Path Dependency, Hysteresis and Macrodynamics

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Abstract

This chapter explores the meaning and application of concepts of path dependency in macrodynamics, with a particular focus on hysteresis. It is argued that hysteresis is a particular type of (rather than a synonym for) path dependency, and that the concept emerges from features of the adjustment dynamics of economic systems, rather than the non-uniqueness of equilibrium. Distinctions are made between stating (or assuming) hysteresis, characterizing hysteresis, and providing a model of hysteresis, and concrete examples of appeals to hysteresis in macrodynamic analysis are used to illustrate these distinctions. Finally, a case is made for retaining linear unit/zero root models of "hysteresis" in macrodynamic analysis, as a useful first approximation and alternative to traditional equilibrium analysis.

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1. Introduction

This chapter explores the meaning and application of concepts of path dependency in macrodynamics. Particular attention is paid to the concept of hysteresis – what it is (and isn't), and how hysteresis can and should be used as an "organizing concept" in macrodynamic analysis. The chapter is thus intended as a "practitioner's guide" rather than as a literature survey. Its purpose is to discuss what serious consideration of path dependency implies for macrodynamic modelling, and to show how hysteresis can and should be incorporated into macrodynamic models – or, in other words, where hysteresis fits into the "toolbox" of macrodynamic model builders.

Two of the central premises of the discussion that follows are that, properly conceived: (a) hysteresis is a particular type of, rather than a synonym or euphemism for, path dependency, the latter being a broader concept with more general implications for the methodology of macrodynamic modelling; and (b) hysteresis emerges from reconsideration of the asymptotic stability properties of purported attractors (such as traditional equilibria) rather than their (non) uniqueness (as in popular unit/zero root models of hysteresis), and involves non-linearities and structural change along the dynamic adjustment path of a system. In what follows, conceptual distinctions are drawn between stating (or asserting) hysteresis, characterizing hysteresis, and providing a model of hysteresis. Concrete examples of appeals to hysteresis in macrodynamic analysis are used throughout to illustrate these distinctions. The relationship between hysteresis and fundamental uncertainty is also investigated, and the potential for reconciling the two is demonstrated. Finally, and despite their having been subject to criticism, a case is made for retaining unit/zero root models of "hysteresis" in macrodynamic analysis.

The remainder of the chapter is organized as follows. Section 2 discusses the role of organizing concepts in model building, identifying hysteresis as one example of a path-dependent organizing concept, and distinguishing hysteresis from the broader concept of path dependency. Section 3 then scrutinizes the concept of hysteresis as it has been used in macrodynamics. Attention is drawn to the distinction between stating (or asserting) hysteresis, characterizing hysteresis, and providing a model of hysteresis. Two main models of hysteresis are presented: linear, unit/zero root models; and non-linear models of "true" hysteresis. The former are shown to provide only a crude approximation of hysteresis, failing to capture some of the most important features of the process – features that are clearly discernable in models of "true" hysteresis. It is also shown that the latter can be reconciled with fundamental uncertainty. In section 4, a case is nevertheless made for *retaining* unit/zero root analysis in macrodynamics. It is argued that, from a pragmatic perspective, unit/zero root models can provide both a useful first approximation of "hysteresis" effects in macrodynamics, and a valuable alternative organizing concept to that of traditional equilibrium. Finally, section 5 concludes.

2. Path dependency, hysteresis and model "organizing concepts"

i) What is *path dependency?*

All formal models are constructed around "organizing concepts," the most common example of which in macrodynamics (and economics in general) is the concept of equilibrium. Organizing concepts make an important contribution to the architecture of formal models, in the context of which *macrodynamic* theories are usually articulated. Concepts of path dependency – such as cumulative causation, lock in and hysteresis –

are, like the familiar concept of equilibrium, best understood as model organizing concepts.[1]

But before we look more closely at specific path dependent organizing concepts – and in particular, hysteresis – it is important to first contemplate a more basic question: what exactly *is* path dependency?[2] Broadly speaking, a dynamical system displays path dependency if earlier states of the system affect later ones – including (but by no means limited to) anything that can be construed as a "long run" or "final" outcome of the system. In other words, path dependency is synonymous with the principle that "history matters". In contrast, path independent systems are ahistorical: their configurations (at least in the long run) are unaffected by events in the past. A good example of a path independent system is any system that embodies a "traditional equilibrium" as its organizing concept. A traditional equilibrium is both defined in terms of exogenous data that is imposed upon the system from without, and displays asymptotic stability (i.e., it is a position to which the system will return following any arbitrary displacement). In other words, traditional equilibrium configurations – or what Kaldor (1934) termed *determinate equilibria* – are "both defined and reached without reference to the (historical) adjustment path taken towards them" (Setterfield, 1997a, p.6).[3] It will immediately be recognized from the foregoing that traditional equilibrium is the canonical organizing concept in economic theory, with which organizing concepts based on path dependency are to be contrasted.

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[1] See Setterfield (1995) for a survey of these concepts of path dependency.
[2] Obviously, this is a counterpart to the more frequently rehearsed question "what is equilibrium?" on which see, for example, Setterfield (1997a, pp.5; 1997b, pp.48-51).
[3] See also Lang and Setterfield (2006-07, pp.198-9) on the concept of traditional equilibrium analysis and Setterfield (1998a) on the contributions of Kaldor (1934).

ii) What *is path dependent?*

Once the possibility of path dependency in dynamical systems is admitted, it is reasonable to ask: what features of a system can be affected by path dependency? Of primary interest in this regard are system outcomes – which in the context of macrodynamics would include growth rates, inflation rates, or, indeed, any "static" macroeconomic variable (such as the level of aggregate output or the general price level) that is understood to result from a prior sequence of adjustments within a macroeconomic system. On this basis, it is tempting to suggest that path dependency is potentially ubiquitous in macrodynamic outcomes – and indeed, this position is defensible. Hence even in formally static models, in which variables are presented as interacting simultaneously and there is no pretense of a temporal ordering accompanying cause and effect statements, it is common to assert that outcomes are the result of a sequential adjustment process. Consider, for example, textbook comparative static exercises performed using the *IS-LM* apparatus, in which the appearance of instantaneous adjustment from one outcome to another is usually accompanied by an intuitive appeal to a series of disequilibrium adjustments that eventually give rise (thanks to asymptotic stability) to the new outcomes of the system. Even models involving rational expectations – in which instantaneous adjustment is conceived as possible on the basis of agents' knowledge of the formal structure of the system, and hence their prior calculation of the consequences of any change – allow for purportedly inter-temporal adjustment processes. The latter arise whenever decision makers need time to learn the "true model" of the system they inhabit, when random shocks create "price surprises" and hence disequilibrium resource allocations that need to be corrected through subsequent adjustments, and/or when systems contain “pre-determined” variables (i.e., variables whose values are fixed at any point in time – such as the capital stock) that constrain the ability of the system to “jump” into its final configuration. Moreover, even in the absence of these mechanisms, it should be noted that, absent shocks and the adjustments (instantaneous or otherwise) they necessitate, the cumulative experience of the *same* outcome creates a “history” that may (in principle) affect the structure of a system and hence its outcomes in the future. Ultimately, then, it can be argued that *all* models postulate sequential adjustment processes of some sort that may give rise to the path dependency of their outcomes (Setterfield, 1995, pp.11-12).

It is important to note at this juncture that the outcomes discussed above as being susceptible to path dependency may take the form of equilibria. Although it is quite possible for a path dependent system to produce outcomes that resemble nothing more than an on-going series of nonequilibrium and non-equilibrating adjustments, it is also possible that a configuration that would ordinarily be associated with a position of equilibrium – such as the “balance of forces” characteristic of market clearing, or the constant rate of expansion over time characteristic of a steady state – could be the outcome of a path dependent process. Of course, said equilibrium configuration will necessarily be a product of the prior adjustment path taken towards it. Nevertheless, what we are suggesting here is that, while the canonical concept of “traditional equilibrium” as defined earlier is clearly incompatible with path dependency, the concept of equilibrium *per se* is not. Suppose, then, that we think of traditional equilibria as configurations that can be identified *a priori* without knowledge of the actual adjustment path taken towards them, and that therefore characterize systems whose dynamics are of secondary importance (because they serve only to guide the system towards a configuration that is independent of the precise sequence of adjustments the dynamics describe). Then following Lang and Setterfield (2006-07, p.200), we can identify "path-dependent equilibria" as having the opposite characteristics. In other words, path-dependent equilibrium configurations are influenced by the specific (historical) sequence of adjustments that a system undertakes in the process of reaching or attaining them, as a consequence of which the system's dynamics are of *primary* importance, since they are intrinsic to the very creation of *any* configuration (including those that can be interpreted as equilibria) that the system experiences.[4]

But is this claim – that path-dependent processes can result in "path-dependent equilibria" – really sustainable? It was stated earlier that path dependency is synonymous with the principle that "history matters". But isn't it the case that the concept of an equilibrium always betrays this principle? Hence consider what achieving a state of equilibrium (of any description) involves. However defined, equilibrium is typically conceived as a state from which there will be no endogenously-generated tendency to deviate. But as noted by Setterfield (1997):

> What this suggests ... is that, once we are in equilibrium, history effectively ends; the future is predetermined by the time path corresponding to the equilibrium that has been achieved. The sequence of outcomes of which this time path is composed does not "matter," because the absence of any endogenous tendency to change dictates that it cannot affect the subsequent outcomes of the system in any way that would cause deviation from the equilibrium time path.
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> (Setterfield, 1997, p.66)

[4] Note that by emphasizing the role of the adjustment path in *creating* (rather than just *selecting*) equilibrium outcomes, the discussion above distinguishes systems with path-dependent equilibria from those with locally stable multiple equilibria. See also Kaldor (1934) and Setterfield (1998a).

In short, it would seem that achieving a state of equilibrium should be regarded as incompatible with the principle of path dependency.

Closer inspection, however, reveals that this need not be the case. Hence it is not essential – and given the potential ubiquity of path dependency, may not be at all prudent – to treat positions of equilibrium as states from which there can *never* be an endogenously-generated tendency to deviate. This is because, as intimated earlier, behavioural change may eventually result as a response to the cumulative experience of "states of rest" themselves. This cumulative experience can eventually promote feelings of boredom or a sense of disappointed aspirations (Witt, 1991, pp.88-9), or (in an environment of non-cooperative interaction characterized by deficient foresight) a perceived need to change behaviour in order to avoid conceding first-mover advantages to others – even when (as perfect foresight would reveal) neither first-mover advantages nor any intent on the part of others to change their behaviour actually exists (Setterfield, 1997, p.67). Any of these factors may create a psychological imperative to change behaviour in response to repeated experience of equilibrium conditions themselves, resulting in an endogenously generated disturbance to the equilibrium (and hence a change in outcomes).[5] It is for this reason that Setterfield (1997, pp.68, 70) recommends that once the possibility of path dependency is recognized, all equilibrium states that are postulated as describing the actual outcomes of economic systems be regarded as temporary or "conditional" equilibria, where "a conditional equilibrium represents a state

[5] Note that this is not the same as contemplating the eventual occurrence of an exogenous shock that disturbs an equilibrium. Hence there is always the possibility of explaining an endogenously-generated behavioural change arising from the cumulative experience of equilibrium conditions in terms of the dynamics of the system itself – even if this is only possible *ex post* (as will be the case when behavioural change involves genuine innovation) rather than as an *a priori* extension of the model of the system (which would allow such change to be predicted). This can never be so in the case of an exogenous shock which, by definition, is imposed upon a system from without.

of rest brought about by ... [a] temporary suspension of the forces of change endogenous to a system" (Setterfield, 1997. p.70).[6] This explicitly allows for the possibility noted earlier – where "the cumulative experience of the *same* outcome creates a "history" that may (in principle) affect the structure of a system and hence its outcomes in the future" – thus reconciling (conditional) equilibrium states with the concept of path dependency.[7]

In short, taking path dependency seriously does not involve dispensing with the notion of equilibrium *per se*.[8] Instead, the possibility of path-dependent systems achieving equilibrium outcomes can be entertained, as long as it is understood that these will be path dependent rather than traditional equilibria, and that all such configurations are necessarily conditional. As we will see in section 3, this observation has been important in the development and use of the concept of hysteresis in macrodynamics. Hence most applications of hysteresis in macrodynamics involve amending the dynamics of traditional equilibrium models, transforming said models into path dependent systems

[6] Strictly speaking, one might argue that the *forces* of change have not been suspended at all – rather, it is simply the case that the *manifestation* of these forces in actual change will be absent for discrete periods (during which a specific conditional equilibrium position is maintained), by virtue of the fact that change results from the cumulative experience, over a discrete interval of time, of a particular "state of rest". Note that the term "conditional" equilibrium as used here is inspired by Crotty's (1994) concept of conditional stability in Keynesian macroeconomic models. See also Chick and Caserta (1997) on the related concept of "provisional" equilibrium.

[7] It should be noted at this point that the "suspension of the forces of change" necessary to generate a conditional equilibrium can also be brought about in an entirely artificial fashion by the analyst him/herself. In other words, it is possible to acknowledge the existence of path dependency in the object of analysis, but choose to set it aside. The purpose of this "locking up without ignoring" path dependency in order to generate a conditional equilibrium is to focus attention on properties of a system other than path dependency (on which see, for example, Kregel, 1976; Setterfield, 1997; Lang and Setterfield, 2006-07). Of course, it is when path dependency is *not* "closed down" in this fashion – so that conditional equilibria arise only from a "temporary suspension of the forces of change" that is endemic to the system that is being studied, that path dependent organizing concepts come into their own as a means for structuring the analysis of a dynamical system.

[8] In and of itself this claim is not at all new – it was effectively made by Kaldor (1934) with the introduction of his concept of a definite-indeterminate outcome. Hence for Kaldor, an outcome may be indeterminate (it cannot be defined and reached independently of the path taken towards it) but nevertheless definite, in the sense that it eventually reaches a (historically contingent) position that has the characteristics of an equilibrium.

with outcomes that are still recognizable as equilibria, but now of the path-dependent variety.[9]

It is not, however, only the *outcomes* of dynamical economic systems that may be subject to path dependency. Other features of such systems, which are traditionally regarded as datum exogenous to their dynamics, may also be affected by the actual sequence of adjustments undertaken by the system over time. These include any "ceiling" or "floor" values of variables that are not defined as a matter of logic,[10] perhaps the most important of which in macrodynamics is the Harrodian natural rate of growth – the maximum rate of growth that an economy can achieve in the long run. The actual rate of growth need not coincide with the natural rate at any given point in time or even in the long run, but (by definition) it is not possible for the actual rate to exceed the natural rate in perpetuity. In other words, the natural rate constitutes a growth "ceiling".

The value of the natural rate of growth can be derived by first defining the maximum or potential *level* of real output that can be produced at any point in time as:

$$Y_p \equiv \left(\frac{N_{\max}}{L} \frac{L}{P} P \right) \frac{Y_p}{N_{\max}}$$

where Y_p denotes potential real output, $N_{\max}$ is the maximum feasible level of employment,[11] L denotes the labour force and P is the total population. If we assume that both the maximum rate of employment ($N_{\max}/L$) and the labour force participation rate(L/P) remain constant, the identity above yields the expression:

[9] These include, *inter alia* (and perhaps most famously), hysteretic models of the "natural" rate of unemployment.
[10] Examples of variables for which ceiling and/or floor values *are* defined as a matter of logic include the unemployment rate and the capacity utilization rate, both of which are bounded above and below by one and zero, respectively.
[11] This maximum level of employment may be determined by labour market conditions (for example, it may coincide with conditions of labour market clearing or full employment) or by a constraint such as the availability of capital in the context of a fixed-coefficient production technology.

$$y_p = n + q \quad [1]$$

where y_p denotes the potential (i.e., natural) rate of growth, n is the rate of growth of the population and q denotes the rate of growth of labour productivity.

There is a long tradition in macrodynamics of regarding the determinants of the natural rate of growth – and hence the natural rate itself – as exogenous.[12] Even in contemporary endogenous growth theories inspired by Romer (1986, 1990) and Lucas (1988), in which technical change (and hence the rate of growth of labour productivity) is explicitly modelled, the ultimate determinants of technical change (such as preferences for the accumulation of human capital) are imposed from without. In other words, the natural rate of growth is typically regarded as invariant to the economy's actual experience of growth: it is treated as being path *in*dependent. But authors in the Kaldorian tradition have long regarded this as a mistake, suggesting that, for example, faster or slower actual rates of growth in the recent past can induce faster or slower population growth (through migration) and/or technical change (through dynamic economies of scale).[13] Suppose, then, that we write:

$$q = \alpha + \beta y_{-1} \quad [2]$$

where y denotes the actual rate of growth. Equation [2] is a version of the Verdoorn law, according to which rapid growth induces technical change and hence increased productivity growth.[14] Substituting equation [2] into equation [1] yields:

$$y_p = n + \alpha + \beta y_{-1} \quad [3]$$

[12] See, for example, Solow (1956) and subsequent analyses of growth in this tradition.
[13] See, for example, Cornwall (1977) and McCombie and Thirwall (1994).
[14] The substance of this claim can be traced back to Adam Smith's famous dictum that the division of labour depends on the extent of the market. See McCombie, Pugno and Soro (2002) for a modern treatment and appraisal of the Verdoorn law.

According to [3], the natural rate of growth is endogenous to the actual rate of growth experienced in the recent past. In other words, the "ceiling" defining the maximum rate of growth that the economy can achieve is now path dependent.[15]

Although the overwhelming majority of research focuses on the implications of path dependency for the outcomes of dynamical systems, what the discussion above illustrates is that other important features of such systems may also be path dependent. In general, then, consideration of path dependency necessitates that, instead of thinking of the adjustment paths of dynamical systems as being circumscribed or contained by path-independent ceilings, floors or point attractors (such as traditional equilibria), we confront the possibility that all such constructs may be subject to endogenous revision in the course of a system's adjustment through time. In other words, we cannot overlook the possibility that ultimately, in any economic system, "the only truly exogenous factor is *whatever exists at a given moment of time*, as a heritage of the past" (Kaldor, 1985, p.61, emphasis in original).

iii) How or why does path dependency arise?

Having established both what path dependency is and what features of a system may be subject to path dependency, we can now investigate more closely *how* or *why* path dependency arises in dynamical economic systems.[16] These issues can and have been addressed philosophically (see, for example, Elster, 1976). But the same issues are also addressed and answered (at least implicitly) by different specific path dependent

[15] See, for example, Leon-Ledesma and Thirwall (2000, 2002) for empirical evidence relating to this idea.
[16] Note that this issue has already been anticipated by our discussion of the natural rate of growth in the previous section.

organizing concepts, all of which purport to show exactly how earlier states affect later ones (including anything that can be construed as a "long run" or "final" outcome).

As intimated earlier, there are numerous concepts of path dependency of which hysteresis is just one. Hysteresis is thus properly regarded as a *particular type of* (rather than a *synonym for*) path dependency – an important point that is, unfortunately, lost on much macrodynamic analysis that uses the term hysteresis. The problem with such analysis is that its use of a specific term (hysteresis) as a synonym for a more general term (path dependency) serves to blur boundaries and obscure the defining features of hysteresis proper.[17] Hence even the otherwise laudable survey by Göcke (2002) begins by identifying hysteresis with the notion that "transitory causes can have permanent effects". This most certainly *is* a feature of hysteresis, but it is by no means a defining feature, since it is also a property of other concepts of path dependency (such as cumulative causation). In anticipation of the discussion in section 3 below, what can be said about hysteresis at this point that helps to set it apart from other concepts of path dependency is the following. First, properly conceived, hysteresis is a form of path dependency that emerges from reconsideration of the asymptotic stability of purported attractors (e.g., traditional equilibria) – and in particular, the assumed invariance of these attractors to the precise adjustment path taken towards them – rather than their non-existence or non-uniqueness. In other words, in terms of the classical triad of equilibrium analysis – existence, uniqueness and stability – our "point of entry" for the study of hysteresis is (or should be) the latter, leading us to focus on properties of the *adjustment dynamics* of a system. Second, properly conceived, hysteresis involves non-linearities and structural change along the dynamic adjustment path of a system. We will also come to see that

[17] See also Amable et al (1993, pp.123-4).

hysteresis can be associated with more specific properties such as *remanence* and selective memory that are not, in general, characteristic of the broader class of dynamical systems in which "earlier states affect later ones" and "transitory causes can have permanent effects" (Amable et al, 1993, 1995; Cross, 1993, 1995).

4. The Concept of Hysteresis in Macrodynamic Analysis

We are now in a position to more fully and thoroughly explore hysteresis and its use is a organizing concept in macrodynamics. The discussion in this section will bear out the assertions made about hysteresis at the end of the previous section by analyzing the various guises in which the concept of hysteresis has appeared in macrodynamics. As will become clear in what follows, it is possible to distinguish between stating (or asserting) hysteresis, characterizing hysteresis, and providing a model of hysteresis. Moreover, with the exception of the analytical "detour" that is created by popular unit/zero root models of dynamical systems, the specific properties of hysteresis alluded to in section 2 become clearer as we progress through this hierarchy of representations of hysteresis.

i) Stating (or asserting) hysteresis

The simplest way of introducing hysteresis into macrodynamic analysis is to simply state or assert its existence in the process of discussing a particular economic phenomenology. A good example of this is provided by Jenkinson's (1987) discussion of "hysteresis" in the natural rate of unemployment or NAIRU.[18] The following simple model summarizes the essence of the claims made by Jenkinson:

$$\dot{p} = -\alpha(U - U_n) \qquad [1]$$

$$U_n = f(U_{-1}) \ , \ f' > 0 \qquad [2]$$

where $\dot{p}$ is the rate of change of inflation, U is the actual rate of unemployment and U_n denotes the natural rate of unemployment or non-accelerating inflation rate of unemployment (NAIRU).[19] Equation [1] is a standard accelerationist Phillips curve, according to which inflation will increase (decrease) over time whenever the actual rate of unemployment is below (above) the NAIRU. Equation [2], meanwhile, posits some functional dependence of the NAIRU on the actual rate of unemployment in the recent past.

The significance of this second equation becomes apparent when we consider the effects of a shock to the rate of unemployment, which raises the latter above the value of the NAIRU. The first impact of this shock is to lower the rate of inflation via equation [1] – in other words, the economy moves along an orthodox, negatively-sloped short-run Phillips curve (SRPC), shown in Figure 1 by the movement from (U_n, p_1) to (U_1, p_2). Conventional NAIRU theory suggests that in response to this situation, the actual rate of unemployment will move back towards the NAIRU. If we assume for simplicity that this adjustment is completed within a single period, the economy will thus arrive at the point

[18] Such discussions were by no means uncommon during the 1980s and Jenkinson's paper is but one example of what is identified here as stating or asserting hysteresis. Indeed, the purpose of singling it out is because judged as an exercise in applied macroeconomics devoted to describing and critiquing NAIRU theory and its implications for macroeconomic policy (rather than as an exercise in identifying the abstract features of hysteresis) it is a model of clarity.

[19] The terms "natural rate of unemployment" and "NAIRU" are used interchangeably in this paper, despite the fact that there are arguably important conceptual differences between the two. Fortunately these differences are not central to the analysis that follows, which is why they are overlooked.

(U_n, p_2) in Figure 2, and the dashed vertical line passing through U_n would be interpreted as the (vertical) long-run Phillips curve (LRPC). But according to equation [2], it is the value of the NAIRU that will respond to the increase in the actual rate of unemployment with which we began. Assuming for simplicity that $f' = 1$ – in other words, that the NAIRU adjusts so as to become equal to the actual rate of unemployment established at the start of the exercise – then the actual unemployment rate U_1 can now be identified as the new value of the NAIRU ($U_n^{'}$ in Figure 1).[20] This means that, *ceteris paribus*, the economy will remain at the point (U_1, p_2) since, with $U_1 = U_n^{'}$, the system in equations [1] and [2] has now reached a steady state where $\dot{p} = 0$. Clearly, the long run or final outcomes of the system depend on events in the past. Had the shock to unemployment with which we began never happened, the economy would still be in equilibrium at (U_n, p_1). But it did happen and, as a result, not only was the economy temporarily displaced from equilibrium, but equilibrium conditions were subsequently recovered at a *different* final equilibrium *position* (the configuration ($U_1 = U_n^{'}$, p_2)). This, it is claimed, demonstrates the workings of hysteresis.

[FIGURE 1 GOES HERE]

[20] Note that if we were to re-write equation [2] to make the change in the NAIRU depend positively on the difference between the values of the actual rate of unemployment and the NAIRU in the previous period, and then add a third equation to our system describing the adjustment of the actual rate of unemployment towards the NAIRU (as posited in conventional NAIRU theory), then the new steady state value of the unemployment rate (the new NAIRU) would lie somewhere between the U_n and $U_n^{'}$ in Figure 1. Figure 1 can therefore be thought of as contrasting two extreme cases – full reversion of the actual rate of unemployment towards the NAIRU, and full adjustment of the NAIRU towards the actual rate of unemployment.

Finally, note that as shown in Figure 1, we can join the points (U_n, p_1) and ($U_1 = U_n^{'}$, p_2) to establish that the shape of the LRPC is negatively sloped.[21] The vertical dashed lines passing through U_n and $U_n^{'}$ have no behavioural meaning – suggesting that NAIRU theory and its associated policy implications (including the purported long run inefficacy of aggregate demand management) do not survive the introduction of hysteresis effects.[22]

But does the model in equations [1] and [2] actually capture the dynamics of hysteresis? The obvious problem that we confront in trying to address this question is that it is not clear what *causes* U_n to depend on U in equation [2]: the function $f(.)$ is a "black box". At this point, authors such as Jenkinson (1987) generally appeal to what may be termed "backstories" to justify the analytical structure of the model in [1] and [2]. For example, suppose that we write:

$$U_n = g(Z) \quad , \quad g' < 0 \qquad [3]$$

$$Z = h(U_{-1}) \quad , \quad h' < 0 \qquad [4]$$

where Z denotes some variable affecting the ability or willingness of workers to find work (and hence the value of the NAIRU). For example, Z may be the value of the insider real wage prevailing in the labour market (assuming that the latter is characterized by insider-outsider relations). It is obvious that substitution of [4] into [3] produces [2]

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[21] Once again, were we to re-write equation [2] to make the change in the NAIRU depend positively on the difference between the values of the actual rate of unemployment and the NAIRU in the previous period, and then add a third equation to our system describing the adjustment of the actual rate of unemployment towards the NAIRU, the LRPC would not be identical to the SRPC (as in Figure 1). Instead, its slope would be steeper than that of the SRPC, the precise slope depending on the relative speeds of adjustment of inflation (in equation [1]), of the NAIRU towards the actual rate of unemployment, and of the actual rate of unemployment towards the NAIRU. However, as long as the NAIRU is at least somewhat sensitive to the actual rate of unemployment, the resulting LRPC will always be negatively sloped.

[22] See also Cross (1995).

(with $f' = g'h'$), but the advantage that equations [3] and [4] confer on the model is that they furnish a “backstory” that appears to un-pack $f(.)$ and thus justify the results associated with the interaction of [1] and [2]. Hence suppose that any increase (decrease) in the actual rate of unemployment from its initial long run equilibrium value entices insiders to restore long run equilibrium in the labour market by increasing (decreasing) the insider wage, so that the latter matches the marginal product of labour at the level of employment associated with the new rate of unemployment. For example, insiders may seek to increase the rents they earn in connection with their employment in the event of an increase in unemployment, or maintain the employed status of newly hired workers in the event of a decrease in employment.[23] These behaviours would remove any incentive for firms to change their pricing and production plans in order to recover conditions of long run equilibrium by adjusting employment (to reconcile the marginal product of labour with a given insider wage) – because conditions of long run equilibrium have already been recovered (by adjustment of the insider wage) at the new rates of employment/unemployment. The new actual rate of unemployment with which we began the exercise is thus enshrined as the new long run equilibrium rate of unemployment, and the NAIRU is described as displaying hysteresis because its value depends on past values of the actual unemployment rate.

But “backstories” of this nature are of little help if our objective is to identify (and successfully apply) the abstract properties of hysteresis in macrodynamics. Hence even at the end of the preceding exercise, we are left only with the observation that an outcome

[23] Obviously such actions depend on a number of conditions, including the ability of insiders to revise the insider real wage without forcing it above the value of the outsider real wage plus turnover costs – an event that would undermine their own status as employees by making them vulnerable to replacement by outsiders.

(in this case, a long run equilibrium) depends on events in the past. Said outcome is, therefore, clearly path dependent – but is it hysteretic? In point of fact we simply cannot know, without we are provided with more details about the system's dynamics that would allow us to point to some feature of these dynamics that can be identified with hysteresis but not with other concepts of path dependency.[24] What this illustrates is that if we are interested in distinguishing hysteresis as a specific concept of path dependency, "backstories" of the sort identified above are no substitute for a full and proper examination of a system's dynamics. It is for these reasons that the claim that we observe hysteresis in the operation of systems such as equations [1] and [2] can be identified as no more than a *statement* or *assertion* on the part of the model builder.

ii) Characterizing hysteresis

The project of characterizing hysteresis is associated with the work of Setterfield (1997a, chpt.2; 1998) and Katzner (1998, chpt.13; 1999). In essence, it comprises an attempt to return to the first principles of economic dynamics in an effort to free the latter from the mechanistic, ahistorical grip of traditional equilibrium analysis and, in particular, the asymptotic stability properties of such equilibria. The concept of hysteresis emerges from this exercise as an alternative (to traditional equilibrium analysis) way of thinking about macrodynamics. In what follows, we focus on the analysis in Setterfield (1997a, chpt.2; 1998), making periodic references to what Katzner (1999) identifies as

[24] Note, for example, that in the model developed above, increasing unemployment today means that unemployment will be higher in the future. Such self-reinforcing tendencies are by no means inconsistent with hysteresis, but they are also characteristic of cumulative causation.

the fullest and most pertinent characterization of hysteresis in economics.[25] As will be made clear, these characterizations are essentially equivalent.

The essential insights of Setterfield (1997a, chpt.2; 1998) can be illustrated in the context of the same NAIRU theory to which we appealed in the previous section.[26] Hence suppose that we begin by re-writing equation [4] as:[27]

$$Z_t = h_t(U_{t-1}) \qquad [4a]$$

Defining $dU_i = U_i - U_{i-1}$ for all i, consider now a series of "cumulatively neutral" changes in U (starting from the initial steady state equilibrium position $U_{t-1} = U_n$), such that:

$$DU = \sum_{t=1}^{n} dU_t = 0$$

---

[25] Katzner (1999) actually identifies *three* "characterizations" of hysteresis – the one alluded to above that is the focus of attention in what follows, a second that corresponds to the concept of path dependency as defined earlier, and a third that corresponds to the property of irreversibility discussed in section 3(iii).

[26] Both of the above-mentioned references are, in turn, based on Setterfield (1992).

It is useful to continue discussing hysteresis in the context of labour market dynamics in general and NAIRU theory in particular for two main reasons. First, NAIRU theory involves a concrete application of adjustment dynamics in macroeconomics that is universally familiar. Second, it is one of the two main literatures in which appeal to the concept of hysteresis was popularized in economics two decades ago (see, for example, Göcke, 2002, p.167). However, it is important to bear in mind two things. First, hysteresis is a dynamical process that could, in principle, affect *any* dynamical system: "the concept of hysteresis refers back to a set of *formal properties*, independently of the various phenomenologies within which it is liable to be encountered (magnetism, ferro-electricity, physical mechanics, various fields of economics, etc.)" (Amable et al, 1993, p.124). Second, our main interest here is in uncovering the abstract properties of hysteresis, rather than exploring its concrete application to any particular phenomenology.

[27] Note that by combining equation [4b] with equation [3] we get:

$$U_n = g(h_t(U_{-1}))$$

which can be written as:

$$U_n = f_t(U_{-1})$$

This is substantively similar to the key equation of motion postulated by Katzner (1999, p.177), which appears as:

$$x_t = f^t(x_{t-1}, \varepsilon_t) \qquad [5K]$$

Indeed, apart from the inclusion of the random disturbance term ε_t, the preceding expression is *exactly* the reduced form that would result from combination of equations [1], [2] and [10] in Setterfield (1998).

In other words, we are forcing *U* along an adjustment path that leads the variable back to its initial value (U_n). The question, however, is whether or not this is still the steady-state value of *U*. In other words, at the end of the series of cumulatively neutral changes in *U* described above, is *U* "back in equilibrium"? Or is U_n now merely a disequilibrium value that we are forcing *U* to attain *en route* to a new long run equilibrium value, $U_n^{'}$?

In order to address these questions, suppose initially that:

(a) $$h_t^{'} \neq 0 \text{ for some } t = 2,...,n+1$$

and:

(b) $$DZ = \sum_{t=1}^{n} h_{t+1}^{'}.dU_t = 0$$

Condition (a) insists that short run changes in *Z* occur as *U* traverses the cumulatively neutral adjustment path described above. But condition (b) insists that these short run changes are, themselves, cumulatively neutral – they "cancel out" over the course of the adjustment path followed by *U*. Hence the long run value of *Z* is unaffected by the sequence of adjustments undertaken by *U*, so that the long run value of *U* is similarly unaffected. In short, when *U* has completed the cumulatively neutral adjustment path described above and thus regained the value U_n, it is back in equilibrium and the system in equations [1] and [2] will achieve a steady state (with $\dot{p} = 0$).

By establishing the path *in*dependence of the NAIRU, this exercise draws to attention an important result: simply making a parameter of a system dependent on the lagged actual value(s) of the systems outcome(s) – as in equations [2] or [4] in the previous sub-section – does not suffice to generate path dependency in the long run or final outcome of the system. This is clearly evident upon closer inspection of the dynamics of equations [1] and [2] discussed earlier. Hence if we assume that $f' < 0$, and then add a third equation describing the lagged adjustment of the actual rate of unemployment towards the NAIRU (as in conventional NAIRU theory), then following the initial increase in unemployment to U_1 depicted in Figure 1, subsequent adjustments will "undo" the revision of the NAIRU that results (in equation [2]) from this initial increase in unemployment, and U will eventually return to its original steady state equilibrium value, U_n. No amount of "backstories" designed to provide a behavioural foundation for equation [2] can resolve this problem, which is intrinsic to the dynamics of the system at hand. These observations reinforce our earlier claim that the existence of hysteresis (indeed, *any* form of path dependency) in the modelling exercise in the previous sub-section is merely an assertion.

But suppose that we now retain condition (a) whilst replacing condition (b) with:

(c) $$DZ = \sum_{t=1}^{n} h_{t+1}^{'}.dU_t \neq 0$$

As before, condition (a) allows for short run changes in Z as U traverses its cumulatively neutral adjustment path. But condition (c) now states that these short run changes are *not*, themselves, cumulatively neutral – they no longer "cancel out" over the course of the adjustment path followed by U. Hence even though, by construction, we observe $U = U_n$ at the end of the cumulatively neutral adjustment path followed by U, U_n is no longer the value of the NAIRU (so $U = U_n$ will not produce a steady-state outcome in equations [1] and [2]). Instead, defining $Z' = Z + DZ$ where the value of *DZ* is given by condition (c), the value of the NAIRU is now given (via equation [3]) by $U_n^{'} = g(Z')$. If we assume that the system adjusts towards $U_n^{'}$ during the periods *t* = *n*+1, ..., *n*+*s*, and that in so doing the associated changes in *U* have no cumulative impact on *Z* (in other words, $DZ = \sum_{t=n+1}^{n+s} h_{t+1}^{'}.dU_t = 0$), then the system will eventually achieve a steady state when $U = U_n^{'}$. Clearly, the long run outcome of the system has been altered by events in the past (experience of the traverse along the cumulatively neutral adjustment path with which we began): we are observing path dependency. Indeed, according to Setterfield (1997 chpt.2; 1998), we are observing *hysteresis* which, in light of the preceding analysis, exists "when the cumulative impact on the structure and hence the long run outcome of a system of movement along a prior disequilibrium adjustment path is non-zero" (Setterfield, 1998, p.292).[28] This structural change is associated with the "adjustment asymmetries" captured by condition (c), which are in turn associated with *threshold effects*. Hence if events propel a system sufficiently far from its current state that it moves beyond an "event threshold", condition (c) is triggered and the long run outcome of the system will be affected. A corollary of this claim is that as long as the system does *not* stray "sufficiently far" from its current state – i.e., as long as an event threshold is *not* crossed – condition (b) will hold and we will observe *no* change in the system's long run

[28] This analysis identifies $h_j^{'} \neq h_k^{'}$ for some $j, k,\ j \neq k$ in condition (a) as the necessary condition for hysteresis to arise, and condition (c) as the sufficient condition for hysteresis. See Setterfield (1998). This parallels Katzner's (1999, p.178) analysis, in which (in the absence of the term ε_t) the outcome in equation [5K] referred to in footnote 27 above would relapse to a traditional equilibrium outcome if we were to observe $f^t = f^{t-1}$ for all *t*. Since Katzner (1999, p.178) describes the latter condition as a "very special situation", it seems reasonable to infer that he would regard condition (b) in much the same way – which coincides exactly with the interpretation of Setterfield (1998, p.292), as discussed below.

outcomes. The upshot of all this is the *possibility* (since not all shocks will trigger condition (c)) of a particular type of path dependency (hysteresis), associated with non-linear adjustment dynamics that give rise to structural change in a system that alters its position of equilibrium, all in response to specific prior adjustment paths. Setterfield (1998) thus argues that the conceptualization of adjustment dynamics in the exercise above is more general than that found in traditional equilibrium economics. Hence traditional equilibrium analysis implicitly treats condition (b) as universal: event thresholds that could trigger condition (c) are not held to exist in the locale of any equilibrium. A more general treatment of adjustment dynamics would take seriously the possibility that such event thresholds do exist (and that research should therefore be devoted to their identification in concrete macrodynamical systems), and that condition (c) may therefore attain – as a result of which what were previously regarded as traditional equilibria would need to be re-interpreted as path-dependent equilibria.

The value of this analysis is that it begins to provide some sense of what is actually involved in generating hysteresis – i.e., how the process of hysteresis actually works. We learn, for example, that hysteresis is associated with structural change ($DZ \neq 0$ in the example above), induced by the concrete (historical) experience of a system, that is discontinuous and therefore non-linear. Hence note that hysteresis as characterized above can be distinguished from other path-dependent organizing concepts, such as (for example) cumulative causation. Cumulative causation arises when the displacement of a system from some initial position gives rise not to negative feedbacks (as a result of which the initial position may subsequently be regained, as in traditional equilibrium analysis) but to positive feedbacks, so that the initial displacement becomes self-reinforcing. Note, however, that in the characterization of hysteresis above, not every displacement from equilibrium will trigger condition (c). As such, history won't *always* matter: some initial displacements from equilibrium will be "self-correcting" – that is, they will restore initial equilibrium conditions in the manner of traditional equilibrium dynamics – so that no trace of the prior adjustment path will be evident in the long run outcome of the system. With cumulative causation, however, any initial displacement becomes self-reinforcing, so *all* history matters. Moreover, feedbacks need not be positive (as in cumulative causation) to generate hysteresis as characterized above. This is illustrated by the following example, which utilizes the sort of adjustment dynamics described in this sub-section. Consider a transitory shock to the supply of a commodity that increases the price of the commodity far above its initial equilibrium value.[29] Suppose now that this sequence of events creates a popular aversion to the commodity, as a result of which many buyers develop a hitherto non-existent preferential attachment to a substitute commodity. This historically-induced change in preferences will cause a decline in demand for the original commodity, so that even when (by hypothesis) initial supply conditions are restored, we will observe a new (path-dependent) equilibrium price for the commodity that lies *below* the original equilibrium price. The market for the commodity will now gravitate towards this new equilibrium price if traditional commodity market dynamics are present (price rises (declines) in response to excess demand (supply)) and the traverse towards equilibrium is cumulatively neutral with respect to the determinants of equilibrium. In this case, an initial increase in price results in a subsequent price *decrease* – i.e., negative feedbacks are operative.

[29] This example is inspired by Georgescu-Roegen (1950).

But having established the value of the characterization of hysteresis outlined above, it is also important to note that it suffers certain drawbacks. For example, it is not clear what explains the event thresholds that trigger condition (c), or where (within the orbit of an initial equilibrium position) we might expect to find them. Nor is it clear what processes are involved in the time-dependent influence of *U* on *Z* summarized in equation [4a] and condition (a). As a matter of logic, allowing that *either* condition (b) *or* condition (c) might hold is more inclusive (and therefore more general) than insisting on the universality of condition (b) (as in traditional equilibrium analysis). But this does not explain why we would expect to observe the event thresholds that distinguish between the applicability of these conditions in economic systems, nor what makes $h_t^{'}$ non- zero.[30] In short, equation [4a], condition (a) and the event thresholds that distinguish between the applicability of conditions (b) and (c) remain "black boxes". It is for this reason that the analysis above is described as *characterizing* hysteresis: it takes seriously the project of illuminating the specific properties of hysteresis, but without providing a complete *model* of the process that shows exactly how hysteresis comes about. As a result, it necessarily remains incomplete.

iii) Modelling hysteresis

We now turn to the project of explicitly modelling hysteresis. It is possible to distinguish between two separate branches of this project – one which focuses on the presence of unit or zero roots in linear dynamical systems, and a second which, starting from contributions to the physical sciences, purports to describe "true" hysteresis. As we

[30] The importance of this last point arises from the fact that, absent condition (a), there is no possibility whatsoever of condition (c) – i.e., the event thresholds discussed above simply would not exist.

shall see, the project of modelling hysteresis has not always succeeded in advancing our understanding of the properties specific to hysteresis as a particular form of path dependency.

a. The unit/zero root approach

The unit/zero root approach to modelling hysteresis involves postulating the existence of unit or zero roots in systems of linear difference or differential equations. In terms of NAIRU theory, the unit root approach to modelling hysteresis can be illustrated if we replace the Phillips curve in equation [1] with:[31]

$$\Delta p = -\alpha(U - \gamma U_{-1}) + Z \qquad [1a]$$

where $\gamma > 0$ and Z captures influences other than the rate of unemployment on Δp. Note that [1a] can be re-written as:

$$\Delta p = -\alpha(1+\gamma)U - \alpha\gamma\Delta U + Z$$

In other words, equation [1a] essentially postulates that both the level *and* the rate of change of unemployment impact negatively on the rate of change of inflation.

Consider now a constant rate of inflation – i.e., $\Delta p = 0$. Substituting into [1a] yields:

$$U = \gamma U_{-1} + \frac{Z}{\alpha} \qquad [5]$$

Suppose further that we set $U = U_{-1} = U^*$ and $Z = \bar{Z}$. Substituting into equation [5] and solving for U^* yields:

[31] Unit root models of hysteresis in the NAIRU can be found in, *inter alia*, Wyplosz (1987), Franz (1990) and Layard, Nickell and Jackman (1991).

$$U^* = \frac{\bar{Z}}{\alpha(1-\gamma)} = U_n \qquad [6]$$

Notice that the value of U^* so-derived is associated in equation [6] with the value of the NAIRU, U_n. It is straightforward to verify that this association is appropriate by substituting the expression in equation [6] into equation [1a] and noting that the corresponding solution to [1a] is $\Delta p = 0$. The upshot of our analysis thus far, then, is that we can identify a unique equilibrium rate of unemployment associated with steady-state inflation. If this unique equilibrium is asymptotically stable, then we are dealing with a path-independent, traditional equilibrium system in which the long run or final outcome (the equilibrium rate of unemployment) is both defined and reached without reference to the path taken towards it.

Suppose, however, that we set $\gamma = 1$. In other words, the first-order difference equation in [5] is characterized by a unit root. This assumption means that equation [6] cannot be solved for U^*. Instead, based on equation [5], we must be content to write:

$$U = U_0 + \frac{1}{\alpha}\sum_{i=1}^{t} Z_i \qquad [7]$$

where U_0 denotes the unemployment rate in some initial period, 0. In equation [7], U depends on the initial unemployment rate, U_0, together with the entire past history of the variable Z (captured by $\sum_{i=1}^{t} Z_i$).[32] This is true even in the "long run", which can now only be interpreted as a period of calendar time observed whenever $t > n$ for some n that is

[32] Note that even if Z remains constant over time (as was assumed in the derivation of [6]) equation [7] will become:

$$U = U_0 + t\frac{\bar{Z}}{\alpha}$$

So past events – specifically, initial conditions U_0 and the time, t, that has elapsed since these initial conditions were observed – will still influence the value of U at any point in time.

deemed sufficiently large. In the unit root approach, the result in [7] – which makes the unemployment rate at any point in time dependent on earlier states of the system described in [1a], and thus involves path dependency – is called hysteresis.

Note that in the more general case where $\gamma < 1$, equation [7] becomes:

$$U = \gamma^t U_0 + \frac{1}{\alpha}\sum_{i=1}^{t}\gamma^{t-i}Z_i \qquad [7a]$$

Equation [7a] may appear, at first, to have the same properties as equation [7]. But closer inspection reveals that this is not the case: in [7a], the dependence of U on its own past history (summarized by U_0 and the history of Z) is strictly transitory. Hence in the limit, the value of U in equation [7a] tends towards the value U^* in equation [6].[33] The result in equation [7a] is referred to as *persistence*. Hence according to the unit root approach, hysteresis in dynamical systems is a special case, arising whenever we observe a unit root (such as $\gamma = 1$ in the example above). We may, nevertheless, observe persistence in the more general case, according to which the value of an outcome will depend on past events over some discrete interval of time. But in the long run persistence disappears, and

[33] To see this, consider the general solution to the first-order difference equation in [5], which may be written as:

$$U_t = \gamma A b^{t-1} + \frac{\overline{Z}}{\alpha(1-\gamma)}$$

where $U_t = Ab^t$. Since the homogeneous function of equation [5] is $U = \gamma U_{-1}$, we can write:

$$Ab^t = \gamma A b^{t-1}$$

from which it follows that:

$$b = \gamma$$

Substituting this result into the general solution of [5] stated above, yields:

$$U_t = A\gamma^t + \frac{\overline{Z}}{\alpha(1-\gamma)}$$

Inspection of this last expression reveals that, since $\gamma < 1$ by hypothesis so that $\lim_{t\to\infty} A\gamma^t = 0$:

$$\lim_{t\to\infty} U_t = \frac{\overline{Z}}{\alpha(1-\gamma)} = U^*$$

as claimed above.

the outcome will converge onto a value (such as U^* above) that is defined and reached independently of the path taken towards it. Ultimately, then, there is no path dependency of any description in systems with persistence – they simply describe the gradual adjustment of traditional equilibrium systems towards their long run or final outcomes.

As intimated earlier, the essential difference between unit and zero root analyses is the choice of discrete or continuous time (i.e., the use of difference or differential equations to explain the motion of a dynamical system). A good example of a zero root (continuous time) system is provided by the following model, based on Lavoie (2006). Hence consider the following system of equations built around the now familiar accelerationist Phillips curve in equation [1]:

$$\dot{p} = -\alpha(U - U_n) \qquad [1]$$

$$U = \beta + \varphi r \qquad [8]$$

$$r = r_n + \delta(p - p^T) \qquad [9]$$

$$\dot{U}_n = \eta(U - U_n) \qquad [10]$$

where r denotes the real interest rate, p^T is a target rate of inflation set by the central bank, and:

$$r_n = \frac{(U_n - \beta)}{\varphi}$$

is the Wicksellian natural rate of interest. Equations [1], [8], [9] and [10] describe a New Consensus model of the economy (see, for example, Clarida et al, 1999; Woodford, 2003). The model consists of an accelerationist Phillips curve accompanied by an IS curve (equation [8]), and a Taylor rule describing the conduct of monetary policy (equation [9]). It is also hypothesized that the NAIRU is endogenous, being sensitive to any deviations of the actual rate of unemployment from the current value of the NAIRU itself (equation [10]).

It follows from combination of equations [1], [8] and [9] that:

$$\dot{U} = -\alpha\varphi\delta(U - U_n) \qquad [11]$$

Equations [10] and [11] together constitute a system of two simultaneous differential equations in two variables (U and U_n). Note that steady-state equilibrium in equations [10] and [11] requires that:

$$\dot{U} = \dot{U}_n = 0$$

This equilibrium condition yields the same isocline from equations [10] and [11], specifically:

$$U = U_n \qquad [12]$$

Note that the result in equation [12] provides us with an equilibrium value for U, while in tandem with equations [8] and [9] it yields $r = r_n$ and hence $p = p^T$. This is the standard equilibrium configuration of a New Consensus model. However, the behaviour of the system when it departs from this equilibrium configuration is less standard, thanks to the operation of equation [10]. Consider, then, Figure 2 below, which depicts the isocline in equation [12]. Assume that the economy begins in equilibrium at point A, but that the economy now experiences a transitory shock to the unemployment rate which, in consequence, rises to $U = U_1 + \varepsilon$. Figure 2 depicts the behaviour of the economy subsequent to this shock, resulting from the operation of equations [10] and [11]. On one hand, the actual unemployment rate moves back towards the initial value of the NAIRU (as shown by the horizontal movement away from point B in Figure 2), as in conventional NAIRU theory. But on the other hand, the value of the NAIRU itself is revised upwards, as a result of $U_1 + \varepsilon > U_{n1}$ in equation [10]. This is captured by the vertical movement away from point B in Figure 2. The upshot of these dynamics is that the economy moves back into equilibrium at point C in Figure 2. Clearly, this involves a new equilibrium value of the unemployment rate (i.e., a new value of the NAIRU, $U_n = U_{n2}$) – an equilibrium position that would not have been attained were it not for the precise prior sequence of events (specifically, the disturbance ε).[34] Equations [1], [8], [9] and [10] therefore describe a path-dependent equilibrium system rather than a traditional equilibrium system. In the zero root literature, this result is called hysteresis.

[FIGURE 2 GOES HERE]

Analytically, the result depicted in Figure 2 can be explained as follows. First, note that equations [10] and [11] can be written in matrix form as:

$$\begin{bmatrix} \dot{U} \\ \dot{U}_n \end{bmatrix} = \begin{bmatrix} -\alpha\varphi\delta & \alpha\varphi\delta \\ \eta & -\eta \end{bmatrix} \begin{bmatrix} U \\ U_n \end{bmatrix} \qquad [13]$$

The Jacobian matrix of the system in [13] is:

$$J = \begin{bmatrix} -\alpha\varphi\delta & \alpha\varphi\delta \\ \eta & -\eta \end{bmatrix}$$

from which we can see that $|J| = 0$ and $Tr(J) = -(\alpha\varphi\delta + \eta) < 0$.[35] Finally, we can calculate the Eigen values or characteristic roots of this matrix as:

$$\lambda = \frac{-Tr(J) \pm \sqrt{[Tr(J)]^2 - 4|J|}}{2} \qquad [14]$$

$$\Rightarrow \lambda_1 = 0, \lambda_2 = \alpha\varphi\delta + \eta$$

[34] Using equations [8] and [9], we can see that the new equilibrium configuration will also involve $r = r_{n2} = (\beta - U_{n2})/\varphi$ and $p = p^T$.

[35] These results suffice to ensure the stability of the system summarized in equation [13] (as illustrated in Figure 2) by appeal to the modified Routh-Hurwitz conditions. See Gandolfo (1997).

As is clear from the solution to [14] above, one of the characteristic roots of the Jacobian matrix in [13] is zero. This is the zero root from which zero root models take their name, and which gives rise to the result depicted in Figure 2 that is associated in these models with hysteresis.

Note, then, that once again, hysteresis is presented as a special case (contingent this time on the existence of a zero root in dynamical systems characterized by linear differential equations). The zero root in [13] will disappear if, for example, equation [10] is replaced with what Lavoie (2006) describes as the conventional "missing" equation of the New Consensus, $U_n = \bar{U}_n$, according to which the NAIRU is an exogenously given constant. The resulting system would be a path-independent, traditional equilibrium system that will converge towards the equilibrium configuration $U = \bar{U}_n, p = p^T, r_n = (\bar{U}_n - \beta)/\varphi$. We will observe *persistence* in this system if adjustment towards its equilibrium configuration is not instantaneous. But the result associated with hysteresis depicted in Figure 2 will disappear.

By formulating explicit models of dynamical systems, unit and zero root analyses claim to locate the exact source of hysteresis in these systems – namely, the existence of unit or zero roots, respectively. While this clarity is, in and of itself, a virtue, it also reveals all of the shortcomings of unit and zero root models of hysteresis – shortcomings that can be readily understood in terms of the characterizations of hysteresis reviewed in the previous sub-section. Hence notice that in the analysis above, so-called hysteresis results ultimately arise from the *non-uniqueness* of equilibrium, rather than from any re-consideration of the traditional *asymptotic stability properties* of equilibrium. This is clearly evident in Figure 2, where the isocline described in equation [12] draws attention to the continuum of equilibrium positions that exists in the dynamical system from which it is derived. Each equilibrium position on this continuum has conventional asymptotic stability properties (albeit within a *very* limited locale of the equilibrium position itself), and the *structure* of the underlying system (and hence the equilibria towards which it can, in principle, converge) is invariant with respect to its adjustment dynamics.[36]

Second, unit and zero root analyses apply to *linear* dynamical systems. No consideration is given to the possibility of non-linearities, which, according to the characterizations of hysteresis reviewed earlier, are essential for generating hysteresis effects. In fact, linearity is the source of the (misleading) result associated with unit and zero root analyses, which suggests that the phenomenon of hysteresis is a special case. This can be illustrated by referring back to conditions (b) and (c) in the previous sub-section, and noting that if $h_i' = h_j' = h'$ for all i, j – which will *always* be the case when *h* (.) is linear – then:

$$DZ = \sum_{t=1}^{n} h_{t+1}' dU_t = h' \sum_{t=1}^{n} dU_t = 0$$

since $\sum_{t=1}^{n} dU_t = 0$ by hypothesis. In other words, absent some discontinuity in the relationship between *U* and *Z* – which cannot exist if *h* (.) is linear – condition (c) is impossible.[37] Instead, a so-called hysteresis result can only emerge if we postulate the special case of a unit or zero root. As was previously illustrated, persistence is the more general case phenomenon associated with unit and zero root models. But persistence is

[36] See also Amable et al (1993, pp.128-30; 1995, pp.169) on this property of unit and zero root systems.

[37] If $h_i' = h_j' = h'$ for all *i*, *j*, then what was identified earlier as the necessary condition for hysteresis is violated.

just non-instantaneous disequilibrium adjustment in an otherwise traditional equilibrium system – it barely merits singling out and naming as a distinct analytical phenomenon.[38]

Finally, unit and zero root systems display irreversibility. In other words, "one may disturb ... [the] system with an exogenous shock of Δx in a control variable x, wait for whatever adjustment takes place and, then, disturb it again with a new shock – Δx, and find that the second end-state does not correspond to the initial one" Dosi and Metcalfe (1991, p.133). This is clearly illustrated in Figure 2, in which the second end-state U_{n2} clearly does not correspond to the first (U_{n1}) despite the fact that the disturbance to the unemployment rate, ε_1, is strictly transitory. The same property is captured in equation [7]. Hence suppose that we observe $Z_i = 0$ for all $i \neq n$ in equation [7]. In other words, we are assuming that $\Delta Z_n = Z_n - Z_{n-1} = Z_n$, and $\Delta Z_{n+1} = Z_{n+1} - Z_n = -Z_n$. Evaluating equation [7] under these assumptions reveals that for all $i \geq n$ we will observe:

$$U = U_0 + Z_n$$

As remarked by Amable et al (1993, p.129), this justifies the claim that in unit and zero root systems, "transitory causes can have permanent affects". But note that a shock/counter-shock sequence in a unit or zero root system, in which both the initial shock and subsequent counter-shock are transitory, of the same magnitude, but of opposite sign, will always completely "wipe" the memory of the system. The initial outcome will be restored leaving no trace of the historical adjustment path of the system. Unit and zero root systems can thus be said to display "super reversibility".

[38] Persistence may, of course, be important in practice if adjustment in a particular traditional equilibrium system is very slow – perhaps even so slow that movement towards equilibrium is slower than the rate at which the data determining the precise position of equilibrium are, themselves, (exogenously) changing. But all of this is already well understood and has been for some time. Hence as Cornwall (1991, p.107) states, "if ... real world change[s] in tastes, technologies and other institutional features are very rapid relative to the rate at which the economy can adjust, the convergence properties of the model take on much less interest and importance than the institutional changes themselves."

Once again, this result can be demonstrated with reference to equation [7]. Hence suppose that we now observe $Z_i = 0$ for all $i \neq n, n+s$ in equation [7], where $Z_n = -Z_{n+s}$. In other words, we assume that $\Delta Z_n = Z_n - Z_{n-1} = Z_n$ and $\Delta Z_{n+1} = Z_{n+1} - Z_n = -Z_n$ as before, but we now also assume that $\Delta Z_{n+s} = Z_{n+s} - Z_{n+s-1} = -Z_n$ and $\Delta Z_{n+s+1} = Z_{n+s+1} - Z_{n+s} = Z_n$. Evaluating equation [7] under these assumptions reveals that for all $i \geq n + s$ we will observe:

$$U = U_0$$

But there is no need to expect super reversibility in systems displaying hysteresis. This is evident from the characterizations of hysteresis discussed in the previous sub-section. Hence in the parlance of the previous section, there is no necessary implication that two consecutive cumulatively neutral sequences of changes in *U*, that begin with a change in *U* of the same magnitude but of opposite sign, will restore the initial long run equilibrium value of *U*. Rather, their exact impact will depend on two things. The first is the initial position of the system relative to the event thresholds that are ultimately responsible for triggering discontinuous changes in the values of long run outcomes in response to prior disequilibrium adjustment paths. Hence despite their symmetry, there is no reason to believe that *both* sequences of cumulatively neutral changes in *U* will necessarily propel the system across event thresholds. The second is the precise quantitative impact of crossing event thresholds on the determinants of the value of the long run outcome. Hence even if event thresholds *are* crossed during *both* the shock and counter-shock sequences of cumulatively neutral disturbances postulated above, there is no reason to believe their impacts on the long run value of *U* will necessarily "cancel out". In short, it is (once again) the *linearity* of unit and zero root systems that gives rise to the "memory wiping" super reversibility result derived above. With non-linearities in the adjustment dynamics of the system, this result disappears.[39]

Of course, none of this remedies the fact that, as noted earlier, the characterizations of hysteresis discussed in the previous sub-section suffer failings of their own. But fortunately, both the super reversibility result discussed above, together with the various other shortcomings of unit and zero root models of hysteresis, are successfully addressed by models of "true" hysteresis. It is to these models that we now turn our attention.

b. "True" hysteresis

Models of "true" hysteresis were introduced into economics by Cross (1993, 1994, 1995) and Amable et al (1993, 1994, 1995), and are based on research in the physical sciences. There are two components to models of "true" hysteresis: the non-ideal relay associated with Krasnosel'skii and Pokrovskii (1989); and the aggregation effects modelled by Mayergoyz (1986). Once again, it is possible to demonstrate the workings of "true" hysteresis in terms of a concrete model of labour market dynamics that has important implications for conventional NAIRU theory.

The model developed below draws on Lang and de Peretti's (forthcoming) hysteretic model of Okun's Law. We begin by specifying a non-ideal relay, describing the employment decision of the i^{th} firm at any point in time, t, which takes the form:

[39] More precisely, it will be observed only as a special case.

$$\begin{aligned} E_{it} &= n_i + 1 \text{ if } E_{it-1} = n_i \text{ and } y_{it} > y_{iu} \\ &= n_i + 1 \text{ if } E_{it-1} = n_i + 1 \text{ and } y_{it} \geq y_{il} \\ &= n_i \text{ if } E_{it-1} = n_i \text{ and } y_{it} \leq y_{iu} \\ &= n_i \text{ if } E_{it-1} = n_i + 1 \text{ and } y_{it} < y_{il} \end{aligned} \qquad [15]$$

where E denotes total employment, n is the initial level of employment, y denotes real output, and changes in employment over time are standardized to the value 1. The second and third rows of [15] describe the conditions under which the individual firm will maintain employment at a constant level from one period to the next. The first and fourth rows, meanwhile, describe the conditions under which the firm will vary employment from one period to the next. It is clear that in all cases, the employment decision depends on the proximity of the actual level of output to two key threshold values, y_{iu} and y_{il}. According to [15], variations in output above or below these thresholds will trigger changes in employment, while variations in output within the same bounds will leave employment unchanged.[40]

The workings of [15] can be illustrated by means of an example, that is depicted in Figure 3 below. Assume, then, that we begin at point A, with output given by y_{i1} and employment by n_i. Now suppose that a shock causes output to rise to y_{i2}. Since $y_{i2} > y_{iu}$, the firm raises employment to $n_i + 1$, and so arrives at point B in Figure 3. But suppose that the shock to output was strictly transitory, and that in the next period, we observe $y = y_{i3}$. Even though output has fallen back to its original level, it has not fallen below the lower threshold value y_{il}. As a result, the firm continues to employ $n_i + 1$ workers, at point C in Figure 3. A transitory shock to output has, therefore, produced a lasting change in the firm's employment – i.e., the non-ideal relay in [15] displays irreversibility.

[40] This implies that there must be some local variation in labour productivity, so that different levels of output can be produced by the same number of employees.

[FIGURE 3 GOES HERE]

Even at this stage of its development, then, our model has succeeded in reproducing the irreversibility property associated with unit and zero root systems. However, the non-ideal real is not the only component of models of "true" hysteresis. Hence consider now the impact of *aggregating* the employment responses of individual firms captured in [15] across all firms in the economy, as output varies relative to the firm-specific threshold values y_{il} and y_{iu}. This aggregation process is captured by the equation:

$$E_t = \iint f(y_u, y_l) E_i(y_{it}) dy_u dy_l \qquad [16]$$

where E_t denotes aggregate employment in period t, the weight function $f(.)$ specifies the relative contribution of firms with specific upper and lower output thresholds (y_u and y_l) to total employment, and E_i (.) denotes the employment functions of individual firms. Total employment is then derived by integrating over the upper and lower output threshold values for individual firms.

The consequences of this aggregation process are best explained in terms of the half-plane diagram (Mayergoyz, 1986) depicted in Figure 4. In Figure 4, we need only pay attention to the area above the solid diagonal line labelled $y_u = y_l$, since $y_{iu} > y_{il}$ for all i by assumption, so that all individual firms are represented by points above this line. Our analysis starts at point A with $E_i = n_i$ for all i. We assume for simplicity that $y_i = y_j$ for all i, j.[41] Consider now a symmetric shock to output, so that we observe $y = y_1$. All firms for which $y_{iu} < y_1$ – i.e., all firms that lie below the horizontal line y_1B in Figure 4 – will now increase their employment to $n_i + 1$. But if a second symmetric shock now reduces output

[41] Note that this does not imply that $n_i = n_j$ because there can be differences in labour productivity between firms.

to $y_2 < y_1$, firms below the horizontal line y_1B *and* to the right of the vertical line y_2C (for which $y_{il} > y_2$) will now reduce employment back to n_i. A third shock that raises output again, to $y_3 > y_2$, will cause firms that lie to the right of the vertical line y_2C *and* below the horizontal line y_3D (for which $y_{iu} < y_3$) to expand employment to $n_i + 1$, and so on. As is clear from this analysis, variations in total employment over time – and hence, by extension, the level of aggregate employment at any point in time – are dependent on the precise sequence of shocks to output. A different historical sequence of shocks over the same discrete interval of time would yield a different historical sequence of changes in employment, resulting in a different aggregate level of employment at the end of the interval. The implications of this analysis are clear. Aggregate employment – and by extension, aggregate unemployment – will not automatically converge towards a traditional (path-independent) long run equilibrium value, as in NAIRU theory. Instead, *ceteris paribus*, the economy will remain at the aggregate level of employment established by the sequence of shocks to output that are traced out in Figure 4 – what may thus be interpreted as the new, path-dependent equilibrium rate of employment. In fact, what we have generated in Figure 4 is "true" hysteresis in aggregate employment.

[FIGURE 4 GOES HERE]

Having constructed a model of "true" hysteresis, we can now identify some of the properties of hysteresis that it draws to attention. We have already noted that in and of itself, the non-ideal relay in [15] displays irreversibility: a transitory shock to output has a permanent effect on employment. It should therefore come as no surprise that this same property of irreversibility arises from the complete model of "true" hysteresis depicted in Figure 4. This is illustrated in Figure 5. Hence suppose that, following an initial increase in output to y_1 (as in the earlier case discussed in Figure 4), we now observe an increase in output to y_2 followed by a decrease in output back to $y_3 = y_1$ (i.e., a transitory shock to the level of output). As illustrated in Figure 5, this sequence of events will *not* restore the *status quo ante*. Instead, all firms within the rectangle y_1y_2BC will have permanently added to employment (since they are characterized by the conditions $y_2 > y_{iu} > y_1$ and $y_3 \geq y_{il}$), as a result of which aggregate employment will be permanently higher (and aggregate unemployment correspondingly lower).

[FIGURE 5 GOES HERE]

Meanwhile, unlike the unit/zero root models reviewed earlier, our model of "true" hysteresis does *not* display "super reversibility". This is illustrated in Figure 6. Suppose that, as in Figure 5, following an initial increase in output to y_1 we subsequently observe an increase in output to y_2 followed by a decrease in output back to $y_3 = y_1$ (i.e., a transitory shock to the level of output). But suppose that we now *also* observe a counter-shock of identical magnitude but opposite sign – i.e., a fall in output to y_4 (where $y_4 - y_3 = -(y_2 - y_1)$) followed by a restoration of output back to $y_5 = y_1$. As illustrated in Figure 6, this shock/counter-shock sequence will once again fail to restore the *status quo ante*. This time, all firms within the rectangle y_1y_2BC will have permanently added to employment (since they are characterized by the conditions $y_2 > y_{iu} > y_1$ and $y_4 \geq y_{il}$), as a result of which aggregate employment will once again be permanently higher (and aggregate unemployment correspondingly lower). Hence models of "true" hysteresis display irreversibility but *not* super reversibility. The "memory" of these systems, and the resulting propensity of past sequences of events to influence future (including long run or "final") outcomes, is clearly different from that of unit/zero root systems. For this reason, the permanent effects of even transitory sequences of past events on outcomes in "true" hysteretic systems are given the special name *remanence effects* (see especially Amable et al, 1995, pp.167-8).

[FIGURE 6 GOES HERE]

The results in Figures 5 and 6 highlight that neither the symmetry of a transitory shock *nor* the symmetry of a shock/counter-shock sequence (where both shock and counter-shock are transitory and of identical magnitude but opposite sign) will automatically restore the *status quo ante* in a model of "true" hysteresis. In so doing, they call attention to the fact that this is because of the adjustment *asymmetries* that characterize this model, arising from non-linearities (specifically, discontinuities caused by event thresholds) in the structure of the non-ideal relay, the assumed heterogeneity of firms in the economy, and the consequent *structural change* that can result from the displacement of a system from any initial state of equilibrium.[42] In other words, in generating the hysteresis results described above, our focus of attention is (properly) on adjustment dynamics and the potential lack of conventional asymptotic stability properties associated with any position that can be construed as an equilibrium, and *not* on the non-uniqueness of equilibrium. Hence note that at any point in time, the equilibrium in a system characterized by "true" hysteresis may, in fact, be unique. But if the system is displaced from this equilibrium configuration, it may not automatically converge back towards it. Instead, the system may settle at a new – and again, unique – position of equilibrium that has been *newly created* by the structural change wrought by

[42] "Structural change" refers here to change in the *composition* of the economy, as measured by the proportion of all firms operating on the upper (rather than lower) branch of the non-ideal relay depicted in Figure 3 – this being a function (as illustrated in Figures 4—6) of the precise historical sequence of adjustments undertaken by the system in the past.

the system's prior adjustment path (see also Amable et al 1993, pp.128-31; 1995, pp.169-72).

Notice that in the discussion above, reference is made to the fact that, following a transitory disturbance, a "true" hysteretic system *may* not automatically converge back towards its original equilibrium position. Whether or not it will depends on the precise nature of the disturbance itself. To be more specific, the "memories" of models of "true" hysteresis are *selective* rather than *complete*, so that what matters for system outcomes are so-called "non-dominated extrema" rather than the entire past history of the system (Cross, 1994).

In order to substantiate these claims, we begin by turning back to Figure 5. Hence suppose once again that starting from y_1, we observe an increase in output to y_2 followed by a decrease in output back to $y_3 = y_1$ (i.e., a transitory shock to the level of output). But suppose now that there are *no* firms within the rectangle y_1y_2BC – i.e., that there are no firms in the economy characterized by the conditions $y_2 > y_{iu} > y_1$ and $y_3 \geq y_{il}$. In this situation, the postulated transitory shock to output will leave total employment unchanged. As intimated earlier, in the event of its being disturbed from an initial position of equilibrium, a "true" hysteretic system *may* not automatically converge back towards its original equilibrium position – but we cannot completely rule out the possibility that it *will*. Clearly, then, not all history matters. Unlike unit/zero models, in which outcomes are sensitive to *all* past events (as in equation [7]), outcomes in models of "true" hysteresis depend on only *some* past events. In other words, "true" hysteretic systems have *selective* rather than *complete* memories.

These properties are further borne out by the events depicted in Figure 7. In Figure 7, we assume that, beginning at point A, the economy has experienced the same sequence of shocks depicted in Figure 4 (y_1, y_2, y_3). But suppose now that this sequence is followed by a further shock, that elevates output to y_4. We will now observe *all* firms for which $y_4 > y_{iu}$ employing at the level $n_i + 1$, regardless of the precise sequence of shocks to output (y_1, y_2, y_3) that occurred in the past. To put it differently, aggregate employment will be exactly the same as it would have been if, starting at point A, output had risen *immediately* to y_4. It is as if the sequence of shocks y_1, y_2, y_3 never happened: the memory of them has been erased or wiped from the system. Once again, then, we are provided with an example of the selective memory of models of "true" hysteresis, as a result of which not all history matters. More specifically, we have discovered the importance of *non-dominated extrema* for the outcomes of "true" hysteretic systems. In the parlance of "true" hysteretic analysis, the shock that raises output to y_4 erases the elaborate effects of what are now the *dominated* extremum values y_1, y_2, y_3 from the system's memory, with the result that aggregate employment depends only on the *non-dominated* extremum value, y_4. Note, then, that just as in unit/zero root systems, it is possible to wipe the memory of a "true" hysteretic system. But the processes involved in this memory wiping are very different (a shock/counter-shock sequence resulting in super reversibility in unit/zero root models; the dominance of previous extrema in models of "true" hysteresis), as befits our previous claims that the "memories" of these systems work in substantively different ways.

[FIGURE 7 GOES HERE]

While models of "true" hysteresis are an advance on unit/zero root models, they are not altogether above criticism. For example, some explanation is required for the event thresholds that are crucial to the non-ideal relay. Fortunately, the analytical role played by these event thresholds is made more explicit in expressions like [15] than in the characterizations of hysteresis discussed earlier. Hence "backstories" justifying their existence in specific applications now suffice to fill the remaining gap. For example, it could be argued that in [15], firms are seeking to avoid sunk costs associated with hiring and firing, and that they therefore adjust employment in response to variations in output discretely – whenever the change in y is "sufficiently large" – rather than continuously.

A more serious problem is that according to authors such as Setterfield (1998) and Katzner (1999), hysteresis in *social* systems must be understood as a property of *historical time*, in which the future is fundamentally uncertain (Davidson, 1991). These are not issues that models of "true" hysteresis typically address – likely by virtue of the fact that they are imported from the physical sciences.[43] But arguably there is still something missing from the model of "true" hysteresis developed above, and that needs to be taken into account when thinking about hysteresis as an organizing concept in macrodynamics.

Fortunately, this omission can again be corrected, once it is recognized that the essence of the problem is methodological. Specifically, models of "true" hysteresis are typically closed systems. But fundamental uncertainty is properly understood in terms of a quite different ontology – one that presupposes that social systems are structured but *open* (see, for example, Lawson, 2006). It is this feature of social ontology that must be taken into account when modelling hysteresis in social systems.

[43] See Cross (1993b) for an exception.

An important feature of the characterizations of hysteresis reviewed earlier is that they show how this can be achieved. This is because they describe hysteresis in terms of systems that lack intrinsic closure – i.e., systems in which causes need not always have the same effects. Hence suppose that, in terms of the analysis in sub-section 3(ii), we observe both conditions (a) (with $h_j^{'} \neq h_k^{'}$ for some $j, k,\ j \neq k$) and (c). In other words, the necessary and sufficient conditions for hysteresis both hold. Suppose further that the inter-temporal variations in $h_t^{'}$ cannot be described *a priori* – there is no "missing equation" that describes changes in $h_t^{'}$ as a time-invariant function of exogenous variables and that could be used to close the system that is being analyzed (Setterfield, 1998, pp.293-5; Katzner, 1999, pp.176-8). In this environment, even if shocks conform to a known stochastic process, it will be impossible to form expectations of (hysteretic) future outcomes without risk of systematic error. Decision makers will find themselves confronting fundamental uncertainty.

Drawing on this analysis, we can now see how models of "true" hysteresis can be reconciled with fundamental uncertainty, understood as a property of structured but open social systems. For example, we could postulate that the event thresholds in the non-ideal relay are time dependent, and insist on the absence of any "equations of motion" that would permit foreclosed explanation (and prediction) of their values (and hence those of aggregate hysteretic outcomes) over time. Allied to the assumed *conditionality* of any path-dependent equilibrium (and hence the possibility that the cumulative experience of equilibrium conditions may eventually disturb a system from an initial conditional equilibrium position) this would result in a model of *evolutionary hysteresis* – that is, hysteresis characterized by endogenously-generated structural change involving novelty (Setterfield, 2002, p.227).

4. The Case for Retaining Unit/Zero Root Analysis in Macrodynamics

In section 3(iii)a, it was argued that many of the properties of unit/zero root models fail to conform to those of hysteresis properly conceived. But despite this, a case can be made for *retaining* unit/zero root models in macrodynamics, as a useful first approximation of hysteresis effects and alternative to traditional equilibrium analysis.[44]

In the first place, unit/zero root systems are easy to construct, and easy to compare and contrast with traditional equilibrium systems (there frequently being little analytical difference between the structure of the two, as was demonstrated in section 3(iii)a). Second, unit/zero root models capture at least *some* of the properties of hysteretic systems – including the key property of irreversibility, according to which transitory causes have permanent effects. They are even consistent with the "errors matter" variant of the theory of decision making under uncertainty. This suggests that any decision made in an environment of risk (where all possible future outcomes and the probabilities with which they will occur are known) that does *not* allow the same decision to be made repeatedly (which would allow the law of large numbers to establish the mathematical expected value of the gamble as the actual average payoff) will be susceptible to the same "second order" psychological influences – confidence, optimism, "animal spirits," etc. – as a decision made under conditions of fundamental uncertainty.[45] The classic example of this is a "crucial" decision that is made only once (for example, betting one's life savings on

[44] See also Dutt (1997) for a sustained argument to this effect.
[45] See, for example, Gerrard (1995) on these "second order" influences on decision making.

one roll of a die). But essentially the same problem will confront decision makers faced with forecasting outcomes based on equation [7]. Suppose that *Z* in equation [7] is constant except for transitory shocks, so that we can write:

$$U = U_0 + \frac{1}{\alpha}\sum_{i=1}^{t}(\overline{Z} + \varepsilon_i)$$

or:

$$U = U_0 + \frac{t\overline{Z}}{\alpha} + \frac{1}{\alpha}\sum_{i=1}^{t}\varepsilon_i \qquad [7a]$$

Suppose further that decision makers understand that $E(\varepsilon_i) = 0$ and are therefore able to calculate:

$$E(U) = U_0 + \frac{t\overline{Z}}{\alpha}$$

The problem is that even a *transitory* shock to *Z* ($\varepsilon_i \neq 0$ for some *i*) will have a *permanent* effect on *U* in equation [7a]. Suppose, for example, that $\varepsilon_i = 0$ for all $i \neq n$. Then for $t \geq n$ we will observe:

$$U = U_0 + \frac{t\overline{Z}}{\alpha} + \varepsilon_n$$

Comparison of this outcome with the expectation described above reveals that the latter will be systematically wrong for all $t \geq n$. In other words, decision makers are vulnerable to systematic expectational error in the event that there is *any* transitory shock to *Z* at *any* point in time over their forecast horizon. Knowing this, decision makers would be unwise to act solely on the basis of the "best forecast" of *U* derived above. Their behaviour will, instead, be influenced by the same psychological influences described by the theory of decision making under fundamental uncertainty.

What this suggests is that a case can be made for retaining unit/zero root analysis as part of the "toolkit" of macrodynamic modelling, based on appeal to "pragmatic instrumentalism". In other words, even if it is understood that unit/zero root systems do not truly reflect the dynamics of hysteresis, they may recognized as providing a useful first approximation for certain specific purposes – as, for instance, when the analyst is attempting to contrast a traditional equilibrium outcome with one in which "history matters" in an otherwise familiar system (for examples, see Dutt, 1997; Lavoie, 2006). This is very much like the strategy that Keynes adopted when holding constant the state of long run expectations in order to facilitate exposition of the principle of effective demand in terms of a traditional equilibrium analysis (Kregel, 1976).

Note that the "pragmatic instrumentalism" described above does not mean that unit/zero root models are *always* justifiable. Instead, it calls for a "horses for courses" approach to macrodynamic modelling. Hence Lavoie's (2006) zero root model discussed earlier is useful for demonstrating certain limitations of and hidden assumptions in New Consensus macroeconomics. But a zero root model would be fundamentally misleading if our purpose is to describe hysteresis effects in real-world labour markets. On this basis, it can be argued that unit/zero root analysis belongs in a "big tent" approach to macrodynamic modelling, designed to maximize the useful contents of the practitioners toolbox.

5. Conclusions

The purpose of this chapter has been to discuss path dependency in dynamical economic systems, and to delineate the features of a specific concept of path dependency

– namely, hysteresis. It has been shown that models of “true” hysteresis are the most acceptable way of using hysteresis as an organizing concept in macrodynamics, by virtue of their superior fidelity to the abstract properties of hysteresis. At the same time, a pragmatic case has been made for retaining unit/zero root analysis, despite its failure to capture some of the most important features of hysteresis properly conceived. Ultimately, then, judicious use of *both* unit/zero root analysis *and* “true” hysteresis best serves to maximize the extent and value of the macrodynamic modeller’s toolkit.

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Figure 1: Asserting Hysteresis in the NAIRU

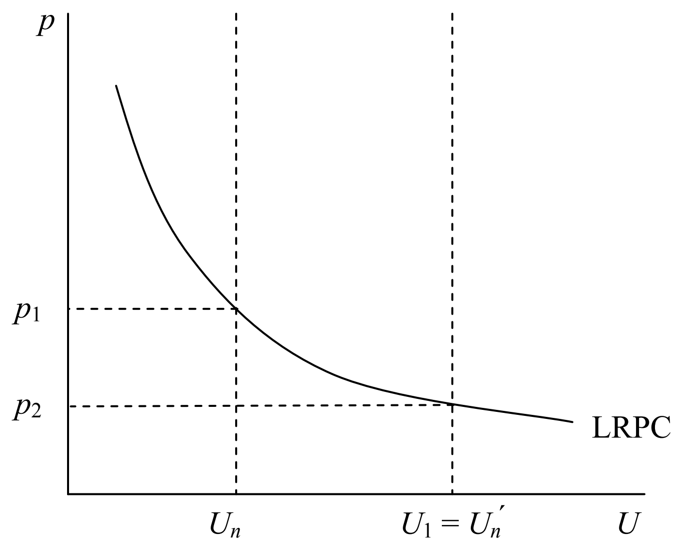

Figure 2: Response to a Shock in a Zero-Root System

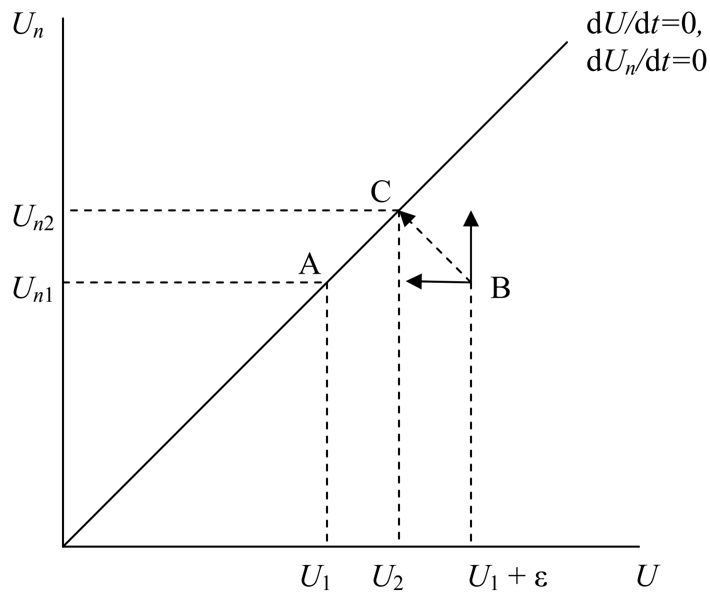

Figure 3: The Non-Ideal Relay

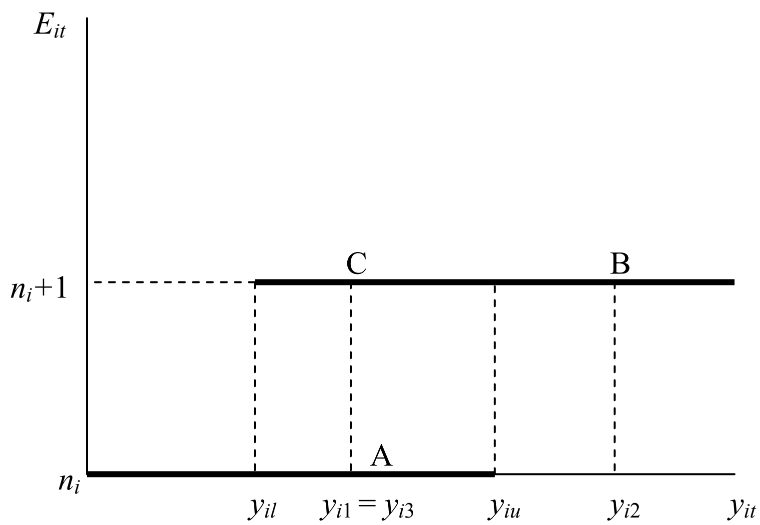

Figure 4: Aggregation Effects and Hysteresis

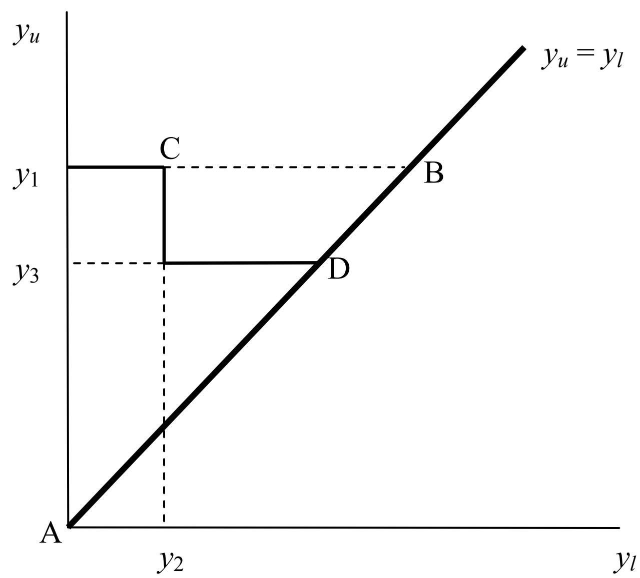

Figure 5: Irreversiblity

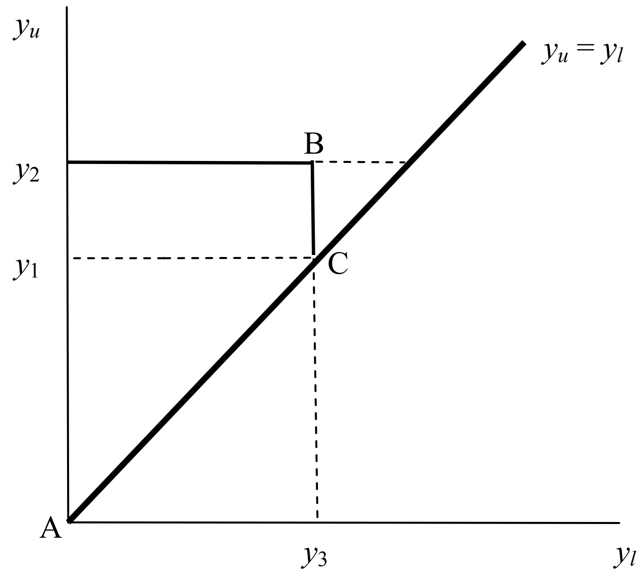

Figure 6: Absence of Super Reversibility

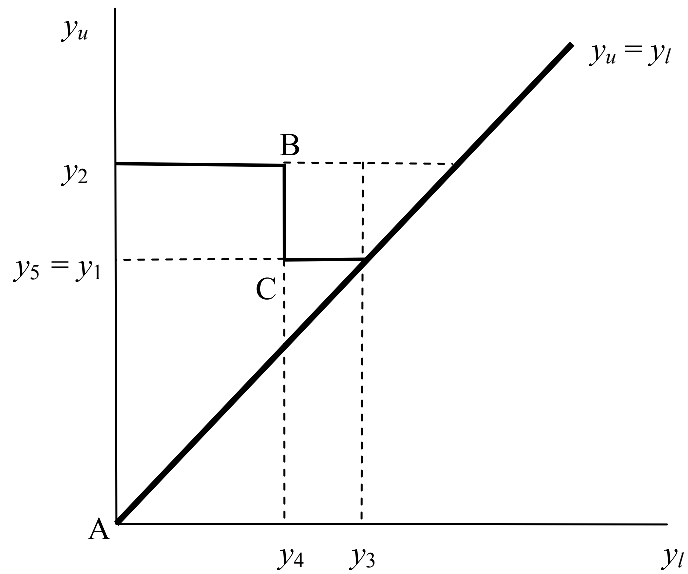

Figure 7: "Wiping" the Memory: the Importance of Non-Dominated Extrema