Decision under Risk

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Most decisions, whether choices to purchase flood insurance, invest overseas, pursue an experimental medical treatment, or steal a base, involve risk. Purchasing insurance is sensible if you believe a flood will happen, but a bad idea if you are convinced it won't. The study of risky decision making has addressed two broad questions. How *should* individuals behave when faced with a risky choice like the ones above? How *do* individuals behave when faced with a risky choice? The first question is *normative*; the second, *descriptive*.[1] Although the first question is clearly important, our aim in this chapter is to provide answers to the second question.

The study of risky decision making has a long, distinguished, and interdisciplinary history. The list of contributors include some of the most prominent figures in economics and psychology, including several Nobel Prize winners in Economics, and these ideas have in turn been applied with great success to business, law, medicine, political science, and public policy.[2] We hope to give the reader an overview of the exciting developments made by these researchers and others. In particular, the goals of this chapter are fourfold: (i) survey the evolution of questions asked by researchers of risky decision making; (ii) review the major intellectual contributions; (iii) summarize the present state of knowledge; and (iv) offer a research agenda for the next generation of research in the field.

We first distinguish between *risk* and *uncertainty* (Knight, 1921). Risk defines decision situations in which the probabilities are *objective* or given, such as betting on a flip of a fair coin, a roll of a balanced die, or a spin of a roulette wheel. Uncertainty defines situations in which the probabilities are *subjective* (*i.e.*, the decision maker must estimate or infer the probabilities), like the decision to invest overseas and the other examples given above. Although most important decisions clearly involve uncertainty rather than risk, we focus primarily on risk in this chapter. We do so, first, because risk is the simpler case and because there is considerably more empirical evidence on risk than uncertainty. But more importantly, we argue that our understanding of the simpler situation of risk readily extends to the more realistic case of uncertainty. In the latter sections of this review, we discuss how research on risk helps us understand decisions under uncertainty.

Before we begin, we point to the many excellent reviews of this sort that have been written over the years, (*e.g.*, Camerer, 1995; Edwards, 1954, 1961; Fox & See, 2003; Luce, 2000; Machina, 1987; Mellers, Schwartz, & Cooke, 1998; Schoemaker, 1982; Starmer, 2000). We encourage those interested in the field to read these reviews; they provide a perspective of how the field has evolved over the years, and also highlight the differences and similarities between how economists and psychologists have approached this field.

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[1] A third category of study is *prescriptive*: how can we get ordinary people to act more normatively? This particular question motivates decision analysis (*e.g.*, Raiffa, 1968). Many decision analysts see the divergence between descriptive and normative models as an argument why prescription is needed (*e.g.*, Bell, Raiffa, & Tversky, 1988).

[2] The list of Nobel Prize winners in Economics who have contributed directly to the study of decision under risk is remarkable and includes Maurice Allais, Kenneth Arrow, Milton Friedman, Harry Markowitz, Daniel McFadden, Paul Samuelson, Herbert Simon, Michael Spence, Joseph Stiglitz, George Stigler, and the most recent winner, Daniel Kahneman. Several others have contributed to the broader study of decision making, including Gerard Debreu, Tjalling Koopmans, Amartya Sen, and Vernon Smith.

Expected Utility

We begin by reviewing the classical model of decision under risk, *expected utility* (EU) theory. Consider a gamble that gives p_i chance at x_i, which we represent $(p_1, x_1; ...; p_n, x_n)$. The expected utility of this gamble is $\sum p_i u(x_i)$, where $u(x_i)$ measures the "utility" of receiving outcome x_i. In expected utility, the burden of explaining risk attitudes falls completely on the shape of the utility function. Risk-averse behavior, such as the purchase of insurance, requires that the utility function be concave, while risk-seeking behavior, such as buying a lottery ticket, is explained by convexity of the utility function. Thus, it is difficult for expected utility theory to explain why an individual simultaneously purchases insurance and lottery tickets. This individual's utility function must be concave for some wealth levels and convex for other wealth levels (Friedman & Savage, 1948; Markowitz, 1952). Nevertheless, economics usually assumes that decision makers are risk averse, the primary justification being diminishing marginal utility: a dollar to a pauper is considerably more useful than a dollar to Bill Gates (*e.g.*, Varian, 1992).

Bernoulli (1738) proposed expected utility in the 18th century as a resolution of the famous St. Petersburg Paradox. The St. Petersburg gamble is a prospect that offers a $1/2^n$ chance at $\$2^n$ for $n = 1,...,\infty$. Although this gamble has an infinite expected value, most people would pay less than \$10 for this gamble. Many concave utility functions, including logarithmic and power utility functions, impose finite bounds on the maximum an individual would pay for the St. Petersburg bet. The contribution of Bernoulli, however, went far beyond reconciling this example. Bernoulli rejected expected value as a criterion for making risky choices, arguing more choices, arguing more generally that two people with different desires and different wealth levels should not necessarily value the identical gamble equally. Although it is unclear whether Bernoulli was making a descriptive argument or normative argument, the generalization of expected value to expected utility was introduced and has remained important to this day.

Expected utility took off in the 1940's and 1950's when von Neumann and Morgenstern axiomatized the model in their *Games and Economic Behavior* (von Neumann & Morgenstern, 1947).[3] Whereas Bernoulli assumed the expected utility representation, von Neumann and Morgenstern provided an axiomatic system: a set of conditions that were necessary and sufficient for expected utility. Axioms have a descriptive as well as normative benefit: they decompose a complex theory into smaller pieces, each of which can be tested empirically or scrutinized as normative principles. The most important axiom became known as the *Independence Axiom* or *Substitution Axiom*, and was reformulated by Marschak (1950) and Samuelson (1952).[4] The basic idea of the axiom is straightforward. If you like gamble *A* more than gamble *B*, then you should prefer the mixture of *A* and some other gamble *C* (in some probabilistic proportion) to the mixture of *B* and *C* (in the same probabilistic proportion). The Independence Axiom can be stated formally: if $A \succ B$ then $pA + (1-p)C \succ pB + (1-p)C$, where "$\succ$" stands for the binary

[3] Although the first edition of *Games and Economic Behavior* appeared in 1944, an axiomatic system did not appear until the second edition (1947). Luce & Raiffa's (1957) critical survey, *Games and Decisions* gives a sense of the enormous amount of activity devoted to understanding individual and strategic decision making in the next ten years.
[4] For a history of the independence axiom, see Fishburn & Wakker (1995). The other two axioms are ordering (either $A \succeq B$ or $B \succeq A$, or both), and continuity (if $A \succ B \succ C$, there exists a *p* such that $pA + (1-p)C \sim B$).

relation “is preferred to” and $pA+(1-p)C$ denotes a probabilistic mixture of *A* and *C*. To illustrate, suppose you prefer a .50 chance at $100 (*A*) to a .80 chance at $50 (*B*). The independence axiom requires that you prefer a .25 chance at $100 to a .40 chance at $50, since these gambles are derived by mixing the antecedent gambles with $0 for sure (*C*) in equal proportions.

In the early 1950’s, there was considerable debate about the normative status of expected utility and the independence axiom (*e.g.*, Manne & Charnes, 1952; Wold, 1952), and how to interpret EU’s utility function (Ellsberg, 1954; Friedman & Savage, 1952). When this debate finished, it was widely believed that expected utility was a compelling normative model (Savage, 1954). Indeed, in its abstract form, the independence axiom is intuitively compelling. Given a choice between $pA+(1-p)C$ and $pB+(1-p)C$, the decision maker receives *A* or *B* if an unfair coin comes up heads, and *C* if the unfair coin comes up tails. If the coin comes up tails, it doesn’t matter what you chose. If the coin comes up heads, you should choose *A* if you like *A* more than *B*, and *B* otherwise. Thus, this logic argues that choosing between *A* and *B* is the same as choosing between $pA+(1-p)C$ and $pB+(1-p)C$.

Subjective Expected Utility

In 1954, Savage published the influential *Foundations of Statistics.*[5] The major contribution was an axiomatic system that extended expected utility from risk to uncertainty. In uncertain situations, probabilities are not given, and outcomes depend on which event obtains. Consider a prospect, $(E_1, x_1; ...; E_n, x_n)$, that offers x_i if event E_i occurs. The *subjective expected utility* (SEU) of this prospect is given by $\sum \rho(E_i)u(x_i)$, where $u(\cdot)$ is a utility function as in standard expected utility and $\rho(\cdot)$ is a *subjective probability measure* that obeys the standard axioms of probabilities. Thus, SEU is the natural generalization of EU from risk to uncertainty.

The critical axiom is Savage’s “Sure Thing Principle”. The Sure Thing Principle shares the same basic intuition as the Independence Axiom, which we illustrate with the following example. Consider a choice between *A* and *B*, where the outcome of the prospects depends on what event is realized:

	E_1	E_2	E_3
A	100	0	0
B	0	100	0

A preference for *A* over *B* can be interpreted as a belief that E_1 is more likely to occur than E_2. Since E_3 shares a common outcome, 0, this event is irrelevant for the choice between *A* and *B*. Thus, a decision maker should have the same preferences if we substitute 50 (or any other outcome) for 0 in E_3:

[5] Savage’s ideas were anticipated to some extent by Borel (1964, originally published in 1921) and Ramsey (1931, originally published in 1926).

	E_1	E_2	E_3
A'	100	0	50
B'	0	100	50

Despite the normative appeal of the independence axiom and the sure thing principle, EU and SEU would soon be challenged as reasonable descriptive models of decision under risk and uncertainty.

The pre-prospect theory era

Edwards' Review Paper

Are the elegant normative frameworks of Savage and von Neumann & Morgenstern descriptively accurate models? In a remarkable review of the decision making literature written only seven years after von Neumann & Morgenstern axiomatized EU, Edwards (1954) summarized the empirical and theoretical literature. There was already mounting empirical evidence that the normative theory of expected utility was descriptively inadequate. One pressure point came from the field, a need to explain the simultaneous purchase of insurance and lottery tickets. There were some impressive attempts to salvage the expected utility framework so that it could conform to this observation. For example, in a famous paper, Friedman & Savage (1948) attempted to account for the simultaneous purchase of insurance and gambling by introducing a utility function that had regions of convexity and regions of concavity (the former accounting for the risk seeking behavior of gambling and the latter accounting for the risk averse behavior of insurance purchase).

Another set of attacks came from the laboratory. Preston & Baratta (1948) found preliminary evidence that people distort given probabilities: small probabilities are overweighted and large probabilities are underweighted. This study, as well as other early empirical endeavors (*e.g.*, Mosteller & Nogee, 1951; Davidson, Suppes, & Siegel, 1957) had various methodological problems, making it difficult to make inferences about the underlying mechanisms. These experiments were the best of the lot, leading Edwards (1954) to remark of the others: "Such experiments are too seldom adequately controlled, and are almost never used as a basis for larger-scale, well-designed experiments…. The results of such pilot experiments too often are picked up and written into the literature without adequate warning about the conditions under which they were performed and the consequent limitations on the significance of the results." (p. 403)

Looking back, it is surprising how much was already known both empirically and theoretically by 1954. What is also surprising is how far away researchers and theoreticians were from a comprehensive model of decision making under risk and uncertainty. Many of the major empirical results that characterized research in the 80's were already known, but the lack of a proper theoretical framework kept researchers from fully understanding these results. As a child may misunderstand the clues given to her about where the last Easter egg is hidden, the researchers at the time were misled by many of the empirical results that were available—not merely because of the methodological deficiencies, but also because of the limited types of inferences the existing theoretical models permitted.

Edwards identified the fundamental problem of decision making research, "(the) development of a satisfactory scale of utility of money and of subjective probability" (p. 403). Indeed, Edwards also anticipated the theoretical problem that would characterize much research in the last 15 years: the composition rule that combines utility with distorted probabilities. In Edwards' words, "it seems very difficult to design an experiment to discover that law of combination" (p. 400).

The Allais and Ellsberg Paradoxes

Allais (1953) posed the first major direct challenge to expected utility. He collected data at a Paris Colloquium in 1952 that was attended by a number of distinguished researchers interested in the foundations of decision theory.[6] One of the problems Allais presented involved two choice pairs. The first pair was a choice between

A) $1 million for sure

and

B) .10, $5 million;
.89, $1 million;
.01, $0.

The modal preference was *A*. The second pair involved a choice between

C) .11, $1 million;
.89, $0;

and

D) .10, $5 million;
.90, $0.

The predominant choice was *D*. This example became famously known as the *Allais Paradox*.

The intuition behind these two choices is the following. In the first choice, the sure thing of $1 million is highly attractive, thus it is not worth risking a chance of nothing for the possibility of winning $5 million. In the second choice, the two probabilities (.10 and .11) appear indistinguishable relative to the difference between $5 million and $1 million (*e.g.*, Slovic & Tversky, 1975).

Choosing *A* and *D* violates the independence condition. The table below illustrates the problem in a transparent manner. Consider a lottery with tickets numbered 1-100. The table below displays each of the four options, with consequences dependent on which lottery ticket is drawn:

[6] Allais (1979) listed the participants of this conference. They included four eventual Nobel Prize winners in economics (Allais, Kenneth Arrow, Milton Friedman, Paul Samuelson), as well as other distinguished contributors to the field, including Bruno de Finetti, Jacob Marschak, Leonard Savage, and George Shackle.

	1-10	11	12-100
A	1	1	1
B	5	0	1
C	1	1	0
D	5	0	0

The table highlights that the main difference between the two pairs is that the tickets numbered 12-100 (representing .89 chance of winning) have a common consequence within each pair—either 1 million in the *A*/*B* pair or 0 in the *C*/*D* pair. Expected utility requires that a change in a common consequence not alter preference. So, if a decision maker chooses *A* over *B*, she should choose *C* over *D*. Note the connection with Savage's Sure Thing Principle discussed earlier. It is interesting that no careful empirical evidence for this choice pattern was collected for over 10 years. MacCrimmon (1965) collected data for his Ph.D. thesis (see also MacCrimmon, 1968), and replications were conducted by others (*e.g.*, Morrison, 1967; Slovic & Tversky, 1974).[7]

There was an analogous violation of the Sure Thing Principle in the domain of uncertainty. Ellsberg (1961) proposed a famous problem that became known as the *Ellsberg Paradox*. Ellsberg did not publish data in that paper (although data appeared in his Harvard Ph.D. thesis), but the violation was soon empirically verified by Becker & Brownson (1964) and then many others (see the review by Camerer & Weber, 1992).

Imagine an urn with 90 balls, 30 of which are red, and 60 of which are black or yellow in an unknown proportion (*i.e.*, perhaps 0 black and 60 yellow, 60 black and 0 yellow, or any combination in between). The following two prospects link payments to whether a red, black, or yellow ball is drawn. Would you choose *A* or *B*?

	red	black	yellow
A	$100	$0	$0
B	$0	$100	$0

Now consider a variation on the above problem where the winnings for the yellow ball in both options are converted to $100 instead of $0. Would you choose *C* or *D*?

	red	black	yellow
C	$100	$0	$100
D	$0	$100	$100

The predominant choice in the first pair is *A*, while the modal choice in the second pair is *D*. In both cases, subjects prefer betting on known probabilities to unknown or vague probabilities (*e.g.*, *A* offers $100 with a known chance of 1/3, while the chance of $100 with *B* may be anywhere

[7] Allais (1979) presented an elaboration of his theoretical model of risky decision making. He also gave an overview of the experiment he conducted in 1952. These data remain unpublished. Allais explained, "Summing up, I have suffered delay and am still behind in publishing the results of the 1952 experiment. Essentially, the reason is the amount of work involved in undertaking a satisfactory analysis of the answers I received to the Questionnaire and the many constraints resulting from my professional duties and my other research and publishing activities." (p. 448)

between 0 and 2/3). However, these choices are incompatible with the Sure Thing Principle. As Ellsberg described the pattern, "The first pattern, for example, implies that the subject prefers to bet 'on' red rather than 'on' black; and he also prefers to bet 'against' red rather than 'against' black" (p. 654).

Prospect Theory

Kahneman & Tversky's (1979) "Prospect Theory: An analysis of decision under risk" is the second most cited paper in economics during the period, 1975-2000 (Coupe, in press; Laibson & Zeckhauser, 1998).[8] The paper's success is probably due to its unique combination of simplicity and depth. Kahneman & Tversky presented convincing empirical demonstrations that highlighted some general descriptive deficiencies with expected utility, as well as a powerful formal theory for organizing these demonstrations. Although the Allais Paradox was now 25 years old, very little data existed challenging expected utility, and there were no theoretical alternatives to the classic model.

The classic model of decision under risk assumes that individuals are generally risk averse (perhaps because of diminishing marginal utility). However, Kahneman & Tversky demonstrated that they are risk-averse and risk-seeking and that the pattern of risk attitudes can be organized in a remarkably simple manner. They found that 84% of subjects preferred $500 for sure to a .50 chance at $1000, but 72% preferred a .001 chance at $5000 to $5 for sure. The first choice demonstrates risk aversion for moderate probabilities, the second risk-seeking for small probabilities. When choices involve losses, the pattern reversed: 69% chose a .50 chance at losing $1000 to losing $500 for sure, and 83% chose losing $5 for sure over a .001 chance at losing $5000. For losses, subjects were risk-seeking for moderate probabilities and risk-averse for small probabilities.

These data are typical of a more general pattern, called the *reflection effect*: preferences tend to reverse when the sign of the outcomes is changed.[10] This pattern has become known as the *four-fold pattern of risk attitudes* and can be summarized in the following table (Tversky & Kahneman, 1992).

	Small Probabilities	Medium to large Probabilities
Gains	Risk-seeking	Risk-averse
Losses	Risk-averse	Risk-seeking

Kahneman & Tversky also presented two direct violations of expected utility. Expected utility can explain the four-fold pattern, by positing a specific utility function, but doing so would strain the theory (Rabin, 2000). A different approach is to construct a direct test of one of the axioms underlying expected utility. A violation of any axiom falsifies the EU model—there can be no utility function that can accommodate the pattern. The Allais Paradox, and Kahneman

[8] Kahneman & Tversky originally envisioned a theory of regret, but "abandoned this approach because it did not elegantly accommodate the pattern of results that we labeled 'reflection'…" (Kahneman, 2000)

[10] But, see Hershey & Schoemaker (1980) and Schneider & Lopes (1987).

& Tversky's common-ratio and common-consequence effects are examples of this methodological strategy.

We begin with the *common consequence effect*, an example that follows the same basic schema as Allais' original demonstration Most subjects preferred \$2400 for sure to a .33 chance \$2500, a .66 chance at \$2400, and a .01 chance at \$0, but preferred a .33 chance at \$2500 to a .34 chance to \$2400. To show that no utility function can reconcile this pattern, it suffices to observe that the first choice under EU reduces to $.34u(2400) > .33u(2500)$, while the second choice simplifies to $.34u(2400) < .33u(2500)$. Indeed, this example was constructed to test the independence axiom directly. One pair is derived from the other pair by substituting a .66 chance at \$2400 for a .66 chance at \$0 (hence the name common consequence effect).

Why do the majority of subjects violate expected utility? The reasoning may go something like this: "In the first choice, it is not worth sacrificing a sure thing for a chance at getting a slightly better outcome. In the second choice, neither gamble is a sure thing. Since .33 and .34 are very similar, it is worth going for the better outcome, \$2500." This pattern demonstrates the *certainty effect*: subjects are willing to pay a large premium to avoid a small chance at receiving nothing.

In the *common-ratio effect*, subjects chose \$3000 for sure to a .80 chance at \$4000, but a .20 chance at \$4000 to a .25 chance at \$3000. This also pattern contradicts expected utility, since the first choice implies $u(3000) > .8u(4000)$, but the second implies $.25u(3000) < .2u(4000)$. The independence axiom is violated in this example as well since the second pair is constructed by mixing a 25% chance of the first pair with a 75% chance of receiving \$0. The effect gets its name because the ratio of the probability of winning \$4000 to the probability of winning \$3000 is the same for both choices.

Kahneman & Tversky proposed a formal theory to explain the four-fold pattern, common-consequence effect, common-ratio effect, and a slew of other demonstrations. The major components were a *value function* and a *probability weighting function*. In expected utility theory, the utility function is defined over final wealth states, an assumption that is known as asset integration. In contrast, the value function in prospect theory, $v(\cdot)$, is defined over changes in wealth rather than absolute wealth levels. The function is concave for gains ($v''(x) < 0$ for $x > 0$), convex for losses ($v''(x) > 0$ for $x < 0$), and exhibits "loss aversion", *i.e.*, the function is steeper for losses than gains ($-v(-x) > v(x)$ for $x > 0$) (Figure 1).

The probability weighting function, $\pi(\cdot)$, captures how different probability levels contribute to the evaluation of a gamble. Tversky & Kahneman presented a schematic weighting function that overweighted small probabilities, and underweighted medium and large probabilities (Figure 2). They also suggested that there might be a discontinuity at the end points of 0 and 1: differences between 0 and 1 either would be ignored, or there would be a categorical distinction between 0 ("impossibility") and some small probability ("possibility") and 1 ("certainty") and some large probability close to 1 ("uncertainty"). The weighting function also exhibits *subcertainty*, $\pi(p) + \pi(1-p) < 1$. Roughly speaking, the weighting function is more below the identity line (for moderate and high probabilities) than it is above the identity line (for low probabilities).

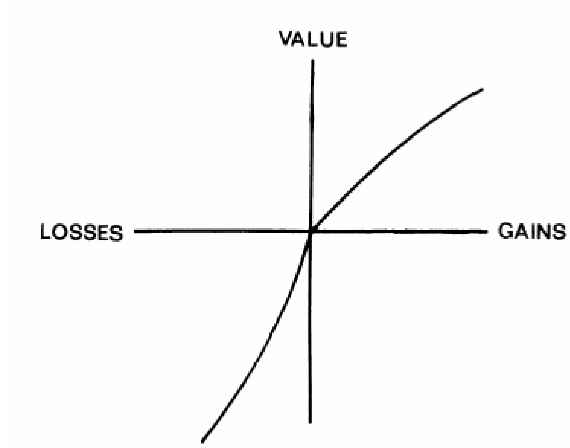

Figure 1: A stylized prospect theory value function from Kahneman & Tversky (1979). The function is defined in terms of gains and losses, is concave for gains and convex for losses, and more steeply sloped for losses than gains.

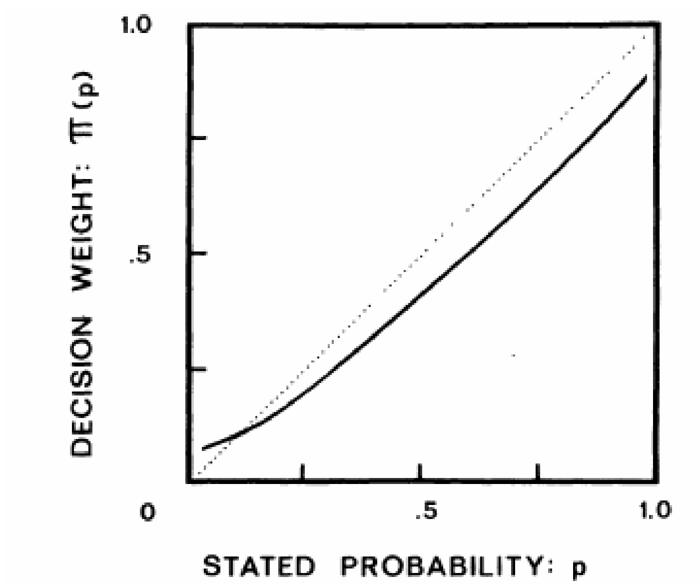

Figure 2: A stylized prospect theory weighting function, from Kahneman & Tversky (1979). The weighting function captures distortions of probabilities, where small probabilities are overweighted and medium to large probabilities are overweighted. The function is not necessarily well-behaved at the endpoints, 0 and 1.

Prospect theory combines the two functions as follows. Consider a gamble $(p, x; q, y)$, where $p + q < 1$ and x and y may both be positive, both negative, or one positive and one negative.[11] The value of such a gamble is given by $U(p, x; q, y) = \pi(p)v(x) + \pi(q)v(y)$. This functional form explains the four-fold pattern of risk attitudes and the common-consequence and common-ratio effects in terms of the qualitative properties on $v(\cdot)$ and $\pi(\cdot)$ discussed above. This form also has the same basic structure as expected value and expected utility. Expected

[11] Gambles with $p + q = 1$ and the same sign are treated differently, and evaluated by $v(y) + \pi(p)[v(x) - v(y)]$. Interestingly, this representation is rank-dependent and coincides with the revised prospect theory (see below).

utility is the sum of utility values weighted by probabilities. Prospect theory generalized this notion by summing utility values weighted by transformed probabilities or decision weights.

Recall that expected utility had a hard time reconciling simultaneous purchasing of insurance and gambling because the utility function had to do all the heavy lifting. The burden of explaining risk attitudes now falls on the value function and the probability weighting function. In prospect theory, insurance purchasing and gambling is explained by the overweighting of small probabilities. Insurance purchasing is risk-averse behavior: thus overweighting of small probabilities has to be large enough to overcome the convexity of the value function in losses. Similarly, gambling represents risk-seeking, which is predicted if overweighting of small probabilities is sufficient to overcome the concavity of the value function in gains.

We also return to Edwards' (1954) question about how to separate distortions of value from distortions of probabilities. The prospect theory representation permits independent inferences about the value and weighting function. For example, restrictions on the weighting function can be inferred from the common-consequence effect problems, and restrictions on the value function (*i.e.*, concavity and loss aversion) can be inferred from other examples presented in the paper.[12]

It is a surprise to many people that many of the ideas from prospect theory existed in previous literatures. In a remarkable paper, Markowitz (1952, p. 154) proposed a utility function very similar to Figure 1, defining utility as changes from present wealth. The Markowitz utility function was an attempt to capture the observation that individuals at just about every wealth were insurance buying gamblers.[13] Preston & Baratta (1948) investigated choices involving varying probability levels and found a pattern remarkably similar to Figure 2 (see also Mosteller & Nogee, 1951; Davidson, Suppes, & Siegel 1957). Edwards (1953) documented probability preferences, preferences to bet on certain probabilities when faced with bets of equal expected value, and argued that descriptive models needed to take account the nonlinear impact probabilities have on decisions. Finally, MacCrimmon (1968) and MacCrimmon & Larsson (1979) presented similar demonstrations of the common-ratio and common-consequence violations, and Williams (1966) documented rejection of fair gambles, consistent with loss aversion (see also, Mosteller & Nogee, 1951; Slovic & Lichtenstein, 1968, p.10).

Thus, many of the pieces of prospect theory, taken alone, were not novel. However, the reputation of prospect theory as one of the most important papers in social science is nevertheless completely deserved. The paper took ideas that had been around, some for as long as 30 years, scattered in different literatures and thought to be unrelated, and constructed a formal model in which all the elements worked together. The paper also produced such compelling demonstrations (some extensions of old findings and some new predictions such as

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[12] The common consequence effect demonstration reveals subcertainty of $\pi(\cdot)$, while rejection of an asymmetric fair gamble, $(x,.50;-x,.50)$ reveals loss aversion, $v(x)<-v(-x)$, and a preference for $(x,p;y,p)$ over $(x+y,p)$ reveals concavity of $v(\cdot)$: $v(x)+v(y)>v(x+y)$.

[13] The Markowitz function actually was convex for small gains, and concave for large gains, concave for small losses, and convex for large losses. Such a shape was needed to explain insurance and gambling purchases (a small loss was barely different from the status quo). Prospect theory would capture this behavior in terms of overweighting of small probabilities.

the isolation and pseudo-certainty effects, and the rejection of probabilistic insurance) that those with no interest in formal theory could nevertheless understand why expected utility was an unsuitable descriptive model and what an adequate descriptive model required. Most importantly, the view that the Allais Paradox was an isolated problem for expected utility was no longer tenable.

The post-prospect theory era

An overview

Kahneman & Tversky attacked the independence axiom and expected utility in an elegant, coherent, and convincing manner. Since the Allais Paradox could no longer be considered an isolated anomaly, prospect theory forced economists to consider EU violations seriously. In this section, we review the quarter-century of research following prospect theory. The phase can best be understood as an ongoing dialogue between alternative models to EU and ingenious tests conducted to discriminate among these alternative models.

In hindsight, the post-prospect theory models appear to be motivated by two different concerns, those of economists and those of psychologists. In general, economists strove for a descriptive theory of decision under risk that was elegant, general and mathematically tractable. This strategy was practical, as much as aesthetic. Expected utility had been applied with great success to many important areas of economics, such as game theory and information economics (*e.g.*, Pratt, 1964; Rothschild & Stiglitz, 1970). The new models relaxed the independence axiom and hence are "generalizations" of EU in the sense that EU is a special case of these models. The common-consequence and common-ratio effect violations presented in the last section posed a minimum standard for a set of new models.

Why did theorists need an alternative to prospect theory when prospect theory explained the basic violations just fine? There are at least three reasons. Many theorists disliked the prospect theory representation since it admitted possible violations of stochastic dominance: $(p, x; q, x-\varepsilon)$ could exceed $(p+q, x)$, even though the second gamble stochastically dominates the first gamble (Fishburn, 1978). To avoid this problem, Kahneman & Tversky proposed an editing operation where subjects spotted dominated alternatives. Second, theorists wanted to have a model that included expected utility as a special case. Finally, prospect theory was limited to two non-zero outcomes.

In contrast, psychologists were generally more concerned with explaining the underlying psychological process. Some alternative models were cognitive, while others considered personality and motivational factors (*e.g.*, Birnbaum & Stegner's (1979) configural weight theory; Lopes' (1987) aspiration-level theory). These models tended to have more free parameters than prospect theory, and therefore were more flexible but less tractable and parsimonious.

At the end of these 25 years of research, prospect theory stands out as the best descriptive model. However, prospect theory, too, evolved as part of this dialogue. One important fruit of this stream of theoretical and empirical work was a refinement of prospect theory, cumulative prospect theory (Tversky & Kahneman, 1992). Although original prospect theory (OPT) was groundbreaking, it had clear limitations, some of which were acknowledged in the original paper. Kahneman (2000) commented on these limitations:

> The goal we set for ourselves was to assemble the minimal set of modifications of expected utility theory that would provide a descriptive account of everything we knew about a severely restricted class of decisions: choices between simple monetary gambles with objectively specified probabilities and at most two nonzero outcomes. Without additional assumptions, prospect theory is not applicable to gambles that have a larger number of outcomes, to gambles on events, or to transactions other than choice; it does not even specify a selling price for monetary gambles.

Not only did cumulative prospect theory overcome the potential violations of dominance that some saw as a flaw of OPT, CPT generalized prospect theory to apply to arbitrary number of outcomes, and uncertainty as well as risk. We will discuss details of CPT later.

One simple device greatly facilitated the dialogue, the probability triangle or simplex, used originally by Marschak (1950) and later adopted by Machina (1982). The unit triangle provided a common "language" and platform for understanding the implications of different models and visualizing and organizing empirical findings. Models could be understood in terms of restrictions placed on the curvature, slope, and fanning property of the indifference curves in the triangle.[14] Figure 3 illustrates indifference curves for some of the major theories.

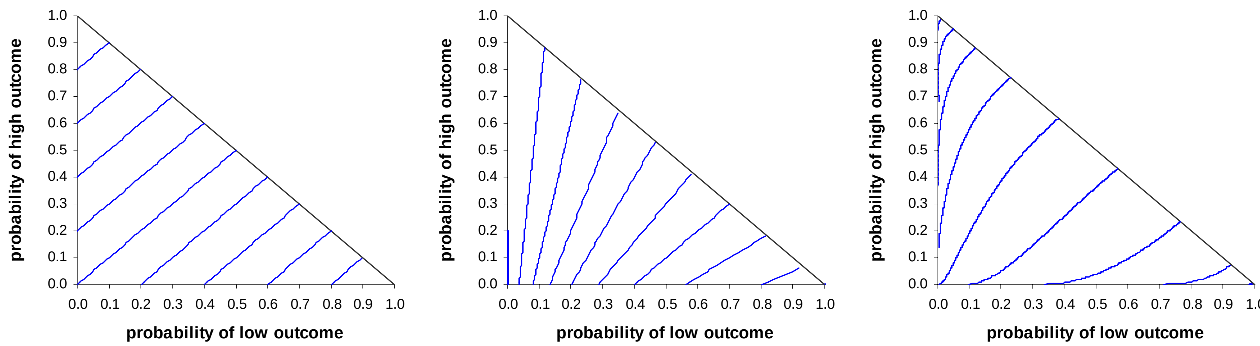

Figure 3: The probability triangle can be used to depict gambles with at most three outcomes. The *x*-axis captures the probability of the lowest outcome, the *y*-axis the probability of the highest outcome. Shown are indifference curves for (A) expected utility theory; (B) weighted utility with indifference curves that "fan out"; (C) rank-dependent utility. All gambles on an indifference curve yield the same utility.

Alternatives to prospect theory

Since the common-consequence and common-ratio effects violate the independence axiom, an adequate descriptive model of decision under risk needs to "relax" the independence axiom in some respect. Many generalized EU models took the strategy of replacing the

[14] However, the unit triangle method is limited to lotteries having at most three outcomes. More complex lotteries therefore cannot be studied within the unit triangle paradigm. For more details on the probability triangle, see Camerer (1989) and Machina (1987).

independence axiom with a weaker form. We discuss several different families of model that were advanced in the 1980's.[15]

Machina (1982) took a non-axiomatic approach. EU requires that indifference curves be linear and parallel. Machina suggested an ingenious hypothesis called the *fanning out hypothesis*. Indifference curves no longer had to be linear or parallel, but just needed to "fan out". Fanning out requires that decision makers became more risk-averse as lotteries improve in the sense of first-order stochastic dominance. Graphically, fanning out hypothesis posits that the indifference curves in the unit triangle become steeper when moved in the northwest direction (see Figure 3b). Fanning out of indifference curves is consistent with the basic common-consequence and common-ratio effects.

One family of models replaced the independence axiom with a weaker form called *betweenness*. Betweenness requires that a probabilistic mixture of two lotteries should be in the middle of the two lotteries in preference, *i.e.*, for any two lotteries A and B: if $A \succeq B$ then $A \succeq pA + (1-p)B \succeq B$. Betweenness is intuitively appealing as well as pragmatic: some important economic applications only require betweenness and not the full force of the independence axiom (Crawford, 1990).

A number of betweenness models were proposed, including weighted utility theory (Chew & MacCrimmon, 1979; Chew, 1983, 1989), implicit weighted utility theory (Dekel, 1986), skew-symmetric bilinear utility theory (Fishburn, 1984), Neilson's (1992) boundary effect hypothesis, and disappointment-aversion theory (Gul, 1991). Graphically, betweenness requires that indifference curves in the probability triangle be straight lines (as with EU), but not necessarily parallel (as required by EU) (see Figure 3a and 3b). With appropriate parameters, these models can accommodate the basic common-consequence and common-ratio effects. These models mainly differ in the fanning properties of the indifference curves, either imposing uniform fanning (all fanning in or all fanning out) or mixed fanning.

Rank-dependent models are variants of prospect theory. In prospect theory, there is a nonlinear transformation of outcomes, as well as probabilities. Original prospect theory permits violations of dominance or monotonicity, a problem that Kahneman & Tversky recognized and dealt with in the editing phase (but see Tversky & Kahneman, 1986). *Rank-dependent* utility (RDU) was an ingenious way of allowing probability distortions, like prospect theory, while prohibiting violations of dominance. The basic idea is to transform *cumulative* probabilities instead of individual probabilities (Quiggin, 1982; see also, Luce, 1988; Yaari, 1987). A prospect, $(p, x; q, y)$, where $x > y$, would be valued by $\pi(p)v(x) + [\pi(p+q) - \pi(p)]v(y)$ or $\pi(p)[v(x) - v(y)] + \pi(p+q)v(y)$. The *decision weight*, the amount that a particular outcome is weighted, depends on the probability of that outcome as well as the rank of that outcome in the gamble. More generally, the value of a prospect, $(p_1, x_1; ...; p_n, x_n)$, where $x_i > x_{i+1}$, is given by

$$\sum_{i=1}^{n}\left(\pi\left(\sum_{j=1}^{i} p_j\right) - \pi\left(\sum_{j=1}^{i-1} p_j\right)\right)v(x_i)$$

or

[15] For a review of models, see Camerer (1992) and Fishburn (1988).

$$\pi(p_n)v(x_n)+\sum_{i=1}^{n-1}\pi\left(\sum_{j=1}^{i}p_j\right)[v(x_i)-v(x_{i-1})]\ .$$

In this general form, the decision weight for an outcome x_i is the probability weighting function applied to the probability of receiving at least outcome x_i minus the weighting function applied to the probability of receiving at least outcome x_{i-1} (for a useful introduction, see Diecidue & Wakker, 2001).[16,17]

Regret theory (Bell, 1982; Fishburn, 1982; Loomes & Sudgen, 1982) took a different approach to generalizing EU. One carrier of value is *regret*, the comparison between the outcome received and the outcome that would have been received under some other choice. Bell and Loomes & Sudgen independently demonstrated how particular forms of regret theory could explain a wide range of phenomena, including purchasing of insurance and lotteries, the reflection effect, the Allais paradox, probabilistic insurance, and preference reversals.

Critical empirical evidence

A slew of tests were designed to discriminate between the various models. The most discerning tests either tested general axioms such as betweenness or general features of preferences such as the fanning-out hypothesis. This approach was efficient: choice patterns inconsistent with these axioms or features ruled out a whole family of models.

Machina's (1982) hypothesis that indifference curves fan out everywhere in the unit triangle stimulated many empirical tests. Conlisk (1989) generalized the Allais Paradox and the common-consequence effect. He found that subjects preferred (.10,$5M;.89,$1M) over (.20,$5M;.78,$1M), but preferred (.98,$5M) over (.88,$5M;.11,$1M). This pattern violates fanning-out, since preferences become more risk-seeking as gambles are improved. This particular problem generalizes the common-consequence effect in the following sense. In Allais' example and Kahneman & Tversky's (1979) original demonstration, probability mass is shifted from the lowest to the middle outcome, which corresponds to a horizontal movement of gamble pairs in the unit triangle. Here, the shift is from the middle to the highest outcome, corresponding to vertical movement of gambles pairs. Similar results showing that indifference curves that fan in vertically are found in a variety of studies (Battalio, Kagel & Jiranyakul, 1990; Camerer, 1989; Starmer & Sudgen, 1989; Wu & Gonzalez, 1998). Prelec (1990) demonstrated fanning-in in a very different part of the triangle. His subjects preferred (.02, $20,000) to (.01, $30,000) but (.01, $30000; .32, $20000) to (.34, $20000). Wu & Gonzalez (1996) found similar patterns of fanning-in along the bottom edge.[18]

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16 We do not discuss the axioms underlying rank-dependent utility models, since they tend not to be transparent or easily tested. There are at least two exceptions, axiom systems based on ordinal independence (Green & Jullien, 1988) and tradeoff consistency (Abdellaoui, 2002).

17 It is noteworthy that the rank-dependent form appeared in Kahneman & Tversky (1979) for two-outcome gambles, $(p,x;1-p,y)$, where $x>y>0$.

18 Another type of common-consequence effect involves shift of probability mass from the lowest to the highest outcome, which corresponds to a diagonal movement of gamble pairs along the hypotenuse (Camerer, 1989). Empirical tests of generally show diagonal fanning-out (Camerer, 1989; Chew & Waller, 1986). See, also, Wu & Gonzalez (1998).

Empirical evidence showing mixed-fanning within the unit triangle ruled out various models assuming uniform fanning-out or uniform fanning-in, such as weighted utility theory, implicit weighted utility theory, and Machina's (1982) fanning-out hypothesis. Other models allowed mixed-fanning, such as Gul's disappointment aversion theory (1991), Neilson's (1992) boundary effect hypothesis, rank-dependent utility, and prospect theory.

Another set of studies investigated betweenness. If betweenness is violated, all betweenness models are falsified. Prelec (1990) documented a stunning violation of betweenness. He found that 94% of subjects preferred *A*=(.34,$20000) to *C*=(.17,$30000) but 82% of subjects preferred *B*=(.01,$30000;.32,$20000) to *A*=(.34,$20000). Since $B = \frac{1}{17}C + \frac{16}{17}A$, betweenness requires that *B* should lie between *A* and *C* in preference. Other empirical tests with gambles located in the southeast corner found similar patterns (Battalio *et al.* 1990; Camerer, 1989, 1992; Camerer & Ho, 1994). Prelec offered a very intuitive way of interpreting his finding. People may find trading a 2% chance of $20,000 for a 1% chance of $30,000 attractive, thus choose *B* over *A*. However, they do not like taking 17 such trades, which means exchanging 34% chance of $20,000 for 17% chance of $30,000.

The direction of violations is also useful for distinguishing between models. Violation of betweenness could be due to either quasi-concave preferences (*i.e.*, convexity of indifference curves) or quasi-convex preferences (*i.e.*, concavity of indifference curves), indicating a preference for or against randomization, respectively (Camerer, 1992). Prelec's example constitutes quasi-concave preferences. Quasi-convex preferences have been found for gambles located in the northwest corner (Battalio *et al*., 1990; Camerer, 1989, 1992; Camerer & Ho, 1994), while both quasi-concavity (Chew & Waller, 1986; Gigliotti & Sopher, 1993) and quasi-convexity (Conlisk, 1989) are found in the southwest corner of the triangle. Finally, betweenness violations tend to be weaker for gambles located inside the unit triangle than for gambles on the boundary.

Are any of the models described above consistent with the pattern of mixed fanning and quasi-concave and quasi-convex preferences? An appropriate model needs to be "nonlinear in probability" in order to capture betweenness violations. It turns out that betweenness data are consistent with both prospect theory and rank-dependent utility models, assuming an inverse S-shaped weighting function (see Camerer & Ho, 1994). Consider the weighting function depicted in Figure 4. The weighting function is concave for small probabilities and convex for medium and large probabilities. This shape generates indifference curves that are convex in the southeast corner, concave in the north corner, and mixed in the south corner (Camerer & Ho, 1994), consistent with the general findings.[19] It also predicts the fanning-in patterns found by Prelec (1990) and Conlisk (1989), located in very different regions (Wu & Gonzalez, 1998). Finally, these models capture the diminished expected utility violations on the interior of the triangle (see Camerer, 1992).

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[19] It is this mixed finding in the south corner that led Camerer & Ho (1994) to conclude that there is no clear relation between the location of indifference curves in the unit triangle and their curvature. But, this is exactly because CPT leaves the curvature of indifference curves in the south corner unspecified.

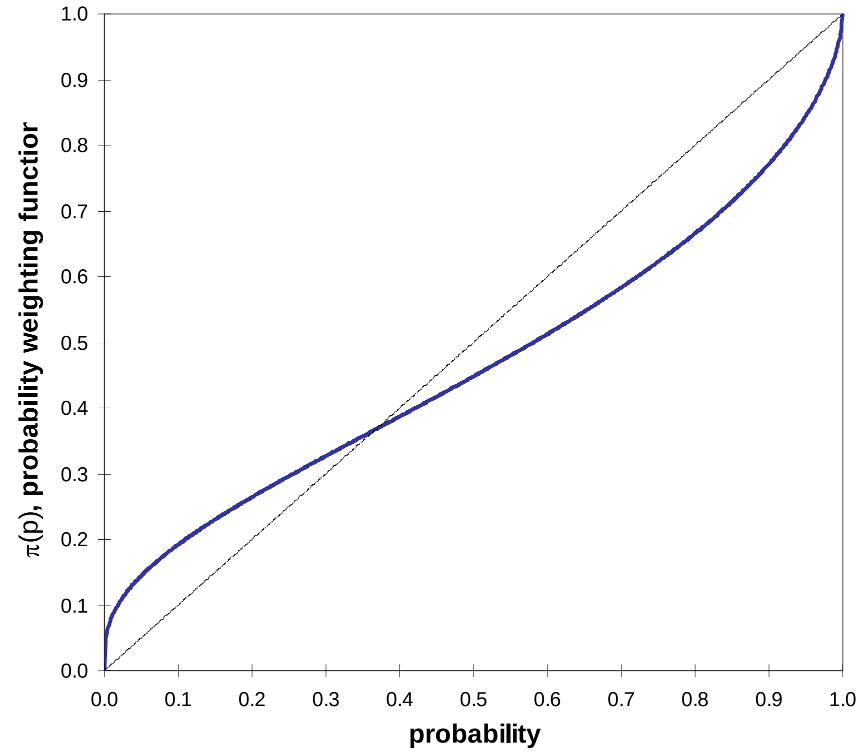

Figure 4: A characteristic probability weighting function. The function is concave for small probabilities and convex for medium and small probabilities.

Tests of regret theory

The primary support for regret theory was a unique prediction called the *juxtaposition effect*: preferences between two prospects displayed in an action/state matrix depend on the way the outcomes of the two prospects are juxtaposed. Participants in one experiment (Starmer & Sudgen, 1989) chose between two lotteries, the outcomes of which were contingent on the particular card they drew from a deck of cards numbered from 1 to 100 (Starmer & Sudgen, 1989, p. 165).

Question 1

	1 80	81 86	87 100
A	0	0	£11
B	0	£7	£7

Question 2

	1 20	21 34	35 100
A′	0	£11	0
B′	£7	0	0

Notice that A and A' have the same distribution of outcomes, as do B and B'. Thus EU requires that individuals should choose either both A and A', or B and B'. Regret theory, on the other hand, predicted that people might also choose B and A', a prediction confirmed by Starmer & Sugden.

The juxtaposition effect was documented by several studies (Loomes & Sudgen, 1982, 1987) but was later found to be driven by *event-splitting* (Starmer & Sudgen, 1993; Humphrey, 1995). Event-splitting occurs when a $p+q$ chance of receiving $x>0$ is split into a p chance at receiving x and a q chance at receiving x. Since it can lead to a violation of dominance, it is not compatible with rank-dependent utility. A second bit of evidence supporting regret theory was the demonstration of a particular intransitive preference pattern predicted by the theory (Loomes, Starmer & Sudgen, 1989). This pattern was found even when the event-splitting effect is controlled for (Starmer & Sudgen, 1998).

Two aspects of regret theory need to be pointed out. First, the effect seems to be very sensitive to the representations of gambles. For instance, Harless (1992) found that the regret effects found in previous studies using matrix displays disappear when a "strip display" is adopted (see also Humphrey, 2001; Starmer & Sudgen, 1998). As Starmer & Sudgen (1998) acknowledge, "…it seems that [regret] … comes into play only when decision problems are framed in ways that make within-event, across-act comparisons particularly salient." Second and more importantly, regret theory maintains the independence axiom and linear probabilities, thus is unable to explain the common consequence effects reported in Tversky & Kahneman (1992) that involve uncertainty, mixed fanning along both the bottom edge and the left edge (Wu & Gonzalez, 1998), and the common-ratio effect when the certainty effect is absent.

Refinements of prospect theory

In 1992, Tversky & Kahneman proposed cumulative prospect theory.[20] The new prospect theory used the same basic building blocks as original prospect theory, a value function, defined over gains and losses, and a weighting function that captured probability distortions. The major technical innovation was to use the rank-dependent form to extend prospect theory to an arbitrary number of outcomes and to uncertainty as well as risk. For risk, a separate rank-dependent transformation was applied to the gain and loss portions of a prospect. The weighting function for gains and losses is also possibly sign-dependent. For uncertainty, CPT used a related model that had been developed for uncertainty, Choquet Expected Utility (Gilboa, 1987; Schmeidler, 1989).

The new theory unified the basic shape of the value and weighting function along one psychophysical principle. Consider the probability weighting function depicted in Figure 4. This particular form of the weighting function explains the betweenness violations as well as the fanning patterns discussed in the last section. For outcomes, concavity of gains and convexity of losses reflects diminishing sensitivity away from the reference point of 0. Concavity of the weighting function for small probabilities and convexity of the weighting function for large

[20] Kahneman and Tversky (1979) wrote: "The extension of equations … to prospects with any number of outcomes in straightforward." (p.288) Kahneman (2000) later wrote: "(O)ur hopeful statement…turned out to be very optimistic. We got to it some thirteen years later … and the extension was not at all straightforward."

probabilities reflects the same principle applied to different reference points: diminishing sensitivity away the boundaries of 0 (impossibility) and 1 (certainty). Consider three individuals endowed with a 0 chance to win *x*, a .33 chance to win *x*, and a .99 chance to win *x*. How might these three individuals view a .01 chance improvement in the chance of winning? An inverse S-shaped weighting function suggests that individuals are most sensitive to changes near the extremes and relatively insensitive to changes in the middle. Thus, the individuals endowed with a 0 chance to win, and a .99 chance to win will view the change much more favorably than the person with a .33 chance to win. In weighting function terms, $\pi(.01)-\pi(0)>\pi(.34)-\pi(.33)$, and $\pi(1)-\pi(.99)>\pi(.34)-\pi(.33)$.

Tversky & Kahneman (1992) presented a comprehensive empirical test of the model. Subjects provided cash equivalents for a number of gambles, differing in the probability and magnitude of the highest outcome, and involving gains and losses. The vast majority of subjects exhibited the four-fold pattern of risk attitudes. A parametric regression analysis of the cash equivalents produced a value function of the form of Figure 1, and a weighting function of the form of Figure 4. Many other studies, using a variety of methodologies, find a similar inverse S-shape (*e.g.*, Abdellaoui, 2000; Bleichrodt & Pinto, 2000; Camerer & Ho, 1994; Tversky & Fox, 1995). The weighting function is found to intersect the identity line somewhere between $.30<p<.40$. It is noteworthy that Kahneman & Tversky's original common-consequence effect demonstration used values that approximately maximize the size of the EU violation.[21]

Wu & Gonzalez used a common-consequence schema to produce a non-parametric trace of the weighting function. They started with a choice between (p,x) and (q,y), where $x>y$. Both gambles were improved in increments by the common consequence (r,y), with r increasing throughout the probability range. Consistent with an inverse S-shaped weighting function, they find the percentage of subjects choosing the risky gamble increases and then decreases. For example, 38% of subjects preferred (.05,$240) to (.07,$200), 65% preferred (.05,$240;.30,$200) to (.37,$200), and 39% preferred (.05,$240;.90,$200) to (.97,$200).

There have been numerous efforts to parameterize the weighting function. Tversky & Kahneman (1992) assumed a one-parameter form of the weighting function, $\pi(p)=\dfrac{p^{\gamma}}{\left((p^{\gamma}+(1-p)^{\gamma}\right)^{1/\gamma}}$, and fit this form to cash equivalent data. The fitted form had the characteristic inverse S-shape, with a crossover point of around $p=.39$. Other exercises have produced similar parameter estimates, even though these exercises varied considerably in terms of the data used (choice versus cash equivalent) estimation techniques (nonlinear regression of cash equivalents versus fitting stochastic choice functionals) (see Abdellaoui, 2000; Camerer & Ho, 1994; Tversky & Fox, 1995; Wu & Gonzalez, 1996).

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[21] To see this, consider the basic common-consequence schema used. One choice consists of a preference for a sure thing, (*y*), over a risky gamble $(p,x;q,y)$. To create the second pair, replace r chance at y with r chance at 0 to get $(1-r,y)$ and $(p,x;q-r,y)$. The standard preference yields $1-\pi(p)>\pi(1-r)-\pi(p-r)$, which is large when $1-r$ is near the weighting function's inflection point.

More recently, Gonzalez & Wu (1999) estimated a two-parameter weighting function to median data and individual subject data, $\pi(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma} + (1-p)^{\gamma}}$. The estimated function for median data resembled previous analyses, but there was considerable heterogeneity at the level of individual subjects. This weighting function has two parameters to capture the relative flatness or steepness of the weighting function ("curvature" through γ) and the relative level ("elevation" through δ) of the weighting function. Adding a second parameter did not appreciably improve the fit for the median data, but both parameters were needed to model the heterogeneity of individual subjects. Although all 10 subjects had inverse S-shaped weighting functions, there was considerable variation along the two dimensions. While the majority of subjects exhibited sub-certainty ($\pi(p) + \pi(1-p) < 1$), some subjects exhibited super-certainty ($\pi(p) + \pi(1-p) > 1$). Some subjects had weighting functions that were close to the identity line, while others had weighting functions that approximated step functions.

The two-parameter weighting function has also proved useful for understanding how literally the reflection effect holds. Tversky & Kahneman (1992) estimated weighting and value functions for gains and losses. The value function had the same coefficient for both gains and losses, $v(x) = x^{.88}$. The weighting function had nearly identical coefficients for gains and losses, $\gamma = .61$ for gains and $\gamma = .69$ for losses, leading Tversky & Kahneman to conclude that: "the weighting functions for gains and losses are quite close, although the former is slightly more curved than the latter." In a different study using a very different elicitation methodology, Abdelloui (2000) estimated nearly identical parameters, $\gamma = .60$ for gains and $\gamma = .70$ for losses. However, Abdellaoui also fitted the two-parameter function used by Gonzalez & Wu (1999) and found significant differences between losses and gains: the weighting function was significantly more elevated for losses ($\delta = .84$) than gains ($\delta = .65$), whereas the curvature parameters were nearly identical. Similar results have been found by Abdellaoui, Vossmann, & Weber (2002). Indeed, a re-analysis of Tversky & Kahneman's (1992) data found the same sign-dependence in elevation. We used Tversky & Kahneman's (1992) median data (Table 3) and assumed the two-parameter weighting function and $v(x) = x^{.88}$ for gains and losses. A nonlinear regression produced estimates of $\delta = .79$ and $\gamma = .60$ for gains and $\delta = .88$ and $\gamma = .67$ for losses.

Extension to Uncertainty

The Allais and Ellsberg Paradoxes led researchers to treat decision under risk and decision under uncertainty differently for many years. More recent evidence suggests a unification of these results. Many of the principles underlying decision under risk apply directly to decision under uncertainty. For example, Tversky & Fox (1995) tested two conditions, lower- and upper-subadditivity conditions, for both risk and uncertainty. Roughly, these conditions can be thought of as capturing the possibility and certainty effects. They found strong support for lower- and upper-subadditivity in both domains, but more subadditivity for uncertainty than risk. The main reason that individuals were less sensitive to uncertainty than risk is that probability judgments were subadditive as well, consistent with support theory (Tversky & Koehler, 1994). This led Tversky & Fox (1995) (see also Fox & Tversky, 1998) to suggest a two-stage model, in which an uncertain prospect (E,x) is valued by $W(E)v(x)=\pi\left(p(E)\right)v(x)$, where $\pi(\cdot)$ is the probability weighting function for risk, and $p(E)$ is the subjective probability of E.

Tversky & Kahneman (1992) generalized the common consequence effect from risk to uncertainty and found qualitatively identical results. More recently, Wu & Gonzalez (1999) extended concavity and convexity conditions from risk to uncertainty and found a nearly identical U-shaped pattern of risk preferences as Wu & Gonzalez (1996). Thus, it seems that the same general principles apply to risk and uncertainty. Below, however, we discuss some ways that risk and uncertainty differ.

Loose threads

In the last ten years, a relatively clear picture of risky decision making and prospect theory has emerged. Violations of expected utility are robust and systematic, and prospect theory seems to explain these violations with the most ease. Basic properties of the value and weighting function, qualitatively as well as quantitatively, can organize these violations, and these basic principles readily extend from risk to uncertainty. Parimonious parametric forms of prospect theory fit choice data well, at both the aggregate level and at the level of individual subjects.

However, the picture is somewhat incomplete, as some parts of prospect theory have received very little empirical attention. We highlight some of these "loose threads" in this section.

<u>Mixed gambles</u>

There has been little research on mixed gambles. This is particularly surprising since most real world gambles involve some possibility of gain and some possibility of loss, at least relative from the status quo. Prospect theory and other bilinear models have problems explaining some of the mixed gamble data collected to date (Chechile & Butler, 2000, 2003). We would like to see more data of this sort collected, particularly data collected to test axioms that permit separability between gains and losses, such as Tversky & Kahneman's (1992) "double matching".

<u>Loss aversion</u>

One of the most striking features of prospect theory is its treatment of gains and losses. Loss aversion suggests that "losses loom larger than gains". In other words, the disutility of a loss, $-v(-x)$, is greater than the utility of a comparable gain, $v(x)$. Loss aversion has been used to explain many real world phenomena (Camerer, 2000). It is thus somewhat surprising that there is very little direct empirical evidence on loss aversion, at least compared to the probability weighting function and the value function.

The common wisdom is that losses loom twice as large as gains. One demonstration comes from riskless decision making, Kahneman, Knetsch, & Thaler's (1990) demonstration of the endowment effect. Students endowed with a mug demanded twice as much to sell their mug as buyers were willing to pay. Tversky & Kahneman (1992) fit a full prospect theory choice model to risky choice data, and estimated a median loss aversion coefficient of 2.25, very close to the loss aversion coefficient inferred from the endowment effect experiments. Beyond that, we know of only one other study. Schmidt & Traub (2002) measured loss aversion, while controlling for curvature of the value and weighting function and allowing these functions to be different for losses and gains and across subjects. They found mixed evidence for loss aversion. A slight majority of subjects were loss averse, but about 40% of subjects were classified as loss seeking. The mixed results may reflect the complicated procedure needed to control for the value and weighting function. The procedure involves several steps, each of which introduces noise.

Clearly, more empirical evidence is needed. How robust is loss aversion at the level of individual subjects? Is loss aversion different for non-monetary attributes, such as health outcomes? Is loss aversion different in dynamic decision situations, such as with the house money effect (Thaler & Johnson, 1990)?

<u>Composition Rules</u>

Tversky & Kahneman (1992) introduced cumulative prospect theory as an advance from original prospect theory. Clearly, it is so in many respects. However, the overall evidence is mixed about which prospect theory does better in explaining the empirical results. The two prospect theories are identical for prospects with one non-zero outcomes, and thus provide the same explanation for the common-ratio effect. The models diverge for more complicated gambles. Both models can explain the original common-consequence violations, as well as generalizations of the common-consequence violations (*e.g.*, Wu & Gonzalez, 1996, 1998), albeit with different restrictions on the weighting function.

Beyond that, there are some patterns that cannot be explained by CPT (Wu, 1994), and some patterns that CPT fits better than OPT (Fennema & Wakker, 1997; Levy & Levy, 2002; Wakker, 2003). In goodness of fit tests using particularly parametric forms of the value and weighting function, the pattern is mixed. OPT sometimes fits aggregate data better (Camerer & Ho, 1994; Wu & Gonzalez, 1996), but CPT fits particular patterns better (Wu & Gonzalez, 1996).[22]

More recently, Gonzalez & Wu (2003) used parameters estimated from 2-outcome gambles, where CPT and OPT coincide, to predict 3-outcomes gambles, where the models diverge. Neither model did particularly well predicting the cash equivalents from the 3-outcome gamble holdout sample. The direction of the misprediction is particularly instructive. OPT tends to overpredict and CPT tends to underpredict 3-outcome gamble cash equivalents. To see why, consider a gamble, $(p, x; q, y)$. The decision weight for y is $\pi(q)$ for OPT and $\pi(p+q)-\pi(p)$ for CPT. If the weighting function, $\pi(\cdot)$, is subadditive, $\pi(p)+\pi(q) \geq \pi(p+q)$, then the decision weight for OPT will exceed that for CPT.

[22] Birnbaum & McIntosh (1996) and Birnbaum, Patton, & Lott (1999) present choice patterns that cannot be explained by CPT, although some can be explained by OPT.

This example illustrates one possible price incurred by rank-dependent models for its mathematical elegance and dismissal of stochastic dominance violations. Under CPT, the relative difference between the evaluation of (p, y) and $(p+\varepsilon, y)$ has to be the same as that (p, x) and $(p, x; \varepsilon, y)$. Stochastic dominance violations are prohibited by imposing some outcome continuity: $\lim_{\varepsilon \to 0}(p, x+\varepsilon; q, x) = (p+q, x)$. Note that OPT does not satisfy this outcome continuity. Outcome continuity, also called coalescing, appears to be violated empirically (Birnbaum & Martin, 2003; Luce, 1998). The reverse operation, breaking an outcome into two pieces, event-splitting, increases the valuation of the gamble (Starmer & Sudgen, 1993; Humphrey, 1995). Some other surprising implications of OPT have been validated empirically (Starmer, 1999). Finally, Wu, Zhang, and Abdellaoui (2004) adapt Abdelloui's (2002) tradeoff consistency conditions to create a critical test of the two prospect theories. They find that CPT is violated for critical test gambles that do not involve a sure thing, while OPT is violated for gambles that involve a sure thing.

Which composition rule is better is an open question. There may be no general answer. The answer probably ultimately depends on a number of factors, including the components of the choice set (Stewart *et al.*, 2003), and whether editing rules can be applied. We discuss some of these issues below.

Future Research Agenda

We close by suggesting some fertile research areas. Some of these research areas, like editing operations, have been open for some time now but largely neglected, while new and provocative findings in others areas have just recently emerged. This chapter has taken you through a tour of the many phases in the history of risky decision making research. We hope that this section provides a preview of what the future has in store.

Economic Implications

Camerer (2000) identified ten real world applications of prospect theory to economic phenomena. Some come from finance (*e.g.*, the equity premium puzzle and the disposition effect), some from consumer behavior (*e.g.*, purchase of telephone wire insurance and state lottery tickets), and some from labor economics (*e.g.*, income targeting for taxi drivers). These applications evoke different features of prospect theory: some use the value function, some the weighting function, and some both functions.

We suspect that this is a mere preview of more to come. Economics and finance are importing psychological theories and models at an increasing rate. This trend includes research that uses these models to generate more general theoretical predictions (Barberis, Huang, & Santos, 2001), as well as research that uses functional specifications in more flexible structural models (Benartzi & Thaler, 1995; Jullien & Salanie, 2000; see, also, Barberis & Thaler's (2003) review of behavioral economics). The parametric work described above offers parsimonious specifications of prospect theory (three parameters: one for the value function, one for the weighting function, one for loss aversion).

Psychological Factors

In the past 50 years of research on decisions involving risk and uncertainty, many alternative models to EU have been proposed. We expect this to continue. We have highlighted the constructive interplay between different theoretical models and empirical tests. While a rich set of empirical findings emerged in the 15 to 20 years after prospect theory, the field sometimes lost sight of the psychological principles underlying decision under risk. Schoemaker (1982, p. 554) cautioned, "…modifications to economic theory … should be based on *cognitive insights* [emphasis added] … rather than ad hoc rationalization or mathematical expedience." To push Schoemaker's point further, we suggest that besides cognitive factors, there is a need to consider the role of motivational, emotional and affective factors. Indeed, we have already seen new and exciting research in this area. In this section, we will briefly review evidence suggesting the important role these factors may play.

Psychological insights are needed at two different levels that reflect different objectives researchers may have. First, psychological insights may help better organize the basic violations of EU and involve choices between simple gambles that are presented in very generic forms. We have reviewed these ideas at some length. Moving forward, additional psychological factors may need to be considered when we apply the model to more complex situations or situations that more closely resemble real life decision problems.

<u>Cognitive factors</u>

Two cognitive principles that have been evoked in understanding risky decision making are perception and information processing.

Prospect theory has already incorporated several important features of our perceptual system. For instance, people are more sensitive to extreme stimuli and stimuli that are closer to reference points. This idea has been incorporated into the probability weighting function, the value function, and the composition rule. Information processing is also reflected in the model through the coding and editing stages. However, so far there has been relatively little research of this last type (see section on editing below).

Perception and information processing both suggest that decisions may be sensitive to the particular way a problem is represented. Indeed, a number of representation effects—context, framing, and information display—have been widely documented in the existing literature. For instance, Moskowitz (1974) found that the degree of EU violations depends on how common consequence problems are displayed (in a decision tree, matrix, or verbally). Similarly, Keller (1985) found similar sensitivity to visual representations: certain visual representations encourage the use of cancellation and hence decrease the incidence of EU violations. However, although EU violations were reduced with some representations, EU violations were in general quite robust.

Some theories demonstrate their validity and power precisely by predicting particular representation effects. Indeed, prospect theory predicts a wide range of representation effects. For instance, the famous Asian disease problem (Tversky & Kahneman, 1981) exploits framing and the reflection effect; the pseudo-certainty effect (Kahneman & Tversky, 1979) exploits isolation and the certainty effect; the violation of dominance (Tversky & Kahneman, 1986) exploits subadditivity of the probability weighting function.

Contrasting perception and information processing, the former is passive and the latter active. What kind of approach dominates in particular situation is not entirely clear. On the one hand, Tversky & Kahneman (1986) found that individuals often treat decision problems in isolation and do not relate them to other surrounding factors, and Slovic (1972) found that people tend to take decision problems as presented. On the other hand, the house money and break-even effects (Thaler & Johnson, 1990) suggest that people actively frame some decision problems (see also, Thaler, 1999).

In summary, representation effects are abundant, yet we lack a comprehensive theory to organize them. Specifically, we need to answer three questions: a) how do different frames influence people's choices; b) when do people engage in active reframing; and c) how do people reframe a decision problem.

Emotional, affective, and motivational factors

Recently, research on the impact of affective and emotional factors on risky decisions has emerged. Some of these are within the framework of prospect theory (Brandstatter, Kuhberger, & Schneider, 2002; Rottenstreich & Hsee, 2001) while some are not (Mellers *et al.*, 1999).

Mellers *et al.* (1999) proposed *decision affect theory* (also Mellers *et al.* 1997) as an alternative to EU. Decision affect theory posits that decision makers consider anticipated emotions, such as elation and disappointment, in addition to the utility of outcomes. In particular, decision makers compare the outcome received to what they would have received had they chosen the other alternative (regret) and to what they would have received had the uncertainty been resolved differently (disappointment). Thus, decision affect theory can be seen as a generalization of regret and disappointment theories (Bell, 1982, 1985).

Mellers *et al.* found that ratings of anticipated emotional responses show elation and disappointment effects, and that these ratings predict choices. However, the experiment is set up in a way that makes salient the comparisons in question and thus might amplify regret and disappointment effects. Moreover, it is not clear how to compare the predictive power of decision affect theory over prospect theory, since decision affect theory uses affective ratings to predict choices, while prospect theory uses choices to predict choices.

Brandstatter *et al.* (2002) proposed a similar idea, but invoked a very different kind of theorizing. They suggested an affective decomposition of the probability weighting function: overweighting of small probabilities and underweighting of large probabilities is due to anticipated elation and disappointment. The smaller the probability of winning, the more elation winning will deliver. Likewise, the larger the probability of winning, the more disappointment will result from not winning. Although their story is appealing, they do not show a direct link between surprise and the weighting function. Do individual differences in the weighting of elation and disappointment correlate with differences in elevation of the weighting function, as their theory would suggest?

These two approaches attempt to show the mediating role of anticipated affect. Although evocative, future research along this line should address some important unresolved issues. First, the link between anticipated affect and decisions need to be formalized. Second, the gain in explanatory power of these models remains to be shown, whether in predicting new violations of EU or improving how well these models predict general behavior.

Rottenstreich & Hsee (2001) took a different approach to incorporating affective and emotional factors into prospect theory. They documented an interesting relationship between the affect richness of outcomes and the probability weighting functions. In one study, most subjects preferred $50 to a kiss from their favorite movie star, but preferred a 1% chance of the kiss to a 1% chance of $50. They argue that small probabilities are more overweighted for affect rich outcomes such as the kiss. In another study, they found that the certainty effect is also stronger for affect rich outcomes, suggesting a weighting function that is more "pinched" at both 0 and 1 for affect rich outcomes.

The full impact of affect richness on the probability weighting functions needs to be spelled out more clearly in the future. For instance, is the documented interaction working through more overweighting and underweighting of probabilities or simply less sensitivity to changes in probabilities? More generally, does affect always work exclusively through curvature of the weighting function, rather than changes in elevation?

Lopes' aspiration level (SP/A) theory (1987) tried to capture the impact of motivational factors on risky choice. The theory has two pieces: aspiration level and mindset (security-minded or potential-minded). The aspiration level seems to mainly influence how outcomes are coded, thus could be potentially modeled through prospect theory's value function. The second piece, an individual difference factor, mainly influences the attention people pay to the best or the worse outcomes, and thus can be modeled through weighting functions. (See Lopes & Oden, 1999, for an exercise of this sort.)

According to Lopes' (1987), aspiration level could be due to cognitive factors, such as the choice set or expectations about the outcomes of a gamble, or to outside motivational factors. For instance, they found that people who are behind tend to choose riskier gambles in the latter rounds of betting. Thus, aspiration level provides an alternative account for the house money and break-even effects.

Editing rules

The original prospect theory representation has some well-known problems. For example, the value of $(p, x; q, x-\varepsilon)$ may exceed $(p+q, x)$, even though the second gamble stochastically dominates the first gamble. To avoid this problem, Kahneman & Tversky proposed that subjects scan "offered prospects to detect dominated alternatives, which are rejected without further evaluation." More generally, Kahneman & Tversky (1979) proposed an editing phase, which was designed "to organize and reformulate the options so as to simplify subsequent evaluation and choice" (p. 274). They proposed six operations: coding, combination, segregation, cancellation, simplification, and detection of transparent dominance. Discussion of editing does not appear until the conclusion of Tversky & Kahneman's (1992) revision of prospect theory, and it is easy to conclude that rank-dependent representation obviates the need for editing operations. However, Tversky & Kahneman conclude with an apt quote:

> Theories of choice are at best approximate and incomplete. One reason for this pessimistic assessment is that choice is a constructive and contingent process. When faced with a complex problem, people employ a variety of heuristic procedures in order to simplify the representation and evaluation of prospects. These procedures include computational shortcuts and editing operations, such as eliminating common components and discarding nonessential differences (Tversky, 1969). The heuristics of choice do not readily lend themselves to formal analysis because their application depends on the formulation of the problem, the method of elicitation, and the context of choice. (p. 317)

We agree with this sentiment. Although the study of editing operations is likely to be considerably less tidy than other aspects of prospect theory, we believe that important questions remain.

Almost no research exists on how decision makers code and represent gambles. Research on managerial decision making has found that managers tend to focus on the best and worst outcomes (the "upside" and "downside") with almost no attention to the probability of the outcomes (March & Shapira, 1987). Beyond this, decision makers may use operations to simplify gambles. It is not well-understood when individuals use within-gamble operations such as combination, or across-gamble operations such as cancellation. Some theories that evoke similarity as an across-gambles operation exist (*e.g.*, Leland, 1998; Rubinstein, 1988), but a more comprehensive theory of editing is out-of-sight at the moment. We suspect that this sort of theorizing is probably the wrong strategy. Instead, we hope that researchers document violations of expected utility that can be plausibly explained by some editing operation (Wu, 1994). When an ample set of findings have been assembled, it might then be possible to build a more comprehensive theory of editing.

It should be noted that editing operations complicate the testing of axioms. Suppose, for example, that a researcher finds that $(p,x;q,y) \succ (p,x;r,z)$ if and only if $(q,y) \succ (r,z)$. Is this evidence for some version of the independence axiom, or does it merely show that subjects obey an editing operation, cancellation, when cancellation is transparent? See, for example, Wakker, Erev, & Weber (1994).

Biological Underpinnings

Pleasure and pain are at the heart of all utility theories. The specific form of how positive and negative evaluations are made, and what carries these utilities, varies across normative and descriptive theories. Interestingly, there is a strong tradition in biological psychology to examine the neural underpinnings of pleasure and pain. Except for a handful of studies there has not been much collaboration between biopsychologists and decision researchers. Recent studies have used brain imaging techniques (such at PET, fMRI, ERP) to examine activation patterns in the human brain while the brain processes decisions under risk and uncertainty. The results are preliminary at this point and not always consistent across laboratories. The stylistic result is that there appears to be localization in the form of hemispheric bilateralism with pleasure typically coded in the left prefrontal cortex and pain coded in the right prefrontal cortex (*e.g.*, Davidson & Irwin, 1999). A particular structure in the brain, the anterior cingulate, which is implicated in general cognitive tasks such as planning and executive control, has also been implicated in the expectation of reward. The ERP signal from the anterior cingulate has been shown to have a higher amplitude in the domain of losses than in the domain of gains (Gehring & Willoughby, 2002). Berridge (2003) has recently distinguished two biological systems that correspond to the behavioral manifestations of liking and wanting, which may make connections to new thinking in utility theory about different types of utility (Kahneman, Wakker, & Sarin, 1997). There has also been some promising behavioral work on animals attempting to test key properties of expected utility maximization in pigeons, rats, and bees (Kagel, Battalio, & Green, 1995; Real, 1991, 1996), suggesting that violations of expected utility may be quite general.

The examination of biological underpinnings may provide deeper insight into the separation of gains and losses, especially if it is shown that different neural systems in the brain process gains and losses. Such results may also illuminate some behavioral observations such as "losses loomer larger than gains". At the very minimum, the biological approach offers a new array of paradigms and dependent variables that may make it easier for behavioral researchers to test their research questions. The new brain imagining techniques provide a window into the brain while it does what it does. For many cognitive psychologists, the ability to trace blood flow in the brain or to examine the electrophysiological signals produced by collections of neurons provide an opportunity to monitor the brain while it works—in a sense, "to look under the hood while the engine is running". However, it still is not clear what these signals and dependent variables mean. The handful of decision making studies that have used these new technologies have many methodological problems and in some cases involve a limited behavioral paradigm (*e.g.*, participants in the Breiter *et al*. (2001) study did not make decisions).

Source Preference

How does the evaluation of a gamble that pays $100 with .7 chance differ from the evaluation of a prospect that offers $100 if the Yankees win tomorrow's game? The difference between these gambles captures the difference between risk and uncertainty. The research reviewed above suggests that these two choices are qualitatively similar. However, we suggest that there are some differences. Of course, uncertainty is more complicated than risk: decision makers faced with the sports bet must assess the likelihood of a Yankee victory. Tversky & Fox's (1995) two-stage model suggests that individuals judge the probability of a particular event and then transform this judgment via a probability weighting function. Under this simple form, a decision maker who judges the likelihood of a Yankee win to be .7 will value the risky gamble and the uncertain gamble the same.

Although the two-stage model is a useful simplification, it fails systematically in some situations. Decision makers may prefer to bet on one source over another, even when the subjective likelihood is equated for the two sources. The clearest example is the Ellsberg Paradox (Ellsberg, 1961). In the two-urn problem, most subjects assign a probability of .5 to both of the two urns, yet nevertheless prefer betting on the objective .5 to the subjective .5 (see Camerer & Weber, 1992, for a review).

Numerous empirical studies have demonstrated some sort of source preference. Heath & Tversky (1991) found that individuals prefer to bet on domains in which they felt particularly competent to domains in which they felt less competent, even when subjective probabilities for the two domains were matched. Fox & Tversky (1995) found that the Ellsberg Paradox was reduced or disappeared in between-subject tests. Evidently, subjects are not averse to ambiguity *per se*, but only when they feel comparatively ignorant (see also, Frisch & Baron, 1988). More recently, Kilka & Weber (2001) measured the degree of source preference directly. German students valued prospects based on a familiar source, the price of Deutsche Bank, a German bank, and an unfamiliar source, the price of Dai-Icho Kangyo Bank, a Japanese bank. The weighting function for the familiar source was significantly more elevated than the weighting function for the more unfamiliar source.

The few studies to date suggest that source dependence acts on the elevation of the weighting function, rather than on the curvature. Illusion of control (Langer, 1975) can be seen as working through elevation of the weighting function. We suspect, however, that there may be effects that work through curvature of the weighting function as well. A decision maker who does not feel particularly knowledgeable about a source, such as politics, may judge one event to be more likely than another, but may attach the same value to a gamble based on the first event compared to a gamble based on the second event. Thus, we hypothesize a flatter weighting function for sources in which subjects feel comparably ignorant.

Source preference complicates matters in one other respect. Decision theorists have assumed that beliefs can be inferred from actions (Ramsey, 1931). For example, suppose that you are indifferent between a risky bet, $100 with a .7 chance, and an uncertain event, $100 if the Yankees win tomorrow's game. Under the standard interpretation, indifference means that you judge the probability of the event in question to be .7. However, a decision-based definition of probability is elusive if decision makers prefer to bet on one source over another (Wakker, 1994).

Intertemporal Effects

Sagistrano, Trope, & Liberman (2002) found an interesting valuation reversal for gambles that were to be played at the end of the experiment ("near") or 2 months in the future ("distant"). Subjects rated and priced gambles with varying probability levels (.1, .3, .5, .7, and .9) and expected values ($4, $6, $8, and $10). Holding expected value constant, gambles with .9 probability of winning were most attractive in the near condition, but least attractive in the distant condition. These data were interpreted in terms of a theory of "temporal construal": desirability looms larger for the distant future, while feasibility is more important for the near future.

These data are also consistent with a weighting function that is mostly flat for prospects to be resolved in the distant future, and characteristically inverse S-shaped for prospects to be resolved in the near future.[23] A weighting function of this sort was posited by Wu (1999) in the

[23] The valuation patterns are generally monotonically increasing for the near future condition, and monotonically decreasing for the distant future condition. However, the .1 probability gamble is sometimes valued higher than

context of delayed resolution of uncertainty. In that paradigm, the value of a gamble is a function of the probability distribution, as well as the timing of uncertainty resolution. Wu suggested that the weighting function becomes more categorical (flatter) as a prospect is delayed. The more general effect of delayed resolution of uncertainty on decision under risk is not known, but see Ahlbrecht & Weber (1997), Chew & Ho (1994), and Lovallo & Kahneman (2000). However, the shape of the weighting function cannot be teased apart from the shape of the value function using simple gambles like the ones used by Sagistrano *et al.* Nonetheless, it is worthwhile understanding the relationship between these two streams of research.

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the .3 probability gamble. Even a strictly monotonic pattern is consistent with an inverse S-shaped weighting function with a crossover around .3, provided that the value function is sufficiently concave (see Prelec, 1998).

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