

# WORKING PAPER

Investment With Uncertain Tax Policy:  
Does Random Tax Policy Discourage Investment?

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## Abstract

We consider the impact of tax policy uncertainty on firm level and aggregate investment, comparing investment behavior when uncertainty is due to a shock following Geometric Brownian Motion (GBM) versus when random discrete jumps in tax policy occur. Expectations of the likelihood of a tax policy switch have an important negative impact on the gain to delaying investment in the latter model and time to investment can fall with increasing tax policy uncertainty. Aggregate investment simulations indicate that capital formation is adversely affected by increases in uncertainty in the traditional GBM model but can be enhanced in the jump process model. We also find that mean preserving spreads are attractive to firms when they have discretion over real behavior. In both models, a mean preserving spread lowers the cost of capital conditional on investment as firms shift investment from high to low cost periods. Ex ante mean preserving spreads in general lead to ex post decreases in the price of capital. We relate this finding to the more general literature on mean preserving spreads.

## I. Introduction

It is often said that nothing is certain in life except death and taxes<sup>1</sup>. While death is undoubtedly certain, there is, in fact, considerable uncertainty with respect to taxes. Tax policy provides a key source of uncertainty about the cost of capital to U.S. firms, for example. The investment tax credit was first introduced in the United States in 1962, and subsequently, has been changed on numerous occasions. In the United Kingdom, major changes in tax policy have changed the cost of capital and returns to capital investment. For example, in a brief span of 7 years, the U.K. shifted from a two-tiered corporate tax system in which retained earnings were taxed at a higher level than distributed earnings to a classical corporate income tax of the type found in the United States and then to an integrated system<sup>2</sup>.

A common view is that policy uncertainty discourages investment. This view is not consistent with the predictions of most investment models, which generally indicate that uncertainty increases investment (e.g. Hartman (1972), Abel (1985)). The view that tax uncertainty harms investment depends importantly on the irreversibility assumption, and the findings that randomness in output prices retards investment (Pindyck (1988)) in such models. This paper extends the literature by considering uncertainty in tax policy. In particular, we consider changes in tax policy for investment tax credits (ITCs).

Uncertainty has typically been introduced in previous work by assuming that some parameter follows a continuous time random walk (Brownian Motion or Geometric Brownian

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<sup>1</sup> This epigram is attributed to Ben Franklin in a letter to M. Leroy (1789): "Our Constitution is in actual operation; everything appears to promise that it will last; but in this world nothing is certain but death and taxes." quoted in Bartlett (1901).

<sup>2</sup> See Poterba (1991) for details on changes in corporate taxation in the United Kingdom.

Motion). When prices follow a random walk, one's rational-expectations forecast for the price at any time in the future is today's price (perhaps adjusted for drift in the process), and the future path of the price is unbounded. Tax parameters, unlike most prices, tend to remain constant for a few years, and then change abruptly to new values. In addition, the jump that occurs is likely to be mean-reverting: When tax credits are high, they are likely to be reduced in the future; when they are low, they are likely to be increased. Finally they range between zero and one. More succinctly, the former process is non-stationary while the latter is stationary.

Given the latter jump process, there is the possibility that the firm might invest today, only to see an ITC introduced, or might delay investment today, only to see an existing ITC repealed. Armed with the knowledge of the expected frequency with which tax policy changes as well as the support of the distribution of tax parameters, firms will delay or speed up investment depending on their perceptions of the probability and magnitude of tax changes. Below, we show that this behavior is crucial to understanding the effects of uncertain tax policy. An analogy may be useful here. One concern with tax-preferred savings vehicles is that they simply shift savings from non-tax-preferred accounts to tax-preferred accounts with no change in aggregate savings. Random tax credit policy raises the same issue: is aggregate investment altered by changes in the frequency or level of credits, or is simply the timing of investment changed as firms shift investment from low to high tax credit periods? In addition, we consider the impact of fluctuating tax policy for government revenue.

These observations on actual tax policy behavior suggest that a Poisson jump process may generate useful insights on the impact of tax policy uncertainty. One set of parameters of a Poisson jump process give the expected duration of a tax policy state but not the actual duration.

Data over the past 30 years for U.S. corporations can help pin down those parameters. For example, Cummins, Hassett and Hubbard (1994) provide a review of post-war U.S. tax changes. Since 1962, the mean duration of a typical state in which a specific ITC is in effect is 3.67 years. The mean duration of the "no-ITC" state is 3.00 years. We will use these average durations later to provide benchmark values of the Poisson jump process parameters. The second set of parameters provide information about the magnitude of the jump among states. Our focus in this paper is on changes in this second set of parameters.

To date, there has been little work addressing the issue of investment behavior and tax policy uncertainty<sup>3</sup>. That tax policy is uncertain is not a new concept; indeed, the notion that investment tax credits may randomly switch on and off was a key argument in Lucas (1976). More recently, Auerbach and Hines (1988) attack this same problem in a discrete-time model in which there is a probability each year that tax policy (an investment tax credit or depreciation allowances) will change. They obtain a tractable solution by making linear approximations around steady-state values of the capital stock. This turns out to be a critical simplification as the use of a first order approximation around the steady state means that the information in the second moment of the distribution of tax policies is eliminated. Thus mean preserving spreads of the distribution of the random tax variable will have no effect on the measures of effective tax rates that they construct.

Bizer and Judd (1989) develop and solve numerically a general equilibrium model that includes random taxation. In this closed-economy model, investment equals saving, and saving

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<sup>3</sup> Jack and Viard (1996) have considered an investment model with temporary tax incentives. But there is no uncertainty over future tax policy in their model.

follows from utility maximization. Thus the results they derive, that fluctuations in output attributable to random taxation lower welfare, are intrinsically related to the curvature of the utility function. In this paper, we follow Pindyck (1988) and Abel (1983) and focus on the investment decision of producers. As a result we are unable to draw the sorts of welfare conclusions that Bizer and Judd do, but we can specify investment behavior more precisely building on an extensive literature on investment behavior.

To be specific, we consider random policy toward an investment tax credit for new capital and investigate the impact on investment of a mean preserving spread.<sup>4</sup> We consider the following question: if a country passed a constitutional (and irrevocable) ban on changes in the investment tax credit, would investment go up or down? An alternative question is whether government can manipulate uncertainty in some revenue neutral way to increase investment (in effect benefitting from a policy free-lunch)? We argue below that such a policy free-lunch is not available and that increases in uncertainty of a mean preserving type will inevitably result in lost tax revenue for the government. In effect, a subsidy will be provided to investment. This follows because firms always have the option to decide *when* to invest. Increasing variation in investment costs brings with it the opportunity to wait out high cost periods and invest in low cost periods. This is the first key finding in our paper. While we develop the argument in the context of tax policy, we argue that the result is more general.

In addition, we find that the impact of tax policy uncertainty on investment depends importantly on whether the stochastic process is mean stationary or not. Using a model of tax

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<sup>4</sup> Focusing on mean preserving spreads follows in the tradition of the investment uncertainty literature (e.g. Hartman (1972), Abel (1985) and others).

policy uncertainty where the uncertainty is in the form of Geometric Brownian Motion, we find that increasing uncertainty slows down investment despite the implicit subsidy arising from the variations in tax credit. However, when tax policy is modeled as a stationary jump process, we find that increasing uncertainty can have the opposite effect. One must therefore be careful before extrapolating the findings of much of the previous literature on uncertainty and investment to the case of tax policy uncertainty.

We turn in the next section to our model of investment and uncertainty. In Section III we present simulations of microlevel investment behavior. We move to simulations of aggregate investment in Section IV. Finally, we conclude in Section V.

## II. Investment with Tax Policy Jumps

We begin with a simple model in which firms choose an optimal rule for when to undertake a project. The amount of capital employed in the project ( $K$ ) and the time at which the project is initiated are at the discretion of the investor. The capital can be used to produce  $F(K)$  units of output per unit of time forever which can be sold at price  $p_t$  at time  $t$ . The production function has the standard properties ( $F' > 0$ ,  $F'' < 0$ )<sup>5</sup>. The price  $p_t$  is an after tax return and is modeled as stochastic. We assume it follows Geometric Brownian Motion:

$$(1) \quad dp_t = \mu_p p_t dt + \sigma_p p_t dz_p$$

where  $dz_p$  is an increment to a Wiener process. The return follows a continuous time random

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<sup>5</sup> Decreasing returns to scale is a necessary but not sufficient condition on the production function. Second order conditions will place the additional restriction on the production function that the ratio of the average to marginal product must increase in  $K$ . This rules out such common production functions as Cobb-Douglas. See Dixit (1993).

walk with volatility  $\sigma_p$  and drift  $\mu_p$ . We model  $p_t$  as stochastic to allow for supply shocks which affect the price level of corporate output.

We next incorporate tax incentives which reduce the cost of capital. Accelerated depreciation and investment tax credits are the two most common forms of cost reduction. As noted in the introduction, tax policy changes are poorly described by Geometric Brownian Motion; they are typically large and discrete changes; moreover the tax policy variable is stationary and bounded. We consider an investment tax credit  $\pi_t \in \{\pi_0, \pi_1\}$  which reduces the price of capital from  $p_k$  to  $(1-\pi_t)p_k$ <sup>6</sup>. The tax credit is assumed to follow a Poisson Process randomly switching between a high level ( $\pi_1$ ) and a low level ( $\pi_0$ )<sup>7</sup>. The actual duration of a particular credit level is unknown although the expected duration is known. Specifically, the tax process follows the equation of motion:

$$(2) \quad d\pi_t = \begin{cases} \Delta\pi & \lambda_{1t}dt \\ 0 & 1-\lambda_{1t}dt & \pi=\pi_0 \\ -\Delta\pi & \lambda_{0t}dt \\ 0 & 1-\lambda_{0t}dt & \pi=\pi_1 \end{cases}$$

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<sup>6</sup> Multiple values for the ITC are certainly possible. Gravelle (1994) describes the history of the U.S. investment tax credit (Appendix A) and details credit rates for numerous equipment classes at a variety of rates (Table B.3). We choose two parameter values for simplicity to focus on the stationary and bounded nature of the process.

<sup>7</sup> It could be argued that other components of the tax code (tax rates, depreciation schedules, minimum tax) give rise to a continuous, bounded process rather than either the jump process we model or Geometric Brownian Motion. In particular, mean reversion is a plausible candidate. For an analysis of investment in a world with mean reverting prices, see our paper, Metcalf and Hassett (1995).

where  $\Delta\pi \equiv \pi_1 - \pi_0 > 0$ . The tax credit randomly switches between  $\pi_0$  and  $\pi_1$  with transition parameters  $\lambda_{1t}$  and  $\lambda_{0t}$ . Since many have argued that tax policy is not exogenous, we allow for the possibility of a covariance between policy response and firm profitability. In particular, we consider a linear relation between the  $\lambda$ 's and output price:

$$(2') \quad \begin{aligned} \lambda_{1t} &= \bar{\lambda}_1 - \alpha_1 p_t \\ \lambda_{0t} &= \bar{\lambda}_0 + \alpha_0 p_t \end{aligned}$$

The probability of shifting from a low ITC to a high ITC state declines with output price while the probability of shifting from a high ITC to a low ITC state increases with output price<sup>8</sup>. The logic behind equation (2') is that government is less likely to increase an investment subsidy when output prices (and firm profits) are high. Conversely, when output prices and profits are high, government is more likely to reduce subsidies to investment.

The transition parameters are informative on a number of counts. The expected duration of a regime with a high ITC ( $\pi_1$ ) is given by  $\lambda_{0t}^{-1}$  while the expected duration of a tax regime with low ITC ( $\pi_0$ ) is given by  $\lambda_{1t}^{-1}$ . In addition, the expected fraction of the time that a high ITC will be in effect is given by  $\frac{\lambda_{1t}}{\lambda_{0t} + \lambda_{1t}}$ . This model is flexible enough to model uncertainty both as a jump process and as Geometric Brownian Motion as others have done (e.g. Pindyck (1988), Dixit and Pindyck (1994)).

Given the randomness in output price and capital costs, the firm wishes to determine the optimal rule for investment to maximize the expected discounted value of the stream of profits

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<sup>8</sup> The parameters  $\bar{\lambda}_i$  and  $\alpha_i$  are chosen to ensure that the  $\lambda$ 's are everywhere positive over the range of prices that firms will face prior to investing.

from the investment net of the cost of the investment:

$$(3) \quad V = \max_{K,T} E \left\{ \int_T^{\infty} p_s F(K_T) e^{-\rho s} ds - (1 - \pi_T) p_k K_T e^{-\rho T} \right\}$$

In effect, we are solving for the optimal stopping time ( $T$ ) as well as the level of investment conditional on stopping ( $K$ ). The rule will provide a stopping time as a function of the current values of the stochastic variables.

There are three regions in the output price space of importance: In region I ( $p < p_1$ ), there is no investment, regardless of the value of  $p$ . In region II ( $p_1 < p < p_0$ ) there is investment if the high ITC is in effect and in region III ( $p_0 < p$ ) there is investment regardless of the level of the ITC (see Figure 1). In Appendix A we describe how we solve the investment problem. In brief we solve for the value function in each region conditional on the value of the tax credit and then

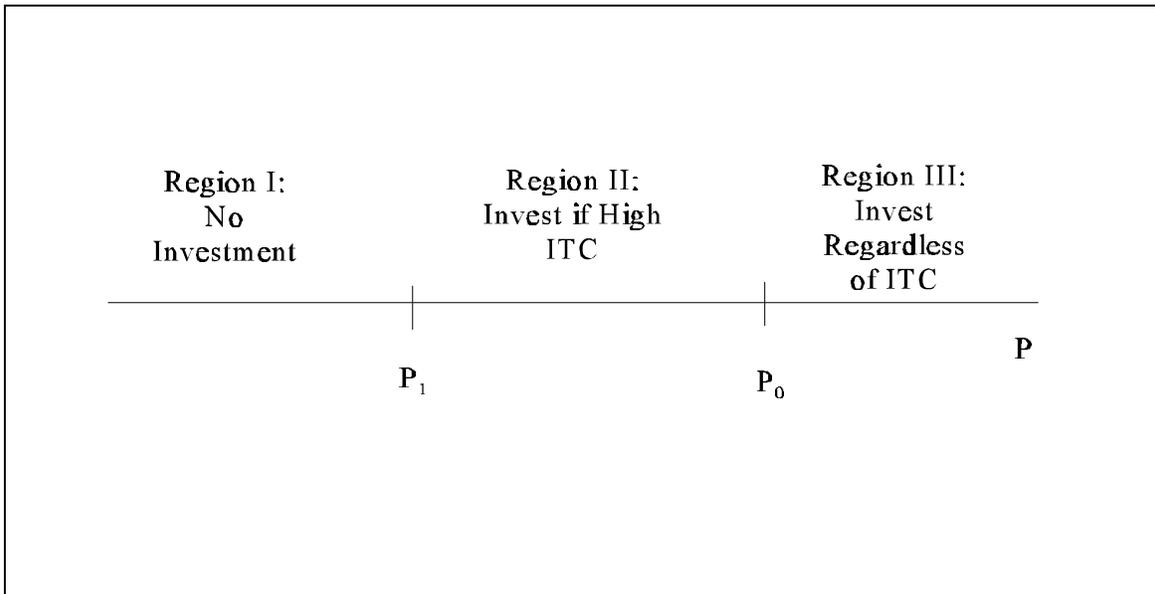


Figure 1: Investment Regions

invoke value matching and smooth pasting conditions at the boundaries between regions. Value matching conditions require that the value function be equal at a boundary while smooth pasting conditions require that the first derivatives also be equal at the boundary. This yields a system of equations that can be solved for the values of the boundary prices ( $p_0$  and  $p_1$ ) along with other variables in the system. See Appendix A for details.

At this point we could do comparative statics to determine the effects of increasing uncertainty on the trigger prices ( $p_0, p_1$ ). However, changes in trigger prices are not necessarily informative about changes in time to investment. Increasing volatility will increase the trigger price which in turn should discourage investment. On the other hand, the increased volatility raises the probability of hitting a favorably high price at which point the firm would like to invest. Whether time to investment increases or decreases is an empirical matter.

Finally, we note that we focus on increasing uncertainty in the form of mean preserving spreads. For the GBM model, increasing uncertainty is characterized by an increase in the instantaneous volatility of the random process ( $\sigma^2$ ). For the Poisson model, we focus on mean preserving spreads by fixing the values of the  $\lambda$ 's and changing  $\pi_0$  and  $\pi_1$  such that the  $E(\pi)$  ex ante is unaffected. For example, consider the case where  $\lambda_{0t} = \lambda_{1t} = \lambda$ . A mean preserving spread results as  $(\pi_0, \pi_1)$  progresses from (.10, .10) to (.05, .15) to (.00, .20). In the first case, there is no uncertainty in tax policy. The ITC always equals 10 percent. In the second case, it randomly switches between 5 percent and 15 percent while in the third case it switches on and off with its value equaling 20 percent when in effect<sup>9</sup>.

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<sup>9</sup> There is a relationship between the duration parameters in the Poisson model and uncertainty although unlike the GBM process, the relation is not monotonic. To see this, let us take the case where  $\lambda_{0t} = \lambda_{1t} = \lambda$  and consider extreme values for  $\lambda$ . If  $\lambda$  equals 0, there is

### III. Investment Simulations

In this section we present Monte Carlo simulations based on the model described in the last section to calculate the average price at which investment occurs. We begin by fixing  $\lambda$  at zero and slightly modifying the model to allow randomness in the capital cost. The first change means that the only uncertainty in the model is due to price uncertainty in the form of Geometric Brownian Motion (GBM). Second, by allowing GBM in capital costs, we can explicitly focus on capital price uncertainty. It can easily be shown that when output price and capital price both follow GBM, only the ratio of output to capital costs matters for investment purposes. Moreover, when  $p$  and  $p_k$  both follow GBM then so does  $p/p_k$ . Thus all of the theoretical derivations in the last section apply directly. We model  $p$  and  $p_k$  by the following trendless uncorrelated processes:

$$(4) \quad \begin{aligned} dp_t &= \sigma_p p_t dz_p \\ &\text{and} \\ dp_{k_t} &= \sigma_k p_{k_t} dz_k \end{aligned}$$

The ratio  $p/p_k$  follows GBM with trend  $\sigma_k^2$  and variance  $\sigma_p^2 + \sigma_k^2$ . In addition, one can easily

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clearly no uncertainty over future tax policy; whatever policy is in effect now will be in effect forever. Now suppose that  $\lambda$  is a very large number; the instantaneous probability of switching between the tax and no-tax states is very close to 1. In that case, the credit will switch on and off every instant. The variation will be extraordinarily high, but there will be almost no uncertainty. At each point in time, you know with great confidence what the credit will be at the next point in time. While there is no uncertainty at boundary values for  $\lambda$ , there is clearly uncertainty at intermediate values of  $\lambda$ . We do not focus on uncertainty of that type in this paper.

show that the required hurdle price ratio increases as the variance of the capital price increases<sup>10</sup>.

The increase in the hurdle price ratio leads one to expect that time to investment will rise.

However, with increasing variance in capital costs, it becomes increasingly likely that capital costs will fall sharply in a short period of time - raising the chances of the hurdle price ratio being hit in a shorter time. Thus whether time to investment goes up or down as the variance of capital costs increases is ambiguous<sup>11</sup>. Therefore we turn to simulations to determine which of these effects dominate. Simulations also allow us to assess the economic importance of changes in uncertainty.

Table 1 illustrates the importance of mean preserving spreads on hurdle prices and investment times. We present results from a Monte Carlo experiment in which we simulate 1500 price paths and consider the investment behavior of a firm facing these prices. We assume a production function of the form  $F(K) = \ln(1+K)$  and use a discount rate of .05 for investment. Output prices are GBM with no trend and variance equal to .01. Capital costs also follow GBM with no trend and variance ranging from 0 to .025. The correlation between the output and capital price series is set to 0<sup>12</sup>.

While simple, the model illustrates the essential points that we wish to make. First we note that the expected time to investment at the firm level is finite only if  $\sigma_k^2 > \sigma_p^2$ <sup>13</sup>. We focus

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<sup>10</sup> See Hassett and Metcalf (1994) for details as well as a complete derivation of the model with output and capital prices following GBM.

<sup>11</sup> If the only uncertainty is in the GBM process, then the mean time to investment can be derived explicitly. See footnote 13 below.

<sup>12</sup> We start the simulations at time 0 with output price equal to 1.40 and capital cost equal to 18. Thus the ratio of prices at time zero equals 7.78 percent.

<sup>13</sup> The expected time to investment equals  $\frac{2(\psi^* - \psi^0)}{\sigma_k^2 - \sigma_p^2}$  when  $\sigma_k^2 - \sigma_p^2 > 0$ , where  $\psi^*$  is the

on the median time to investment in this analysis. We do this for two reasons. First, it facilitates comparisons with the jump process model. In that model, there is no explicit equation for the expected time to investment. Second, the mean is not well representative of the central tendency of the distribution of stopping times in the GBM model. There is considerable skew in the distribution due to the possibility of long periods in which the price ratio may drift away from the hurdle price ratio. While such a realization may not occur frequently, the realized stopping time can be quite large. For example, the median time to investment when  $\sigma_k^2$  equals .02 is just under 80 years while the mean time is over 150 years.

Column 2 in Table 1 presents the hurdle rates. As noted above, the hurdle price ratios increase as  $\sigma_k^2$  increase. If capital costs are certain, the optimal time to invest is when output price exceeds 9.7 percent of the price of capital. The ratio increases to 15.1 percent when the instantaneous variance of  $p_k$  rises to .01. The hurdle price ratio rises and reaches 44.1 percent at  $\sigma_k^2$  equal to .025. The next column of Table 1 gives the median time to investment. If there is no uncertainty in capital costs, the median time is 14.1 years<sup>14</sup>. The average output price at investment equals 1.75, an increase from time zero of 25 percent.

What happens when the capital price variance increases? As can be seen in the table, increasing  $\sigma_k^2$  from 0 to .01 increases the median time to investment. The required price ratio has increased from 9.7 percent to 15.1 percent and the median time to investment is now 54.1 years. This pattern of higher hurdle rates and longer median time to investment persists as the variance

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log of the hurdle price ratio and  $\psi^0$  is the log of the starting price ratio. We thank a referee for pointing this formula out to us.

<sup>14</sup> Standard errors of the median time to investment are based on Koenker and Bassett (1982) as implemented in STATA.

of  $p_K$  rises.

Table 1 illustrates another important point. Note that the hurdle rate can rise either as a result of the output price rising or the capital cost falling. The last two columns indicate that the firm hits the hurdle rate primarily through a fall in capital cost rather than through a high realized output price. The fall in average capital cost conditional on investment is dramatic; as the variance rises from 0 to .015, cost falls by nearly 50%. When the variance rises to .025, the capital cost falls by 75 percent while the output price has only risen by 13 percent. While we have not linked the variation in capital price variability to tax policy, it is straightforward to do so. Let the pre-tax cost of capital be constant and equal  $\bar{p}_k$  and let the tax credit follow the process:

$$(4') \quad d\pi_t = -\sigma_k(1-\pi_t)dz_k$$

Assuming  $p_K = (1-\pi)\bar{p}_k$ ,  $p_K$  is GBM with zero trend and volatility  $\sigma_K$  (see equation (4)). Thus, in Table 1, mean preserving spread in the capital cost can be generated by mean preserving spreads in the tax credit.

The crucial result is that mean preserving spreads bring with them increased credit generosity as evidenced by the lower average realized price of capital conditional on investment. The explanation for this result is quite simple. Firms simply wait for a good state (low capital cost) and concentrate their investment activity in those periods. For the GBM process, the value to waiting is quite high and the result is an increase in waiting time as the variance of the capital price increases.

This link between mean preserving spreads and credit generosity in this model will be present in any model that allows firms to adjust to changing conditions. Consider the following

simple example. Figure 2 illustrates a firm's upward sloping firm-specific supply curve. When output price equals  $P_0$ , the firm sells  $Q_0$  units of output. Now let the firm be subject to random output price fluctuations between  $P_1$  and  $P_2$  such that the expected price continues to equal  $P_0$ . Output for the firm will fluctuate between  $Q_1$  and  $Q_2$  and average profits will increase by the area  $(A-B)/2$ . The increase in

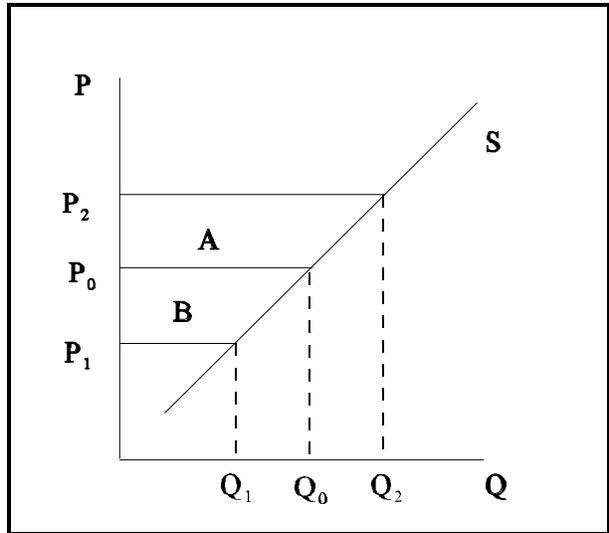


Figure 2: A Simple Example

profits can be interpreted as a result of the fact that average price weighted by output goes up.<sup>15</sup> The firm shifts production from a bad state (low output price) to a good state (high output price). This is precisely analogous to the situation for the firm in Table 1 except that in the simple example above the firm is a seller whereas in Table 1 the firm is a buyer (of capital). In Table 1, the good realization is a high tax credit (low capital cost). The analogous shift is in investment from the high price state to the low price state.

Continuing with the simple example in Figure 2, consider a fluctuation in output price arising from a government policy that shifts randomly between a tax on output (lowering the price received by firm from  $P_0$  to  $P_1$ ) and a subsidy to output (raising the price received by the firm from  $P_0$  to  $P_2$ ). While the tax/subsidy policy increases the variance of price received by the firm while leaving the mean price unaffected *ex ante*, it also inevitably leads to a subsidy to the firm since the

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<sup>15</sup> Mathematically, the increase in profits is a direct result of Jensen's Inequality and the fact that the profit function is convex in price.

firm shifts production from the tax state to the subsidy state. Increasing the tax/subsidy policy would only increase the overall subsidy received by the firm. In effect, a close link between increases in uncertainty (in the form of a mean preserving spread) and generosity of the tax system is inevitable. This link will be even more apparent in the jump process to which we next turn.

The model of random tax policy with a jump process is the same as the one used in the Monte Carlo experiment for Table 1 except that capital costs are now fixed at 20 and there is a Poisson Process for the tax credit. Again, we begin the output price process at 1.40.

Furthermore, we assume that  $\pi = \pi_0$  at time zero.

Table 2 presents results for a mean preserving spread in the tax credit. Panel A in Table 1 assumes that the jump process parameters ( $\lambda_0$  and  $\lambda_1$ ) are constant and equal to 0.35. This means that the unconditional probability of being in either the low or high ITC state will equal 0.50 and the expected duration of either state will be  $(0.35)^{-1}$  or just under three years, roughly the post-war U.S. experience. The first row in Panel A of Table 2 sets  $\pi_0 = \pi_1 = 0.10$ . This corresponds to the first row of Table 1. We then increase the spread between the high and the low tax credit to 10 percentage points (.15 - .05) and then to 20 percentage points (.20 - .00).

In the case where tax credits fluctuate between 5 percent and 15 percent, investment is now delayed in the low ITC state until the hurdle price ratio reaches 12.2 percent (2.31/19). This represents an increase of roughly 26 percent over the hurdle price ratio in the certainty case. In the high ITC state the hurdle price ratio falls to 9.2 percent (1.56/17), a drop of about 5 percent from the certainty price. The table also reports that on average the ITC switches states 3.23 times before an investment occurred. Most noteworthy is the result that median time to investment falls from 14 to 8 years with the mean preserving spread.

The third row increases the variance in the ITC variable by setting  $(\pi_0, \pi_1)$  equal to  $(0, .20)$ . In the low ITC world, investment is now delayed until the hurdle price ratio reaches 13.4 percent  $(2.68/20)$  while in the high ITC world, investment occurs when the ratio reaches 9.1 percent  $(1.46/16)$ . The median time to investment continues to fall; it is now 4.4, a fall of 69 percent from the certainty case.

There are both similarities between Panel A of Table 2 and Table 1 as well as differences. First, note that investment piles up in high tax credit periods. As the spread between the low and high rates widens, the fraction of investment occurring in the high credit period rises. When the ITC vector equals  $(.05, .15)$  the net price of capital can equal 19 or 17. The average net price of capital equals 17.04 which implies that 98 percent of the time the investment occurred when the high tax credit was in effect. When the ITC vector is  $(0, .20)$  investment occurs in the high ITC state 99.5 percent of the time. This corresponds to the result in Table 1 that as the variance of the capital cost rises in the GBM example, the average cost of capital conditional on investing falls. In both cases the increase in the spread costs the government revenue as investment piles up in the high credit period.

Next, note that the median time to investment in Panel A of Table 2 falls in contrast to Table 1. This is explained both by the shift from a non-stationary to a stationary process as well as the discrete nature of the Poisson process. In a "good" state in the GBM world, there is an equal possibility of the price falling further (the state gets even better) and the price rising. Because the change is small, there is little cost to waiting for a better realization in contrast to the Poisson process. In the latter case, the value of waiting in a good state is very low since

stationarity implies that the firm can only transit to a lower credit state from a high credit state.<sup>16</sup> Moreover, the cost of not moving increases with the spread since if the firm doesn't invest, the credit may shift to a sharply lower level as the spread goes up, a cost made more dramatic by the discrete nature of the process. The result is a fall in the median time to investment as the spread between the low and high credit values increases.<sup>17</sup> There is a strong "use it or lose it" force at work in the Poisson process model due in main to stationarity. The discrete nature of the process only serves to magnify this force. If we were to add more tax credit states to the model, this "use it or lose it" force would be attenuated somewhat, being more important when the credit is in states near the edges of its range.

The next panel in Table 2 allows for a covariance between policy and firm profitability<sup>18</sup>. The Poisson parameters still equal 0.35 at the ex ante expected price ( $p = 1.40$ ) but  $\frac{d\lambda_1}{dp} = -.12$  while  $\frac{d\lambda_0}{dp} = .12$ . The probability of switching from a low (high) ITC state to a high (low) ITC state goes down (up) as output price increases. The first row yields identical results to the first row in Panel A of the table since the low and high tax credits are the same. When the low credit

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<sup>16</sup> This would be true for any stationary process for a state in which price exceeds the mean. Note that the value of waiting when in the high credit state is not zero. Firms wish to combine a high credit state with a high output price. As can be seen in the last column of Table 2, firms on average sit through at least one high credit state before investing.

<sup>17</sup> The optimality of speeding up investment can also be seen by considering the hurdle rate for the high credit state (the state in which investment is most likely to occur). As the spread between the low and high credit values increases from 0 to 0.20, the hurdle rate (equal to  $p^*/((1-\pi_1)p_K)$ ) falls from .098 to .091.

<sup>18</sup> The absolute value of the covariance between  $\lambda$  and  $p$  is  $\alpha \text{Var}(p)$ . Since  $\lambda$  is a linear function of  $p$ , the correlation is equal to either positive or negative one. We use  $\alpha$  as a proxy for the relation between firm profitability and government tax policy with higher values of  $\alpha$  indicating a higher responsiveness of tax policy to changes in the economic environment.

falls from 10 to 5 percentage points while the high credit increases from 10 to 15 percentage points, the median time to investment increases sharply from 14 to over 23 years. The increase is even more dramatic when compared to the second row in the Panel A of this table where the tax credits are the same as in this row. In that case, median time to investment rises from 8 to 23 years. The key to understanding the lengthened time to investment lies in the behavior of  $\lambda_0$  and  $\lambda_1$  in the second investment region in which the firm only invests if the high ITC is in place.  $\lambda_1$  varies between 0.30 and 0.22 in that regime while  $\lambda_0$  ranges from 0.40 to 0.48. The expected duration of the low ITC state can be as high as 5.8 years in this region and the expected probability of being in the low ITC state ( $\lambda_0/(\lambda_1 + \lambda_0)$ ) ranges from .55 to .75. Most investment in this model occurs in region II and the lower probability of being in the high ITC state in this region lengthens the median time to investment.

The interaction between the mean preserving spread and policy covariance is evident in the last row of Panel B of Table 2. If the tax credits range between zero and 20 percent, the median time to investment is only slightly lower than in the case in which there is no ITC variation (first versus third row of Panel B). We can view the move from the first to the third row in Panel B in two steps. First a move from the first row of Panel A (same as first row of Panel B) to third row of Panel A. This shows how the mean preserving spread reduces median time to investment. Then we move from third row of Panel A to third row of Panel B. The increase in covariance of policy response brings with it an increase in median time to investment from 4 to 13 years.

Panel C focuses on the impact of increasing covariance between output price and policy. The parameter  $\alpha$  (equals  $\alpha_0$  equals  $\alpha_1$ ) is ranged from 0 to 0.12 in the case where the tax credits can either be zero or 20 percent. At  $\alpha$  equals 0.04, there is a small increase in median time to

investment. In the second investment region, the probability of being in the low ITC state only rises modestly from 0.50 to 0.57. Median time to investment increases by less than 1 year. At  $\alpha$  equals 0.08, the probability of being in the low ITC state now rises to as much as two-thirds in the second region and the impact on median time to investment is significant. And as noted above, if  $\alpha$  equals 0.12, the fraction of time spent in the low ITC state can be as much as 75 percent with a consequent lengthening of median time to investment.

Summing up, we find that unlike the model in which capital prices move according to Geometric Brownian Motion, the ex post average hurdle price ratio and median time to investment both fall with increases in uncertainty in the Poisson Process model for constant  $\lambda$ s. Increases in the relation between tax policy and firm profitability ( $\alpha$ ) blunt this fall in time to investment and for sufficiently high values of  $\alpha$ , the time to investment actually rises and then falls with mean preserving spreads.

While we have developed a model in terms of an individual firm's decision to make a one-time investment, it turns out that this is less of a restrictive modeling approach than might appear at first glance. Pindyck (1988) has developed a model of an individual firm making incremental investments. As he notes in an appendix to that paper, the model of incremental investment in that paper gives rise to a set of equations quite similar to the equations resulting from a model of one-shot investment (like ours) in which size and timing of investment are chosen. Moreover, as Dixit and Pindyck (1994) point out, one can view incremental investment as a sequence of distinct projects and build up an capital investment solution from the model developed above<sup>19</sup>. The

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<sup>19</sup> "The most noteworthy feature of the solution [to the incremental investment problem] is that we can regard the successive marginal increments to capital as distinct little projects, each contributing its marginal product independently of the others" (Dixit and Pindyck, p. 366).

model above distinguishes boundary points ( $p_0$  and  $p_1$ ) that distinguish among the three investment regions. A more formal model of aggregate capital accumulation would replace the boundary points with boundary regions in  $(p,K)$  space. While the mathematics becomes more complicated, the results are similar to those we obtain through our aggregation approach. Moreover, Caballero, Engel, and Haltiwanger (1995) demonstrate how such a model can be used to describe microlevel behavior and then aggregated up to develop aggregate predictions. Thus one should not view the model as unnecessarily restrictive or removed from the capital accumulation problem that is of interest to macro and public finance economists. In the next section, we move from the individual to the aggregate level.

#### IV. Aggregate Analysis

The previous section focuses on behavior at the individual level. What happens in the aggregate as policy uncertainty changes? We modify the production function in equation (3) to allow for firm heterogeneity.  $F(K) = \delta \ln(1+K)$  where  $\delta$  is normally distributed with mean 1 and standard deviation .25. One can think of the economy as a continuum of one project firms each of which has a productivity parameter  $\delta$ . Firms wait until output price relative to the after tax price of capital rises sufficiently high at which point they invest<sup>20</sup>. The capital price ( $p_k$ ) is fixed and the output price follows GBM with trend .01 and variance  $\sigma^2$ .<sup>21</sup>

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<sup>20</sup> As noted in the last section, this is a convenient way to model aggregate investment building on the firm level investment decision developed in the previous section. Modeling either a single firm with variable (but irreversible) capital stock (as in Pindyck (1988)) or a continuum of single project firms in an industry setting (as in Caballero and Pindyck (1996)) will not lead to qualitatively different conclusions.

<sup>21</sup> This is consistent with both prices being GBM with no trend. Recall that in that case (and if the processes are uncorrelated, the trend in  $p/p_k$  equals  $\sigma_k^2$ ).

We begin by fixing the investment tax credit at 10 percent and considering how changes in the volatility of the price process affect cumulative investment. We vary  $\sigma^2$  from 0.005 to 0.015.<sup>22</sup> Results are presented in Table 3. The first column presents results when the variance is .005 (this corresponds to an instantaneous volatility on the order of 7 percent of price). The initial level of investment based on the price starting at 8.75 percent of capital costs is .435. By the end of the first year, the median investment more than doubles to .958 and by year 10 equals 1.245. Increasing the volatility of the price process decreases aggregate investment at any point in time. Increasing  $\sigma^2$  from .005 to .015 decreases the initial investment from .435 to .008. The rate of growth of the capital stock increases but the level at year 10 with  $\sigma^2$  equal to .015 is roughly a fifth of the level at the lower variance. These results are quite robust to parameter choices: if prices follow GBM, aggregate capital stocks are lower as the price uncertainty increases. The higher hurdle price ratio significantly dampens capital formation.

Table 4 illustrates results from the model with discrete changes in the capital cost due to an investment tax credit. The first column simply repeats the results from the last table (column 2). The second column of Table 4 introduces policy uncertainty in the form of an investment tax credit that randomly switches between .05 and .15 with Poisson parameters equal to 0.35. With a value for the  $\lambda$ 's equal to .35, the expected time in any ITC state is roughly 3 years. We start the process with the initial level of the ITC random with an equal probability of being in the low or high state<sup>23</sup>. Comparing the first two columns, we see that the median investment is higher in any

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<sup>22</sup> A value of  $\sigma^2$  equal to 0.015 represents instantaneous volatility on the order of 12 percent of price.

<sup>23</sup> At time zero, there are only two possible values for the capital stock depending on the initial value of the tax credit. We report the average of these two values in this row. All other

of the years which we analyze. At the end of year 10, the capital stock is 45 percent higher with the policy uncertainty.

An important issue is the initial value of the ITC. The increase in investment from the ITC rising from 10 percent to 15 percent is substantially more than the decrease in investment from the ITC falling from 10 percent to 5 percent. We investigate the importance of the starting values in the next column where we start the ITC at the lower value of 5 percent. Initially, there is no investment and the capital stock lags behind the capital stock when the ITC is fixed at 10 percent for the first several years. By year 5 however, capital accumulation has caught up and by year 10, the capital stock is 17 percent higher.

The last column of the table increases the spread between the high and low tax credits from 10 percentage points to 20 points. Comparing columns 2 and 4, the impact of the increase in uncertainty is to once again increase capital stocks, with the increase by year 10 on the order of 30 percent<sup>24</sup>.

The results in Table 4 indicate that policy uncertainty in the form of a fluctuating ITC may not reduce capital formation. The key factor explaining this is the difference in investment when the ITC increases relative to when it decreases. Investment piles up in the high ITC periods and more than offsets the lack of investment in low ITC periods. In effect, the policy uncertainty brings with it a subsidy to investment since firms can time investment to take advantage of the high credit states of nature. One objection to these results is that it is unlikely that the probability

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rows report median stocks.

<sup>24</sup> If we start the process with a tax credit of zero, investment begins more slowly but once again catches up within 5 years relative to the constant tax credit case.

of regime shifts would be unaffected by firm profitability. We address this issue in the next table.

In Table 5, we now let the  $\lambda$ 's vary linearly with output price. Recall that  $\lambda_0 = \bar{\lambda}_0 + \alpha_0 p$  and  $\lambda_1 = \bar{\lambda}_1 - \alpha_1 p$ . We range  $\alpha_0 = \alpha_1 \equiv \alpha$  from 0 to 0.12. At every point in time, we find that increasing the covariance between the Poisson change parameters and output price reduces aggregate investment. There are two forces at work, both of which act to reduce investment. First, as  $\alpha$  increases, region II (the region in which investment only occurs if the high tax credit is in effect) shifts to the right. The lower boundary moves from 1.61 to 1.98 as  $\alpha$  increases from 0 to 0.12. Similarly, the upper boundary moves from 3.14 to 3.50. This shift will delay investment as it will increase the expected time required for output price to move to this region. Second, investment only occurs in this region if the high tax credit is in effect. As  $\alpha$  increases, the unconditional probability of being in a high ITC state falls and can be as low as 1/3. Both these factors serve to retard investment and offset the increase in capital accumulation that results from the increase in the spread between the low and high credits. Comparing the last column of Table 5 with the first column of Table 4, we see that these two effects effectively cancel each other out so that capital accumulation patterns are roughly the same in a world with a constant 10 percent tax credit and a world with either a zero or 20 percent credit but in which  $\lambda$  is affected by output price with  $\alpha$  equal to 0.12.

## V. Conclusion

When tax policy uncertainty leads to capital costs following a continuous time random walk in logs, increasing uncertainty delays firm-level investment and leads to lower levels of investment. This result follows directly from work by Pindyck (1988) and others. However, when tax policy follows a stationary and discrete jump process that is more similar to actual

historical experience, we find that increasing uncertainty can have the opposite effect, speeding up the time to investment, and increasing the amount of capital purchased conditional on investing. At the aggregate level, the result is a higher capital stock both initially and after a number of years. The increase in investment is blunted to the extent that tax policy is related to firm profitability ( $\alpha_0$  and  $\alpha_1$ ). Estimating these parameters is an important topic that is left for future research.

The difference in results between the two models follows from the bounded and discrete nature of the jump process model. In the "good" credit state, the value of waiting to invest is low and the loss from a shift from a "good" credit to a "bad" credit can be substantial. In contrast, the value of waiting to invest in the GBM model does not fall as sharply as capital costs fall. The non-stationarity of the process implies that investors are not subject to a "use it or lose it" effect to the extent found in the stationary process model. The first conclusion we draw from this research is that the impact of uncertainty on investment depends to a large extent on the underlying stochastic process. We believe that the process used in much of the literature to model uncertainty and investment is inappropriate for thinking about tax policy uncertainty.

Our second main conclusion is that there is an inextricable link between uncertainty and tax revenue. While the models differ in their implications for investment, they both share the common feature that increases in uncertainty bring with them an increased loss of tax revenues to the government. Hence, to the degree that governments are unable to commit to a fixed tax policy for investment, the uncertainty acts as an implicit subsidy to investment. The implicit subsidy may or may not lead to increased investment. When uncertainty follows a GBM process, for example, the subsidy is not large enough to offset the value of waiting as the variance

increases and so investment is delayed.

Our conclusions are subject to a number of cautionary remarks. First, we note that the pre-tax price of capital is constant in our model. This follows if the supply of capital goods is perfectly elastic and there are no adjustment or other costs incurred when investment purchases swing wildly from booms to busts. For many countries, the implicit "smallness" assumptions built into our model may be reasonable. Second, the results in this paper should be interpreted as positive rather than normative. The model and results are an important first step towards understanding the effect of randomness in tax policy on aggregate investment. Further work remains however before conclusions can be drawn as to the welfare implications of tax policy uncertainty.

## Appendix A. Solving the Poisson Model

As noted in the text, there are three investment regions of interest. Let  $p_1$  denote the boundary between regions 1 and 2 and  $p_0$ , the boundary between regions 2 and 3. Consider first region 1 below  $p_1$ . In this region, no investment is made regardless of the level of the ITC. Let  $V^1$  represent the value function when the high tax credit is in effect and  $V^0$  the value when the low tax credit is in effect. An arbitrage argument can be made that<sup>25</sup>

$$(A1) \quad \rho V^0 dt = E(dV^0)$$

$$(A2) \quad \rho V^1 dt = E(dV^1)$$

Using Ito's Lemma (and ignoring terms of order  $dt^2$ )

$$(A3) \quad E(dV^0) = (0.5\sigma_p^2 p^2 V_{pp}^0 + \mu p V_p^0 + (\bar{\lambda}_1 - \alpha_1 p_t)(V^1 - V^0))dt.$$

Similarly,

$$(A4) \quad E(dV^1) = (0.5\sigma_p^2 p^2 V_{pp}^1 + \mu p V_p^1 - (\bar{\lambda}_0 + \alpha_0 p_t)(V^1 - V^0))dt.$$

Substituting these expressions into (A1) and (A2), dividing through by  $dt$  and letting  $dt$  go to zero yields a system of partial differential equations in  $p$ :

$$(A5) \quad \rho V^0 = .5\sigma_p^2 p^2 V_{pp}^0 + \mu p V_p^0 + (\bar{\lambda}_1 - \alpha_1 p_t)(V^1 - V^0)$$

$$(A6) \quad \rho V^1 = .5\sigma_p^2 p^2 V_{pp}^1 + \mu p V_p^1 - (\bar{\lambda}_0 + \alpha_0 p_t)(V^1 - V^0)$$

The expectation on the right hand sides of equations (A3) and (A4) are composed of two parts. The first is the expected gain in  $V^0$  as prices evolve. The second part reflects the capital gain

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<sup>25</sup> This also follows from the Bellman equation.

(loss) if the tax credit is increased (reduced) which occurs with probability  $\lambda_{1t}$  ( $\lambda_{0t}$ ).

To solve these equations, let  $Z = V^1 - V^0$  and  $X = \lambda_{1t}V^1 + \lambda_{0t}V^0$ . Making this change of variables yields the two independent differential equations in  $Z$  and  $X$ :

$$(A7) \quad \rho Z = .5\sigma^2 p^2 Z_{pp} + \mu p Z_p - (\bar{\lambda}_1 + \bar{\lambda}_0 + (\alpha_0 - \alpha_1)p_t)Z$$

$$(A8) \quad \rho X = .5\sigma^2 p^2 X_{pp} + \mu p X_p$$

A solution for  $Z$  is given by the power series:

$$(A9) \quad Z = A_1 \left\{ p^{\gamma_1 + \sum_{n=1}^{\infty} \frac{(\alpha_0 - \alpha_1)^n}{\prod_{i=1}^n \phi_1(i)} p^{n+\gamma_1}} \right\} + A_2 \left\{ p^{\gamma_2 + \sum_{n=1}^{\infty} \frac{(\alpha_0 - \alpha_1)^n}{\prod_{i=1}^n \phi_2(i)} p^{n+\gamma_2}} \right\}$$

where  $\phi_j(i) = \frac{1}{2}\sigma^2(i+\gamma_j)(i+\gamma_j-1) + \mu(i+\gamma_j) - (\bar{\lambda}_1 + \bar{\lambda}_0 + \rho)$  for  $j=1, 2$  and  $\gamma_1$  and  $\gamma_2$  are roots to the equation  $\phi(0) = 0$  and  $\phi(i)$  is analogously to  $\phi_j(i)$ <sup>26</sup>. It can easily be shown that the roots are real and of opposite sign. We arbitrarily define them such that  $\gamma_1 < 0 < \gamma_2$ . The limiting behavior of  $Z$  as  $p$  approaches zero provides information about  $A_1$ . Since zero is an absorbing state for  $p$ ,  $Z$  must equal zero if  $p$  ever goes to zero. Since  $\gamma_1 < 0$ , the first term in the expression for  $Z$  would explode unless  $A_1$  equals 0.

Equation (A8) is a ordinary differential equation of the Euler type. Its general solution is given by  $X = B_1 p^{\beta_1} + B_2 p^{\beta_2}$  where the  $\beta$ 's are the roots to the quadratic equation

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<sup>26</sup> Details are available upon request from the authors. This solution is valid so long as  $r_1$  does not differ from  $r_2$  by an integer (or zero). In all cases that we consider this condition is met. The power series can be shown to converge for all positive prices so long as  $\sigma^2$  is positive.

$Q(x) = .5\sigma^2x(x-1) + \mu x - \rho$  defined such that  $\beta_1 < 0 < \beta_2$ . Again we use a limiting argument as  $p$  approaches zero to determine that  $B_1 = 0$ . Substituting the expressions for  $Z$  and  $X$  into their definitions and solving for  $V^0$  and  $V^1$  gives us the following equations:

$$(A10) \quad V^0 = \frac{1}{\lambda_1 + \lambda_0} \left\{ c_1 p^{\beta - \lambda_1} c_2 \left( p^\gamma + \sum_{n=1}^{\infty} \frac{(\alpha_0 - \alpha_1)^n}{\prod_{i=1}^n \phi_2(i)} p^{n+\gamma} \right) \right\}$$

$$(A11) \quad V^1 = \frac{1}{\lambda_1 + \lambda_0} \left\{ c_1 p^{\beta + \lambda_0} c_2 \left( p^\gamma + \sum_{n=1}^{\infty} \frac{(\alpha_0 - \alpha_1)^n}{\prod_{i=1}^n \phi_2(i)} p^{n+\gamma} \right) \right\}$$

where  $\beta$  is the positive root to the quadratic equation  $Q(x) = .5\sigma^2x(x-1) + \mu x - \rho = 0$ ,  $\gamma$  the positive root to the quadratic  $R(x) = .5\sigma^2x(x-1) + \mu x - (\rho + \bar{\lambda}_1 + \bar{\lambda}_0) = 0$  and  $c_1$  and  $c_2$  are constants of integration.

In region 2, the firm only invests if the high level of the tax credit is in place. The arbitrage argument used above is modified slightly to account for the investment in the presence of the high tax credit:

$$(A12) \quad \rho V^0 dt = E(dV^0)$$

$$(A13) \quad V^1 = \frac{p_T F(K_1)}{\rho - \mu} - (1 - \pi_1) p_k K_1$$

where  $p_T$  is the output price at time of investment,  $p_k$  is the price of the capital good,  $K_1$  is the

level of investment conditional on  $\pi = \pi_1$ ,  $F(K_1)$  is output at that level of capital and  $T$  is the time at which investment occurs. Equation (A13) says that the value of the project if the investment is made equals the expected present discounted value of the flow of profits from the investment (discounted at rate  $\rho$ ) less the cost of the project. Upon application of Ito's Lemma and limiting behavior as  $dt$  approaches zero, Equation (A12) becomes

$$(A12') \quad \frac{1}{2}\sigma^2 p^2 V_{pp}^0 + \mu p V_p^0 - (\bar{\lambda}_1 - \alpha_1 p + \rho) V^0 = (\alpha_1 p - \bar{\lambda}_1) \left( \frac{pF(K_1)}{\rho - \mu} - (1 - \pi_1) p_k K_1 \right).$$

The general solution to equation (A12') is given by

$$(A14) \quad V^0 = d_1 \left\{ p^{\eta_1} + \sum_{n=1}^{\infty} \frac{(-\alpha_1)^n}{\prod_{i=1}^n \psi_1(i)} p^{n+\eta_1} \right\} + d_2 \left\{ p^{\eta_2} + \sum_{n=1}^{\infty} \frac{(-\alpha_1)^n}{\prod_{i=1}^n \psi_2(i)} p^{n+\eta_2} \right\} + \sum_{n=0}^{\infty} a_n p^n$$

where  $\psi_i(k) = \frac{1}{2}\sigma^2(\eta_i+k)(\eta_i+k-1) + (\eta_i+k)\mu - (\bar{\lambda}_1 + \rho)$ , the  $\eta$ 's are positive and negative roots to  $\psi(0) = 0$  such that  $\eta_1 < 0 < \eta_2$ ,  $d_1$  and  $d_2$  are constants of integration, and

$$(A15) \quad \begin{aligned} a_0 &= \frac{\bar{\lambda}_1(1-\pi_1)p_k K_1}{\xi(0)} \\ a_1 &= \frac{-\alpha_1}{\xi(1)} a_0 - \frac{1}{\xi(1)} \left\{ \frac{\bar{\lambda}_1 F(K_1)}{\rho - \mu} + \alpha_1(1-\pi_1)p_k K_1 \right\} \\ a_2 &= \frac{-\alpha_1}{\xi(2)} a_1 + \frac{\alpha_1 F(K_1)}{(\rho - \mu)\xi(2)} \\ a_n &= \frac{-\alpha_1}{\xi(n)} a_{n-1} \quad n = 3, 4, 5, \dots \end{aligned}$$

where  $\xi(i) = \frac{1}{2}\sigma^2 i(i-1) + i\mu - (\bar{\lambda}_1 + \rho)$ . As in region I, it can easily be shown that these series converge for all positive prices assuming  $\sigma^2$  is positive.

Finally, in region 3, investment is made regardless of the ITC value. The value functions are given by

$$(A16) \quad V^0 = \frac{p_T F(K_0)}{\rho - \mu} - (1 - \pi_0) p_k K_0$$

$$(A17) \quad V^1 = \frac{p_T F(K_1)}{\rho - \mu} - (1 - \pi_1) p_k K_1$$

where  $K_0$  is the level of investment conditional on  $\pi = \pi_0$ .

Value matching and smooth pasting arguments can be invoked to complete the system.

Value matching implies that the value function  $V^0$  and  $V^1$  must be equal at the boundaries of the regions. These imply equations

$$(A18) \quad \frac{1}{\lambda_1 + \lambda_0} \left\{ c_1 p_1^\beta - \lambda_1 c_2 \left( p_1^\gamma + \sum_{n=1}^{\infty} \frac{(\alpha_0 - \alpha_1)^n}{\prod_{i=1}^n \phi_2(i)} p_1^{n+\gamma} \right) \right\} \\ = d_1 \left\{ p_1^{\eta_1} + \sum_{n=1}^{\infty} \frac{(-\alpha_1)^n}{\prod_{i=1}^n \psi_1(i)} p_1^{n+\eta_1} \right\} + d_2 \left\{ p_1^{\eta_2} + \sum_{n=1}^{\infty} \frac{(-\alpha_1)^n}{\prod_{i=1}^n \psi_2(i)} p_1^{n+\eta_2} \right\} + \sum_{n=0}^{\infty} a_n p_1^n$$

$$(A19) \quad \frac{1}{\lambda_1 + \lambda_0} \left\{ c_1 p_1^\beta + \lambda_0 c_2 \left( p_1^\gamma + \sum_{n=1}^{\infty} \frac{(\alpha_0 - \alpha_1)^n}{\prod_{i=1}^n \phi_2(i)} p_1^{n+\gamma} \right) \right\} = \frac{p_1 F(K_1)}{\rho - \mu} - (1 - \pi_1) p_k K_1$$

$$\begin{aligned}
(A20) \quad & d_1 \left\{ p_0^{\eta_1} + \sum_{n=1}^{\infty} \frac{(-\alpha_1)^n}{\prod_{i=1}^n \psi_1(i)} p_0^{n+\eta_1} \right\} + d_2 \left\{ p_0^{\eta_2} + \sum_{n=1}^{\infty} \frac{(-\alpha_1)^n}{\prod_{i=1}^n \psi_2(i)} p_0^{n+\eta_2} \right\} + \sum_{n=0}^{\infty} a_n p_0^n \\
& = \frac{p_0 F(K_0)}{\rho - \mu} - (1 - \pi_0) p_k K_0
\end{aligned}$$

In addition to the value matching conditions, smooth pasting conditions must be met. Equations (A18) and (A19) are differentiated with respect to  $p$  and evaluated at  $p_1$  while equation (A20) is differentiated with respect to  $p$  and evaluated at  $p_0$ . Marginal investment conditions must also be met: the expected present discounted value of marginal revenue product must equal the net (of tax) price of capital. Equations (A18)-(A20) plus the five smooth pasting equations yield a system of eight equations in the eight unknowns:  $p_0$ ,  $p_1$ ,  $K_0$ ,  $K_1$ ,  $c_1$ ,  $c_2$ ,  $d_1$ , and  $d_2$ .

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Table 1. Mean Preserving Spreads  
with Continuous Policy Uncertainty

$\sigma_K^2$	$\left(\frac{p}{p_K}\right)^*$	$t_{\text{med}}$	$E(p)$	$E(p_K)$
0.00	0.097	14.10 (1.24)	1.75 (0.00)	18.00 (0.00)
0.0025	0.108	26.46 (1.76)	1.80 (0.01)	16.66 (0.09)
0.005	0.120	32.81 (2.64)	1.81 (0.01)	15.05 (0.14)
0.010	0.151	54.07 (3.42)	1.82 (0.02)	11.99 (0.14)
0.015	0.199	64.03 (3.52)	1.88 (0.03)	9.43 (0.14)
0.020	0.279	79.52 (3.41)	1.85 (0.04)	6.61 (0.13)
0.025	0.441	98.68 (2.02)	1.98 (0.05)	4.47 (0.12)

This table presents results from a simulation in which output prices ( $p$ ) and capital costs ( $p_K$ ) follow Geometric Brownian Motion. The series have zero trend and are uncorrelated. The instantaneous variance of the output price equals 0.01. The discount rate equals 0.05. The second column presents the optimal ratio of output to capital prices for investment (the hurdle ratio). The next column gives the median time to investment while the final two columns give average prices conditional on investment. There are 1500 replications for each simulation. Standard errors are reported in parentheses.

Table 2. Mean Preserving Spreads  
in Jump Processes

$\alpha$	$\pi_0$	$\pi_1$	$p_1^*$	$p_0^*$	$t_{med}$	$E(p)$	$E((1-\pi)p_K)$	$E(\Delta\pi)$
<b>A. No Covariance Between Policy Changes and Output Price</b>								
0.00	0.10	0.10	1.76	1.76	14.10 (1.24)	1.75 (0.00)	18.00 (0.00)	-
0.00	0.05	0.15	1.56	2.31	7.99 (0.46)	1.63 (0.01)	17.04 (0.01)	3.23 (0.12)
0.00	0.00	0.20	1.46	2.68	4.36 (0.22)	1.54 (0.01)	16.02 (0.01)	2.30 (0.08)
<b>B. Covariance Between Policy Change and Price</b>								
0.12	0.10	0.10	1.76	1.76	14.10 (1.24)	1.75 (0.00)	18.00 (0.00)	-
0.12	0.05	0.15	1.77	2.51	23.11 (1.48)	1.86 (0.01)	17.10 (0.01)	5.08 (0.15)
0.12	0.00	0.20	1.66	2.92	13.37 (0.93)	1.72 (0.00)	16.02 (0.01)	3.91 (0.13)
<b>C. Impact of Covariance</b>								
0.00	0.00	0.20	1.46	2.68	4.36 (0.22)	1.54 (0.01)	16.02 (0.01)	2.30 (0.08)
0.04	0.00	0.20	1.49	2.73	4.97 (0.27)	1.57 (0.00)	16.01 (0.01)	2.71 (0.10)
0.08	0.00	0.20	1.55	2.80	8.55 (0.36)	1.62 (0.00)	16.01 (0.01)	3.09 (0.11)
0.12	0.00	0.20	1.66	2.92	13.37 (0.93)	1.72 (0.00)	16.02 (0.01)	3.91 (0.13)
<p>This table presents hurdle prices and average prices for output and capital cost (net of tax credit) at which investment takes place. In addition, it reports the expected number of changes in the ITC before investment occurs and the median time to investment. The probability of switching an ITC from a low (high) value to a high (low) value is fixed at .35. The index zero (one) indicates the low (high) ITC state. There are 1500 replications for each simulation. Standard errors are reported in parentheses.</p>								

Table 3. Median Aggregate Capital  
Stocks: Fixed  $\pi$

$\sigma^2$ Year	0.005	0.010	0.015
0	0.435 --	0.098 --	0.008 --
1	0.958 (0.003)	0.277 (0.004)	0.039 (0.001)
2	1.012 (0.005)	0.338 (0.006)	0.061 (0.003)
5	1.120 (0.007)	0.474 (0.012)	0.134 (0.007)
10	1.245 (0.010)	0.686 (0.019)	0.269 (0.017)

Results from authors' simulations based on 1,500 replications. Production function is  $F(K) = \delta \ln(1+K)$  where  $\delta \sim N(1, 0.25)$ .  $\pi$  equals 0.10. Standard errors are reported in parentheses.

Table 4. Median Aggregate Capital  
Stocks by Year: Random  $\pi$

$\pi$	0.10	0.05, 0.15	0.05, 0.15	0.00, 0.20
$\lambda$	-	0.35	0.35	0.35
$\pi$ at t=0	0.10	random	0.05	random
Year				
0	0.098 --	0.164 (0.004)	0.000 --	0.256 (0.007)
1	0.277 (0.004)	0.644 (0.008)	0.005 (0.001)	1.004 (0.044)
2	0.338 (0.006)	0.686 (0.007)	0.085 (0.020)	1.027 (0.010)
5	0.474 (0.012)	0.799 (0.011)	0.509 (0.010)	1.106 (0.013)
10	0.686 (0.019)	0.994 (0.016)	0.800 (0.023)	1.278 (0.024)

Results from authors' simulations based on 1,500 replications. Production function is  $F(K) = \delta \ln(1+K)$  where  $\delta \sim N(1, 0.25)$ . Mean capital stock is reported at time t=0. Standard errors are reported in parentheses.  $\lambda_0 = \lambda_1 \equiv \lambda$  and does not depend on output price.

Table 5. Covariance in Policy Response

$\alpha$ Year	0.00	0.04	0.08	0.12
0	0.256 (0.007)	0.223 (0.006)	0.179 (0.004)	0.055 (0.001)
1	1.004 (0.044)	0.878 (0.014)	0.692 (0.006)	0.228 (0.004)
2	1.027 (0.010)	0.910 (0.009)	0.724 (0.008)	0.261 (0.005)
5	1.106 (0.013)	1.007 (0.015)	0.836 (0.012)	0.364 (0.009)
10	1.278 (0.024)	1.177 (0.017)	1.020 (0.017)	0.570 (0.024)

Results from authors' simulations based on 1,500 replications. Production function is  $F(K) = \delta \ln(1+K)$  where  $\delta \sim N(1, 0.25)$ . Mean capital stock is reported at time  $t=0$ . Standard errors are reported in parentheses.  $\alpha_0 = \alpha_1 \equiv \alpha$  while  $\pi_0 = 0.00$  and  $\pi_1 = 0.20$ .

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