer, the individual had an obligation to stay with his or her studio. Alchian argues informally that the right of first refusal serves the purpose of providing incentives for efficient specific investment.

There is a literature that explores how the complexity of the *ex post* environment makes it impossible to write an efficient contract (Segal (1999) and Hart and Moore (1999)). In these papers it is assumed that *ex post* there are a large number of potential goods that may be traded, but it is optimal to trade only one of these. When the nature of this good cannot be specified *ex ante*, as the number of possible goods approaches infinity the optimal contract is a fixed price contract, which in turn implies that the level of investment in the relationship is inefficient. This result demonstrates how environmental complexity can cause individuals to optimally choose an incomplete contract, though this result is not an implication of bounded rationality and cognition *per se.* Both papers assume that contracting parties anticipate correctly the consequences of any mechanism they choose, hence do not explore the implications of unforeseen contingencies, and are rather concerned with "indescribable contingencies" (see Maskin and Tirole (1999b) for a further discussion of these points).

Hart (1990) further argues that hold-up models provide an inadequate foundation for the study of the implications of human cognition for organization and contract design. For example, suppose there is an unforeseen event ω' for which it is efficient that the asset be sold to the market. *Ex post* renegotiation ensures that this indeed will be the outcome. However, given that specific investments have been sunk at the time individuals learn about ω', the occurrence of this event plays no role in setting *ex ante* incentives. Structuring relationships to efficiently deal with unforeseen contingencies is one of the motivations for Simon (1951)'s original model of the employment relationship.

2.3 Authority

Simon (1951)'s model of employment is concerned with the role played by authority. His idea is that in a complex world, rather than planning for all future events, one might gain by delaying decision making until after an event occurs. The formal timing for his model is illustrated in figure 3. After the contract is signed the principal is able to observe the state of nature, denoted by $\omega = \{\alpha, \beta\} \in \Omega$, where Ω is the set of possible states, and can direct the agent to perform a task y as a function of this information (without loss of generality we set $q = 1$). In Simon's model giving the principal authority imposes costs on the agent *ex post* since he may be asked to carry out tasks with large private costs, $C(y, \beta)$. Simon supposes that the authority relationship is characterized by a wage, W, and a set of tasks $\mathbf{Y}^o \subset \mathbf{Y}$ from which the principal may choose. Giving the principal more authority corresponds to choosing a larger set of tasks, $\mathbf{Y}^o$, that the employee may be asked to carry out in exchange for a higher wage. Notice that since control is specified in terms of $\mathbf{Y}^o$, and not states, then the model incorporates a well defined protocol to be followed when an unforeseen event occurs.

If this set is a single action, i.e. $\mathbf{Y}^o = \{y\}$, then Simon calls this a sales contract and the concept of authority has no relevance. Simon shows that the optimal contract gives the principal some authority over the agent when the benefits of flexibility outweigh the costs. Notice that the potential for renegotiation changes this result. Suppose that the agent accepts any sales contract $\{W^*, y^*\}$ satisfying $W^* - E\{C(y^*,\beta)\} \geq 0$, then it will follow that the expected utility of the agent is at least zero. After the event $\omega = \{\alpha, \beta\}$ occurs, under the sales contract the agent receives $U_A^*(\beta) = W^* - C(y^*, \beta)$ *ex post.* Suppose that the principal has all the bargaining power. In this case, she can offer a new *efficient* contract

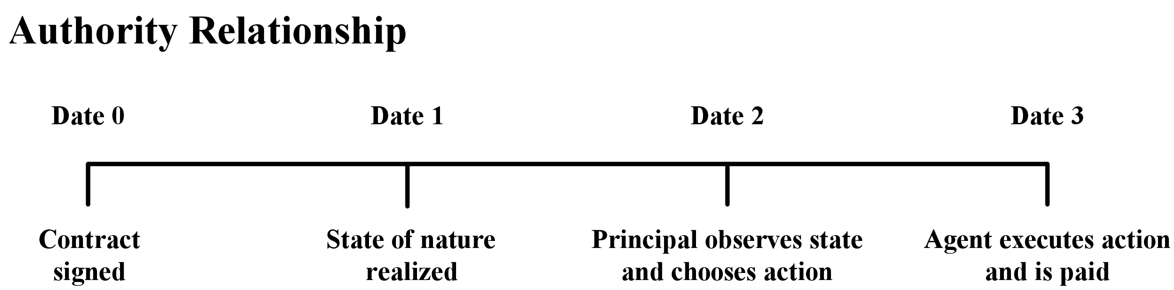

Figure 3: Time Line for the Authority Relationship

that would be accepted by the employee as long as the utility is at least $U_A^*(\beta)$. Hence we have the following result:

Proposition 1 *If renegotiation before the agent chooses his action is possible, then the sales contract results in the first best.*

For this contracting problem the allocation of bargaining power is not important, rather the key ingredient is the hypothesis that renegotiation can occur between the time the state is observed and the agent selects her action. In contrast to the hold-up problem, the addition of renegotiation in this case increases, rather than decreases, efficiency. However, there are a number of situations for which the hypothesis of renegotiation is not reasonable. For example firemen must make second by second decisions on how to respond to a burning building, teachers need to be able to deal with new and unexpected questions and events in the class, surgeons must be able to deal with unexpected events during an operation. While not stated explicitly, it is likely that Simon had in mind situations such as these for which renegotiation to an efficient action in real time is not possible. Certainly, this is a case that is clearly not considered to be part of the standard hold-up model where renegotiation is assumed to be possible.

However, when renegotiation is not possible, the exercise of authority may also be imperfect. Alchian and Demsetz (1972) make this point when they argue that in employment relationships there is typically no real authority. The agent follows the principal's directives because he believes that he will be rewarded in the future. If the agent is dissatisfied then he is free to leave for another employment relationship. Alchian and Demsetz argue that the key point is the ability to *monitor* the agent's actions in order to be able to choose the appropriate level of compensation. Yet when performance is non-contractible, and the agent is unable to renegotiate her contract, she faces the prospect of taking a personally costly action, without any assurance that she will be rewarded because the principal can always claim that existing compensation is sufficient. This leads to a contracting hazard that I call *ex post hold-up.*

2.4 Ex Post Hold-up

In the contracting problems we have considered thus far, either the principal can observe the state of nature before the agent takes an action (authority) or the state of nature is revealed after the agent selects

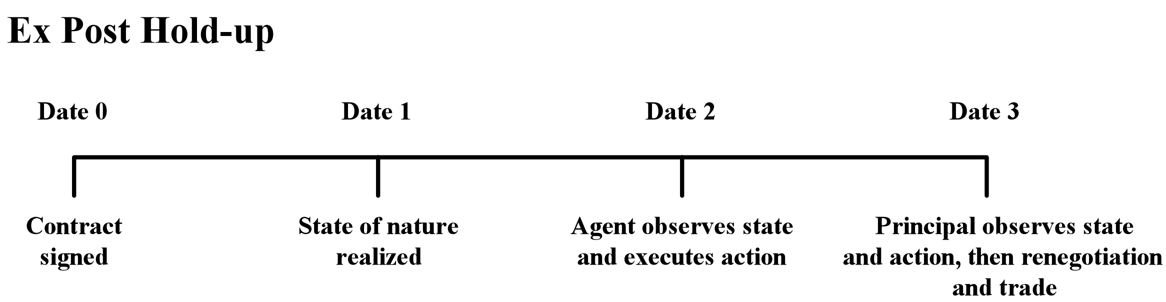

Figure 4: Time Line for Ex Post Hold-up

her action (agency and hold-up). A case that has not been considered, but is ubiquitous in many employment relationships, is one where the agent is expected to respond to uncertainty before the principal has knowledge of the event or can guide the agent in selecting the appropriate action. I have already mentioned the case of fire fighters and surgeons, but this case also includes many employment situations where the employee is expected to internalize the objectives of the principal, and make decisions on the principal's behalf.

The hazard of *ex post holdup* arises from the need to have an agent respond appropriately to events as they occur in the absence of an explicit and enforceable contract. The time line for this contracting problem is illustrated in figure 4. A defining feature of employment relations facing *ex post* hold-up is the need for the agent to allocate activity among a number of different tasks in response to the costs and benefits of the different tasks. More formally, suppose that the agent is facing a multi-tasking problem parameterized as follows:

1. There are k tasks: $\mathbf{y} \in \mathbf{Y} = \left\{ \left\{ y^1, y^2, ..., y^k \right\} | y^1 + y^2 + ... + y^k \leq T \right\}$, where T is the agent's total time available to allocate between tasks.

2. The cost function takes the form: $C(\mathbf{y}, \boldsymbol{\beta}) = \sum_{i=1}^{k} c\left(y^i, \beta^i\right)$, where $c\left(y^i, \beta^i\right)$ is the cost of allocating effort to task i. If y^i is zero, then this cost is zero, otherwise it is $\beta^i y_i^2 + f$. The cost parameter β^i is a random variable that can take on one of m discrete values $\{d_1, ..., d_m\}$.

3. The benefit function is assumed to take the form: $B(\mathbf{y}, \boldsymbol{\alpha}) = \boldsymbol{\alpha}^T \mathbf{y}$, where $\boldsymbol{\alpha}^T \mathbf{y} = \alpha^1 y^1 + \alpha^2 y^2 + ... + \alpha^k y^k$ is the benefit to the firm from the agent's effort. The marginal benefit of task y^i is α^i, a random variable that can take at most n values: $\{a_1, ..., a_n\}$.

In this parameterization, the state space is given by the possible benefit and cost parameters: $\Omega = \{\{a_1, ..., a_n\} \times \{d_1, ..., d_m\}\}^k$. For each $\omega \in \Omega$, the optimal response is defined by:

$$\mathbf{y}^*(\omega) = \arg\max_{\mathbf{y} \in \mathbf{Y}} B(\mathbf{y}, \boldsymbol{\alpha}) - C(\mathbf{y}, \boldsymbol{\beta}). \tag{6}$$

An important assumption I make for the rest of the paper is that both the benefit and cost measures are themselves non-contractible. In the case of the benefits, consider for example a secretary in a large firm.

His or her typing output is important to the firm, but there is no way to attach relative values to say typing versus filing. Similarly, the dollar value of a research paper written by a professor, or an hour devoted to seeing students is not known in practice. If the benefits were contractible, then the provision of an incentive contract would be straightforward. Similarly costs represent dis-utility to the agent, and hence are also difficult/impossible to verify accurately in practice.

2.5 State Contingent Contracts and Complexity

Though a single measure of performance may not be available, it may be reasonable to suppose that the principal can observe, or put into place a system that evaluates an agent's response to a specified state in a verifiable way. One way to avoid the potential for opportunistic behavior when an agent is simply told vaguely to do a good job is to outline explicitly what is expected for certain contingencies. For example, one may require a secretary to explicitly stop what he or she is doing if a client comes in and needs attention. Such an explicit condition may be necessary when an employee faces conflicting goals, for example if the secretary must decide between completing a typing task immediately or addressing the needs of a client. For each possible state ω suppose there is an appropriate response, denoted $\mathbf{y}^* (\omega)$. Given that the agent is risk neutral, one may use a forcing contract that rewards the agent if and only if he achieves a satisfactory performance. This can be formally represented by the *judgement* function:

$$J : \Omega \times \mathbf{Y} \rightarrow \{0, 1\}, \tag{7}$$

where $J(\omega, \mathbf{y})$ is 1 if the choice of $\mathbf{y}$ given ω is satisfactory, otherwise it is zero. In the case of an optimal complete contract, the principal defines the judgement function by $J^* (\omega, \mathbf{y}) = 1$ if $\mathbf{y} \geq \mathbf{y}^* (\omega)$, and zero otherwise, offers a contract $\{w, bJ^* (\omega, \mathbf{y}^* (\omega))\}$, where w is a fixed payment and $bJ(\omega, \mathbf{y}^* (\omega))$ is the bonus payment. This forms an optimal contract if it satisfies the individual rationality constraints and the incentive compatibility constraints:

$$w + b - E\{C(\mathbf{y}^* (\omega), \beta)\} = 0, \tag{8}$$

$$w + b - C(\mathbf{y}^* (\omega), \beta) \geq w - \min_{\mathbf{y} \in \mathbf{Y}} C(\mathbf{y}, \boldsymbol{\beta}). \tag{9}$$

With no restrictions on the sign of w, as long as costs are bounded then there always exists a contract satisfying these conditions.

Notice that in order to implement this contract one is required for every event ω to specify *ex ante* the expectations for the agent, and to reward the agent if these expectations are met. However, when the number of tasks is moderately large this is simply impossible. In this model the number of tasks is k, and the number of productivity and cost levels are respectively m and n. The *complexity* of the contract is a measure of the costs of designing, writing and implementing the contract as a function of the data describing the relationship. Suppose that the cost of agreeing upon a contingency ω is γ, then since the number of possible events is $n^k m^k$, the cost of a complete contingent contract is $n^k m^k \gamma$. Since these costs are exponential in the number of tasks, they quickly rise to an astronomical level. For example, suppose that $\gamma = 1$ cent, and that the number of cost and performance levels is the same ($n = m$), then the following table presents the cost of a complete contract as a function of the number of tasks and effort levels.

Number of Cost and Performance Levels	Number of Tasks 2	5	10	15
2	$0.16	$10	$10,000	$10 million
3	$0.81	$600	$35 million	$2 trillion
4	$2.56	$10,000	$11 billion	$11,000 trillion
5	$6.25	$100,000	$1000 billion	$10 million trillion
Cost of considering a contingency:	1 cent			

Table 1: Cost of a Complete State Contingent Contract

As one can see from table 1, when there are several tasks, even with just two performance levels, the cost of even thinking about a complete state-contingent contract would be astronomical. Observe that it is the multi-tasking that increases the complexity costs, and not the number of cost and performance levels (the discreteness of the state space). In other words if the benefits and costs vary in a number of dimensions, then it is simply impossible to create a contingency plan for every possibility. This example illustrates the point made by Williamson (1975), and earlier still by Savage (1972), that in any realistic environment the number of possible contingencies is so large that complete state-contingent planning is impossible.[11] In particular, it is worth emphasizing that thinking in terms of human bounds on rationality is not helpful in these cases, rather one faces fundamental limits that make it impossible to construct complete contingent plans and contracts. To deal with this complexity, humans have developed algorithms and techniques for decision making in complex environments that can be used for the design of more efficient contracts.[12]

3 The Sales Contract Revisited: Ex ante governance

Even though the contracting parties cannot consider every possibility, they can still write a complete contingent contract, of which Simon (1951)'s sales contract is an extreme case. The sales contract is a form of *ex ante governance* requiring the agent to perform $\bar{\mathbf{y}}$, *regardless* of the state of nature, and represents the polar opposite contract, in terms of complexity, to a complete state-contingent contract. Dye (1985) proposes that one endogenizes the complexity of the contract by specifying actions for a limited set of events. For example the event might be that there is a need to have a paper typed, which is then associated with the action 'type the paper today'. This event and response may not be efficient because demanding the paper be typed immediately may lead to mistakes, or there may be more pressing tasks. The optimal contract trades off the quality of the contract against the cost of increased complexity.
More formally, let $\Pi_N = \{E_1, E_2, ..., E_N\}$ be a partition of the state space Ω, and let $\mathbf{Y}_N = \{\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_N\}$ be the associated actions. This defines a contract of complexity N, under which the agent in exchange for a

[11] See Dekel, Lipman, and Rustichini (2001) for an interesting axiomatic approach to modelling decision making in complex environments.

[12] See Churchland and Sejnowski (1993) for a good review of computational neuroscience exploring the algorithmic foundations of human decision making.

wage W agrees to carry out the following actions:

$$c(\omega|\Pi_N, \mathbf{Y}_N) = \mathbf{y}_i, \text{ if and only if } \omega \in E_i, \tag{10}$$

Though this contract is complete in the sense that it defines an action for every state, it is not efficient. This is because all states in a single event E_i are associated with the same action, which many not necessarily be efficient.

For purposes of this example suppose that for each N the principal and agent agree upon a particular partition Π_N. Further suppose that if $N' > N$, then for every $E' \in \Pi_{N'}$, there is an $E \in \Pi_N$ such that $E' \subset E$. That is if we agree upon a more complex contract, it refines the events of less complex contracts. Let $c_N^*(\omega)$ denote the optimal contract relative to Π_N defined by:

$$c_N^*(\omega) = \arg\max_{\mathbf{y}\in\mathbf{Y}} E\{B(\mathbf{y}, \alpha) - C(\mathbf{y},\beta)|E_\omega\}, \tag{11}$$

where $E_\omega \in \Pi_N$ is the unique event such that $\omega \in E_\omega$. Under these assumptions we have the following proposition, whose proof is straightforward.

Proposition 2 *The* ex ante *surplus generated by* $c_N^*(\omega)$,

$$S_N^* = E\{B(c_N^*(\omega), \alpha) - C(c_N^*(\omega), \beta)\}, \tag{12}$$

is an increasing function of N.

Notice that this expression is strictly increasing when going from N to $N+1$ if and only if the additional partition causes the optimal action to change for some events. This reflects that well known fact that information is valuable only when it causes a change in one's decision. For the multi-tasking problem of the previous section this is true for a generic choice of parameters α and β. The surplus *net* of transaction costs from the optimal contract of complexity N is $S_N^* - \gamma N$, where γ is the cost of adding a contract contingency. As illustrated in table 1, even if γ is very small, transaction costs for a complete state-contingent contract may be very large, and hence we are unlikely to observe such a contract. Suppose that the agents choose the complexity of the contract to solve

$$\max_{N\geq 1} S_N^* - \gamma N, \tag{13}$$

then we have the following result:

Proposition 3 *Suppose that* $\gamma \times \#\Omega > S^*$ *where* $\#\Omega$ *is the number of states and* S^* *is the maximum surplus under a complete contingent contract then:*

1. *The optimal contract complexity is decreasing with contracting costs* γ.

2. *Keeping the transaction cost* γ *fixed, then a proportional increase in the value of trade:* ζS_N, $\zeta > 1$, *increases the optimal complexity of the contract.*

This result highlights the fact that increasing transaction costs lowers the complexity of a state-contingent contract. Secondly, as the value of trade rises, then so does the complexity of the contract, a result that is consistent with Macauley (1963)'s observations regarding the commercial contracts. The benefit of *ex ante* governance is that the agent knows and understands exactly what is expected for every event E_i. However, it is precisely the fact that the contract is well defined and binding that the principal faces the hazard of opportunism. Consider the following example from the Lincoln Electric Case in which the firm attempted to expand its system of piece rates to secretarial staff. Let ω denote the correspondence to be typed in a particular day, and suppose that task i is the number of times that one strikes a particular letter. To improve productivity the company decided to reward individuals as a function of the number of keystrokes hit or $\sum y^i$. Clearly the intent is that the secretary type a particular text at a higher speed, but what occurred is in one case a secretary repeatedly hit the same key during her lunch break to improve her earnings![13]

This is a rather stark example of Williamson (1975)'s concept of opportunism. If the terms of employment simply specify the payment as a function of the number of keystrokes without mention of the quality of output, then even if the output is useless, the explicit terms call for payment to the secretary. The firm would argue (probably successfully) that the intent in this case is that the secretary produce useful documents, however the secretary could argue that this sophisticated firm had written an explicit contract and should be held responsible for its decisions. Unfortunately, organizations often make this kind of mistake, as highlighted in the famous article by Kerr (1975) who outlines several examples of workers responding to incentives in undesirable ways. As Kerr points out, many organizations are "rewarding A while hoping for B".

Yet, propositions 2 and 3 suggest that in principle a sufficiently contingent contract would be close to the first best, a view point that has led many economists to promote the increased use of pay for performance contracts (see for example Milkovich and Wigdor (1991)). Moreover, as table 1 illustrates, the complexity of jobs involving multi-tasking is such that even very sophisticated firms may not be able to anticipate all the consequences of a contract. As Kerr observes, an explicit contract creates an incentive for the agent to discover ways to improve measured performance rather than a firm's performance, a behavior that is reinforced by the legal presumption that explicit contracts are legally binding. This point is illustrated in the case of **?**). In this case a salesperson, Wakefield, was employed on an explicitly at will basis, but was also paid commissions for sales in his office. After several years of employment, he was dismissed just before he was to receive a commission payment from a significant sale. Northern Telecom did not pay this commission, arguing that the at will nature of employment relieved it of this obligation. However, the court ruled that employment at will did not absolve the firm from its explicit obligation to pay a commission, and established the protection of explicit performance pay, highlighting the risk that a firm faces when using a poorly constructed contract.

In principle increasing the complexity of a contingent contract should enhance performance. However, not only does the complexity of the environment imply that a complete contract is impossible, it may also be the case that the contract provides incentive for an individual to discover *unanticipated* actions that are

[13] See Irrgang (1972), page 13.

Pareto inefficient but, under the terms of the explicit contract, are in the interests of the employee to implement. The next section discusses how subjective evaluations may be used to address this issue.

4 Judgement and Subjective Performance Evaluation

An important insight of Simon (1951)'s model is the idea that actions should be decided upon *after* the state of nature is revealed. Even when the determination of the appropriate action, given ω, is of low cost, the large number of potential states make such contingent planning impossible, a complexity that is dramatically reduced by delaying decision making until after the state is revealed. The difficulty is that now we face the problem of the agent being held up. If he takes an appropriate, but costly, action how can he be sure that the principal will reward him appropriately?

Secondly, given that our maintained hypothesis is that there is no univariate measure of performance, in the absence of an *ex ante* agreement, how is the agent going to know what is appropriate performance, and how is the principal going to judge such performance? As Prendergast (1999) observes, in many cases both the principal and agent engage in subjective evaluations based upon *human* capabilities that cannot be replicated by any mechanical system. For example, the owner of a restaurant judges the performance of a chef by tasting the food. At the moment there is no known device that can automate such a process. When deciding upon whether to accept a paper for publication in a journal, once the referee has decided that the results are correct, the final decision turn upon the notoriously vague criteria of "importance" or "contribution to the literature".

In these examples, evaluation depends upon the *superior* performance of human versus mechanical evaluations of performance. From the cognitive science literature we know that humans have remarkable pattern recognition abilities that we are only just beginning to understand and model. The formal link of incentives to pattern recognition can be modeled with the introduction of a judgement function $J(\omega, \mathbf{y})$. Formally this function is a *classifier* that divides the set $\Omega \times \mathbf{Y}$ into two subsets:

$$A = \{(\omega, \mathbf{y}) \in \Omega \times \mathbf{Y} | J(\omega, \mathbf{y}) = 1\}, \text{ and} \tag{14}$$

$$U = \{(\omega, \mathbf{y}) \in \Omega \times \mathbf{Y} | J(\omega, \mathbf{y}) = 0\}, \tag{15}$$

where A denotes 'acceptable performance' and U denotes 'unacceptable performance'. When there is multi-tasking, then the state space Ω is very large making a complete state contingent contract impossible. Given that the classification problem simply involves dividing a space into two sets, then this is an easier problem than writing a state-contingent contract. This is in fact not the case. Notice that any contract can be written as specifying whether or not performance has occurred in a state, and hence the complexity of a classifier is the same as the original contracting problem. Moreover, the seminal work of Minsky and Papert (1988) has proven that the identification of a classifier is a "hard" problem, a point that Anderlini and Felli (1994) have made explicitly in the context of contract formation.

While classification is a hard problem that challenges even the most sophisticated computing machines, research in cognitive science has found that the brain is specifically designed to be very good as pattern recognition (see for example Churchland and Sejnowski (1993)). Though human classification is not perfect, it is the case that individuals can learn to be good at categorizing inputs. For the purposes of this paper, the aspect of categorization I wish to emphasize is the ability to judge whether performance is acceptable or not (as opposed to providing a numerical measure of its quality). In the next section it is shown that as long as the employer and employee have judgements that are correlated, then it is possible to construct contracts that are *not* explicitly state-contingent, yet nevertheless result in high performance.

4.1 Subjective Contracting

Consider a situation for which a principal and an agent agree to a contract that requires the agent to formulate a response to a large number of events. When an event occurs, the agent is assumed to choose effort λ that determines the probability of good performance for that event. We do not explicitly model either the underlying state space, nor the set of possible actions. Rather, motivated by the previous discussion, it is assumed that both the principal and agent evaluate the response to the event, and decide whether or not performance is acceptable. Given that these evaluations are both non-contractible and that *ex post,* it is not possible to write a screening contract, this greatly constrains the set of possible performance contracts. In particular, it is shown that if judgement is not perfect, then the optimal contract necessarily entails the potential for conflict between the principal and agent.

More formally, suppose that the cost of effort $\lambda \in [0,1]$ to the agent is $c(\lambda)$, where $c(0) = 0$ (cost of no effort is zero), $c', c'' > 0$ (more effort costs increase at an increasing rate) and $c'(\lambda) \to \infty$ as $\lambda \to 1$ (perfection is impossible). When success occurs, then a reward B^* is produced, otherwise there is no return. Hence the expected net surplus of the relationship for this reduced-form model is given by:

$$S(\lambda) = \lambda B^* - c(\lambda), \tag{16}$$

with the first best level of effort, λ^{fb}, satisfying $B^* = c'\left(\lambda^{fb}\right)$

Let us assume that these parameters are commonly known, and that if success does not occur, then this is commonly known by both parties (this assumption can be relaxed at the cost of greatly complicating the analysis). Subjective evaluation is modelled by supposing that when success does occur, then the principal and agent may or may not agree upon this. In the event of objective success, let β_{ij}, $i, j \in \{A, U\}$, be the probability that the principal believes quality is i and the agent believes quality is j, where A and U denote "acceptable" and "unacceptable" respectively. Thus if the good outcome occurs, then β_{AA} is the probability that both principal and agent agree on this. It is assumed that the signals are positively correlated, that is $\beta_{AA}\beta_{UU} - \beta_{UA}\beta_{AU} > 0$. If the beliefs of the principal and the agent are perfectly correlated then $\beta_{AU} = \beta_{UA} = 0$.

Due to the complexity of the relationship it is not possible to write a contract conditional upon the objective characteristics of output, nor can it be made binding upon the beliefs of the individuals. However the agents can agree to a contract that makes payments conditional upon messages sent by the principal and agent. Formally the contract between the principal and agent is given by:

$$c_{ij} = \{\pi_{ij}, w_{ij}\}, \tag{17}$$

where π_{ij}, w_{ij} are the payments to the principal and agent under the contract as a function of the message

$i, j \in \{A, U\}$, satisfying the constraint $\pi_{ij} + w_{ij} \leq 0$.[14] This constraint allows the total payments to be less than zero, a possibility that will prove to be crucial. The *ex post* hold-up problem has the following sequence of moves:

1. The principal makes a take-it-or-leave-it contract offer to the agent, who accepts or rejects.

2. An event $\omega \in \Omega$ occurs.

3. The agent selects $\lambda \in [0, 1]$, which is his level of effort, in response to this event, to produce an observed response $\mathbf{y}$.

4. The principal and agent observe $\{\omega, \mathbf{y}\}$ and form subjective judgements regarding the success of the agent's action and simultaneously send messages from the set $\{A, U\}$ to the third party enforcing the contract.

5. The payoffs are determined.

I assume that the principal is able to select the most efficient incentive-compatible contract. The payments under the contract to the principal and agent when they report k, but their true state is l are respectively:

$$\pi(k|l) = \pi_{kA}\beta_{lA} + \pi_{kU}\beta_{lU} / (\beta_{lA} + \beta_{lU}), \tag{18}$$

$$w(k|l) = w_{Ak}\beta_{Al} + w_{Uk}\beta_{Ul} / (\beta_{Al} + \beta_{Ul}). \tag{19}$$

The principal's problem is to maximize expected payoff subject to the agent's individual rationality and incentive compatibility constraints:

$$\max_{\lambda, c} \lambda B^* + \lambda\pi(c) + (1 - \lambda)\pi_{UU} \tag{20}$$

subject to

$$\lambda w(c) + (1 - \lambda) w_{UU} - c(\lambda) \geq U^o \tag{21}$$

$$w(c) - w_{UU} = c'(\lambda) \tag{22}$$

$$\pi(l|l) \geq \pi(k|l), k, l \in \{A, U\} \tag{23}$$

$$w(l|l) \geq w(k|l), k, l \in \{A, U\} \tag{24}$$

$$\pi_{ij} + w_{ij} \leq 0, \; i, j \in \{A, U\} \tag{25}$$

where $\pi(c) = \sum_{i,j\in\{A,U\}} \pi_{ij}\beta_{ij}$ and $w(c) = \sum_{i,j\in\{A,U\}} w_{ij}\beta_{ij}$ are the expected transfers to the principal and agent respectively when the good outcome occurs. Constraint 21 requires the agent to earn at least his outside payoff, constraint 22 is the requirement that the agent select effort to maximize his payoff at stage 2. Constraints 23 and 24 are the stage 3 incentive compatibility constraints ensuring that the principal and agent truthfully report their subjective judgements to the third party enforcing the contract. The final constraint is the budget balancing constraint for the contract.

[14] From the revelation principal (e.g. Myerson (1979)) we know that without loss of generality we can identify the message space with the information that is private to each individual.

Notice that if the contract is budget balancing, $\pi_{ij} + w_{ij} = 0$ for all $i, j \in \{A, U\}$, then the contract defines a constant sum game at the message stage between the principal and agent. Such games have a unique value, and hence the payoff cannot depend upon subjective information. Thus in order that a subjective evaluation system induce positive effort on the part of the agent it is necessary that in some states there be a net loss to the relationship.[15] The next result provides a complete characterization of the optimal contract when we relax the budget breaking requirement.

Proposition 4 *Suppose that $\beta_{AA}\beta_{UU} - \beta_{AU}\beta_{UA} > 0$ then optimal contract with subjective performance evaluation has the form:*

		Agent's Report A	Agent's Report U
Principal's Report	A	$(-b-w, b+w)$	$(-b-w, b+w)$
	U	$(-P-w, w)$	$(-w, w)$

Table 2: Contract Payoffs

where

- *The optimal effort λ^* solves $c'(\lambda^*) = B^* - \frac{\beta_{UA}}{\beta_{AA}}(\lambda^* c''(\lambda^*) + c'(\lambda^*))$, where $\beta_{A*} = \beta_{AA} + \beta_{AU}$ is the probability that the principal believes performance is acceptable.*
- *The bonus satisfies: $b^* = c'(\lambda^*)/\beta_{A*}$.*
- *The fixed wage satisfies: $w = U^o + c(\lambda^*) - \lambda^* c'(\lambda^*)$.*
- *The penalty satisfies $P = c'(\lambda^*)/\beta_{AA}\beta_{A*}$.*

The proof of this proposition is in the appendix. The optimal contract has the property that the agent's payment is independent of his report, and hence he has no incentive to misrepresent his self-evaluation. The principal provides the agent with effort incentives by paying him a bonus whenever she believes that he has provided acceptable performance. Given that we expect subjective evaluation to be used when explicit contracts are more expensive, then this implies that the incidence of bonus pay should be greater in jobs of greater complexity, an implication that has some empirical support, as shown by Brown (1990) and MacLeod and Parent (1999).

If the agent reports unacceptable performance when the agent reports acceptable, then she must pay a penalty P. It is the prospect of paying a penalty when the reports from the agent and principal differ that provides the appropriate incentives for truthful revelation by the principal. When correlation is imperfect and $\beta_{UA} > 0$, there is a positive probability that the principal will pay the penalty. Given that the size of the penalty depends upon the size of the bonus promised, the lack of correlation increases the marginal cost of providing incentives. This is reflected in the term $\frac{\beta_{UA}}{\beta_{AA}}(\lambda^* c''(\lambda^*) + c'(\lambda^*))$, the amount by which

[15] This is a recurrent theme in the theory of incentives. The early work of Groves and Ledyard (1977) and Holmström (1978) explicitly recognized the need for some budget breaking mechanism to ensure the truthful revelation of preferences. See Moore (1992) for a review of this literature, and the implications for contract theory.

the marginal benefit from effort is reduced in the optimal contract. Thus if the probability of the principal having an unacceptable evaluation while the agent has an acceptable self-evaluation is zero we obtain the first best. This result shows that the optimal contract is structured so that the principal's evaluation determines whether or not the agent receives a bonus, while the role of the agent's evaluation is to provide the necessary incentives for the principal to be truthful.

MacLeod (2001) extends this result to the case of risk averse agents and multiple signals of performance. In that case, the optimal contract with subjective evaluation entails a compression of the rewards to performance, relative to the optimal contract with objective measures of performance. The pooling is more extreme as the correlative between the principal's and agent's evaluations decreases. In the extreme case of no correlation in beliefs, Levin (1998) in the case of a risk neutral agent, and MacLeod (2001) in the case of risk aversion, have shown that the optimal contract pools all evaluations into two levels, acceptable or not.

4.2 Relational Contracts

Goetz and Scott (1981) define a *relational contract* as one for which "parties are incapable of reducing important terms of the arrangement to well-defined obligations", a case that includes the problem of contracting with subjective evaluation studied here. They argue informally, as I formally do above, that such contracts arise when the number of contingencies is so large that it is not possible to write a complete contingent contract, creating problems for the interpretation and enforcement of contract terms and conditions.[16] This definition of a relational contract is not however a universally accepted definition. The term originates with MacNeil (1974), for whom the term refers to the complex set of behaviors and norms characteristic of individuals engaging in long term commercial transactions.

Following Axelrod (1981), the prisoner's dilemma problem is often viewed as capturing the essence of relational contracts. In this game two individuals simultaneous decide whether to cooperate or not each period. The model can capture the essence of the contracting with subjective evaluation when beliefs are perfectly correlated. In that case, the strategy cooperate can correspond to truthfully reporting one's evaluation. In these models it is typically assumed that budget balancing is imposed, and hence directly imposing a cost P is *not* possible. Since the principal has an incentive to report low performance if a bonus payment is required, then the only equilibrium in the one-shot game is to not pay the bonus, and hence the agent would choose low effort.

Equilibria with high levels of effort are constructed using a *self-enforcing contract,* modelled formally as a repeated game (see Bull (1987) and MacLeod and Malcomson (1989)). The agent agrees to work hard, and in return the principal agrees to paying a bonus if the agent works hard. The relationship is terminated should either person renege. MacLeod and Malcomson (1989) provide necessary and sufficient conditions for the existence of a high effort equilibrium in such a contract: it must be the case that the value of the relationship is strictly greater than the value of their next best alternatives by an amount exactly corresponding to the value of the penalty P derived above.

This result, in common with much of the literature on repeated games, takes the game from as given and

[16] See Schwartz (1992) and Scott (2000) for recent discussions of relational contracts that argue against too much court intervention.

then analyzes the set of possible equilibria[17]. These equilibria all share a common feature, namely in any given period there are a number of possible equilibria that can be played. Performance incentives are generated by a *norm of behavior* (equilibrium play) in which agents agree to move to an equilibrium specifying a lower payoff to any agent that cheats in the pervious period. The maximum punishment that can be inflicted upon an individual will therefore depend upon the structure of the constituent one stage game. This approach creates a complex relationship between the structure of the game and the set of possible equilibria. See in particular Kandori and Matsushima (1998) and Levin (1998).

To better understand the role of cognition and contract incompleteness for the structure of the optimal contract, I have instead assumed that contracting parties have unlimited punishment ability. The result above illustrates a number of features of relational contracts that appear to be consistent with observed practice. The first is that the potential for conflict and disagreement that can generate a cost P, is a necessary ingredient of any productive relationship when subject evaluations are used and beliefs are not perfectly correlated. Given that organizational conflict is a ubiquitous phenomenon, this result is in some sense heartening because it implies that observed behavior is consistent with this theory! Moreover, as management consultants emphasize, such conflicts can be reduced when individuals have shared values, and there is general consensus that the system of evaluation is fair.[18]

Conflict is not the only mechanism that can generate such a cost. When disagreement results in the termination of a relationship, costs can also arise due to unemployment (Shapiro and Stiglitz (1984)) or the loss of relationship-specific investments (Becker (1975) and Williamson, Wachter, and Harris (1975)). Other market mechanisms include reputation effects (Kreps, Milgrom, Roberts, and Wilson (1982) and Bull (1987)), tournaments (Malcomson (1984) and Carmichael (1983)), wages attached to jobs (MacLeod and Malcomson (1988)), social networks (Kandori (1992) and Kranton (1996)) and gifts (Carmichael and MacLeod (1997)). In addition, the value of a relationship be affected by the use of explicit pay for performance contracts, that can affect the set of self-enforcing agreements, as explored in Pearce and Stacchetti (1998) and Baker, Gibbons, and Murphy (1994).

The common feature of these labor market institutions is that they can be seen as market responses to the problem of contract incompleteness arising from the use of subjective evaluation, which in turn is used to induce high performance in the case of *ex post* hold-up. This is a distinctively different problem from the standard hold-up model, whose implications for the theory of the firm have recently been explored in the work of Baker, Gibbons, and Murphy (1997) and Bolton and Rajan (2000). One suspects that ultimately a complete theory of the firm will entail an integration of the problems of *ex ante* and *ex post* hold-up.

5 Discussion

Contract incompleteness is a ubiquitous phenomenon, yet the welfare theorems of economics require complete markets and contracts to ensure the existence of an efficient equilibrium.[19] Hence, a complete

[17] See Abreu (1988)'s seminal contribution characterizing the set of equilibria in a repeated game, that the survey of cooperation and repeated game theory by Pearce (1992).

[18] See Milkovich and Newman (1996), chapter 10.

[19] See Magill and Quinzii (1996) for a comprehensive review of general equilibrium theory with incomplete markets.

understanding of the efficiency of observed economic institutions depends upon understanding both why contracts are incomplete, and the extent to which such incompleteness generates inefficiencies. The traditional answer to this question follows from the research of Herbert Simon and Oliver Williamson, who argue that complexity and bounded rationality are the central ingredients of a complete theory. Yet, as Oliver Hart (1990) has argued, complexity considerations do not play an important role in the determination of the optimal contract for the hold-up model, a situation that corresponds to non-contractible investment decision being made before resolution of uncertainty.

Moreover, there is a growing literature that demonstrates that in many situations contracting parties *choose* to write incomplete contracts. When there are costs for including contract terms, Shavell (1984) argues that in the case of low probability events it is cheaper to let courts fill in the gaps. While Dye (1985) explicitly derives the optimal risk sharing contact in this case, work has recently been extended to dynamic contract formation by Battigalli and Maggi (2000). The example in section 3 illustrates that costly contingent contracting is a reasonable hypothesis when performance is multi-dimensional. In contrast, Ayres and Gertner (1989) and Bebchuk and Shavell (1991) show that the presence of asymmetric information may lead individuals to choose incomplete contracts, even when transaction costs for including additional terms are zero. Bernheim and Whinston (1998) demonstrate that strategic ambiguity can result in a similar effect.

In contrast, in the case of the hold-up model, renegotiation can introduce inefficiency, as emphasized by Hart and Moore (1999). For example, Che and Hausch (1999) show that renegotiation in a hold-up model with cooperative investments may result in an optimal contract that is incomplete, but not the first best. Segal (1999) shows that one obtains a similar result when the good being traded is complex in the sense that one cannot describe the good *ex ante,* while Schweizer (2000) derives necessary and sufficient conditions for efficient allocation to be implementable in a hold-up model with renegotiation. When renegotiation is not possible, Maskin and Tirole (1999a) have shown that one can achieve an efficient allocation even when the good is not describable *ex ante.*[20]

These conflicting results suggest, not that incomplete contracts are unimportant, but that the term itself is possibly too encompassing of the different problems that arise from contract design. Rather, the main point of the paper is to suggest that the extent to which complexity affects the form and efficiency of a contract is very sensitive to the timing of uncertainty and decision making in a relationship. The problem of *ex post* hold-up follows naturally from Simon's model of the employment relationship, and refers to situations for which it is not possible for an agent to renegotiate her contract between the time she learns the parameters of her decision problem and the time at which an action must be taken. The complexity of the environment make a complete contingent contract impossible, and hence performance incentives depend upon *ex post* evaluation and reward by the principal.

My second point is that the focus upon *human* cognitive limitations is misplaced. In the case of *ex post* hold-up I have argued that the contracting problem is complex in an absolute sense. That is, complete contracts are physically infeasible, and thus not dependent upon constraints imposed by (very real) human

[20] Though, Maskin and Tirole also show that one can relax the renegotiation constraint with risk averse agents and the introduction of lotteries *ex post.*

cognitive limitations. In contrast, I suggest that the use of subjective evaluation is a way to harness the superior pattern recognition abilities that humans possess. The quality of the contract in this case is an increasing function of the correlation between evaluations of the principal and agent.

Finally, I have suggested that the hazard of *ex post* hold-up, or what the legal scholars refer to as the problem of relational contracting, can provide an economic explanation for a number of observed features of the employment relationship. These include the importance of corporate culture to ensure employees have a shared set of values[21], the use of rank order tournaments, bonus pay rather than explicit pay for performance, up front gifts during recruiting in the form of dinners etc.. Though in the end when appropriate incentives for employer performance do not exist, it may simply be optimal to lose one's temper when the boss gives you an unfair evaluation![22]

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[21] See Hermalin (1999) for a review of this literature.

[22] On the role of emotions and contracts see Hirshleifer (1987), Frank (1988) and Posner (1997).

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