# The Cyclical Behavior of Equilibrium Unemployment and Vacancies with Worker Heterogeneity*

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**Abstract**

In this paper, we extend the basic Mortensen-Pissarides search and matching model along two dimensions. First, we allow for ex-ante heterogeneity between workers. Second, two technology shocks, neutral and investment-specific, are the driving forces of the economy. Specifically, we integrate the framework of Krusell, Ohanian, Ríos-Rull, and Violante (2000) - a production function with capital-skill complementarity and two skill-groups - into a business-cycle search and matching model. We calibrate the model extending the approach in Hagedorn and Manovskii (2006) and find that the model accounts well for the cyclical behavior of labor market variables in the aggregate and for each demographic group. We show that the response of unemployment to changes in taxes or unemployment insurance benefits is substantially reduced in the model with worker heterogeneity.

JEL Classification: E24, E32, J41, J63, J64

Keywords: Search, Matching, Business Cycles, Heterogeneity, Labor Markets

*First version: April 2007. This version: April 2008. We would like to thank seminar participants at the 2007 NBER Summer Institute meeting for useful comments. This research has been supported by the National Science Foundation Grant No. SES-0617876, NCCR-FINRISK and the Research Priority Program on Finance and Financial Markets of the University of Zurich.

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# 1 Introduction

The Mortensen-Pissarides (MP) search and matching model (Mortensen and Pissarides (1994), Pissarides (1985, 2000)) has become the standard theory of equilibrium unemployment. It provides an appealing description of the labor market and has been found relevant in quantitative work. For example, Merz (1995) and Andolfatto (1996) have shown that the performance of the real business cycle model can be improved significantly when the MP model is embedded into it. Hagedorn and Manovskii (2006) proposed a calibration strategy of the model and found that the model is successful in matching the data on the cyclical behavior of unemployment and vacancies.[1]

These papers build on the representative agent model that is the workhorse of modern business-cycle theory. The homogeneity assumption makes the analysis tractable, both computationally and theoretically. A simple framework also enhances our understanding of the economic forces underlying the MP model. In a homogeneous worker framework, we can investigate whether wages fluctuate less than productivity, we can understand the role of the flow value of non-market activity in generating unemployment fluctuations and we can conduct policy experiments theoretically.

However, a simple empirical finding suggests that our understanding of labor markets might be enriched by moving beyond the representative agent model. Both the levels and the magnitude of business-cycle fluctuations in labor market variables (e.g., unemployment rates) vary quite significantly across subgroups in the population. We believe that understanding why some groups fluctuate more than others could help to assess the quantitative performance

[1] Many other authors have explored whether a search model is consistent with business cycle facts, including Lilien (1982), Abraham and Katz (1986), Blanchard and Diamond (1989), Merz (1995), Cole and Rogerson (1999), Den Haan, Ramey, and Watson (2000), Gomes, Greenwood, and Rebelo (2001), Fujita (2004), Pries (2004), Bruegemann and Moscarini (2007), Costain and Reiter (2008). The literature has also suggested several modifications of the basic MP model that help the model generate a high volatility of unemployment. Farmer (2005) and Shimer (2004) suggest that some rigidity in wage formation may be necessary. In Hall (2005) and Gertler and Trigari (2006) a form of social wage norm renders wages not responsive to productivity changes. Hall and Milgrom (2008) modify the bargaining game to limit the influence of labor market conditions on wages. Kennan (2007) and Menzio (2005) endogenize wage rigidity by modeling asymmetric information about productivity. See Hornstein, Krusell, and Violante (2005), Mortensen and Nagypal (2007), Pissarides (2007) and Yashiv (2006) for excellent recent surveys of the literature.

of the MP model for at least two reasons. First, this empirical observation can serve as an additional test of the quantitative performance of the MP model. We can ask whether a reasonable parameterization that can account for aggregate fluctuations can also account for the different fluctuations across various groups. Second, it would be useful to know whether the model, for a reasonable parameterization, can account for the fluctuations of some groups even if it cannot account for aggregate fluctuations. This could provide guidance as to which modifications to the theory might be required.

Incorporating worker heterogeneity might also be key for reconciling the strong response of unemployment to cyclical fluctuations in labor productivity and a relatively weak response of unemployment to changes in policies such as taxes or unemployment benefit levels and durations. In a very influential paper, Costain and Reiter (2008) have suggested that there is an inherent contradiction in the MP model in being simultaneously consistent with both responses.[2] We show below, first theoretically, and later numerically, why allowing for worker heterogeneity helps reconcile the model with these features of the data.

We believe that incorporating worker heterogeneity along several dimensions into the model might be quantitatively important. In this paper, however, we make only a small step of considering workers of two types: high-skilled or low-skilled. We restrict our attention to such a coarse partition for the following reasons.

First, the driving force of the fluctuations in the MP model are fluctuations of labor productivity. We see no clear reason to assume that the productivity of various types of workers evolves similarly over the business cycle, even if the only driving force of fluctuations is aggregate shocks. Thus, to understand the cyclical behavior of labor market variables for different groups of workers it is essential to identify the cyclical behavior of their productivities.[3] The celebrated aggregate production function estimated by Krusell, Ohanian, Ríos-Rull, and Violante (2000) provides the natural way to do so. This production function accounts

[2]This issue has received a lot of attention in the literature and is discussed at length in recent surveys by Hornstein, Krusell, and Violante (2005), Mortensen and Nagypal (2007) and Pissarides (2007).

[3]We cannot use wages to infer the cyclical behavior of productivity because wages are not equal to the marginal product of labor in a search model. In most parameterizations of the MP model, including the one in this paper, the level of wages is very close to average productivity. The cyclical properties of wages, however, are different from the cyclical properties of productivity.

exceptionally well for the trends in wages of skilled and unskilled workers at least over the last several decades. It thus appears to be a natural candidate to provide an accurate and parsimonious way to also measure the business-cycle properties of the marginal productivities of the two labor inputs it considers: high-skilled and low-skilled workers. To be able to use this framework we consider the same two labor inputs. This naturally restricts our flexibility but provides a sufficient amount of heterogeneity for our analysis to yield interesting results. Measuring the evolution of worker productivity using this production function, we find that the (endogenously determined) marginal product of high-skilled workers is considerably more volatile over the business cycle than the marginal product of low-skilled workers.

One important reason for this finding is that Krusell, Ohanian, Ríos-Rull, and Violante (2000) estimate that high-skilled workers and capital equipment are complements in the production process. The standard MP model assumes that TFP shocks are the only stochastic driving force. This provides a limited picture of the underlying technology processes. Fisher (2006), for example, finds that investment-specific shocks, in addition to neutral technology shocks, are an important contributor to business-cycle fluctuations. Thus, to be able to use the framework of Krusell, Ohanian, Ríos-Rull, and Violante (2000) to understand the cyclical properties of the productivity process for high- and low-skilled workers, we must incorporate shocks to the price of capital equipment into the model.

While we find that the cyclical behavior of productivity is quite different across the two demographic groups, documenting the behavior of other labor market variables for them is important for assessing the model. In the data we find that relatively low-skilled workers account for most of the cyclical fluctuations in employment. Their unemployment rates are relatively high and strongly countercyclical. Costain and Reiter (2008) and Hagedorn and Manovskii (2006) have shown that this is consistent with the MP model if their productivity in the market is sufficiently close to their productivity at home. The key puzzle for the researchers was the fact that for highly skilled workers, whose productivity in the market is likely to be considerably higher than at home, and whose unemployment rates are low, the cyclical volatility of labor market statistics is as high as that for the low-skilled workers.

We calibrate the model following the strategy of Hagedorn and Manovskii (2006) and find that the two-skill version of the MP model is consistent with the cyclical volatility of its key variables in the data. Consistent with the common prior, we find that the worker's value of non-market activity is substantially smaller (relative to the wage) for high-skilled than for low-skilled workers. The unemployment rates of high-skilled workers are very volatile in percentage terms over the business cycle, however, because of high volatility in their productivity.

The restriction of ex-ante heterogeneity to just two groups has an advantage in that it allows us to maintain the simplicity of the analysis to the maximum possible extent. Restricting the model to just two types of workers makes it amenable to a theoretical analysis of the effects of labor market policies, which, we think, yields important insights. For example, we show that introducing curvature in the production side of the MP model is not sufficient *per se* to dampen the effects of policies. It is only if the production function includes heterogeneous and imperfectly substitutable labor inputs that the effects of policies will be dampened relative to the effects of cyclical movements in productivity.

The paper is organized as follows. A discrete time stochastic version of the Pissarides (1985, 2000) search and matching model with two skill groups and capital-skill complementarity is laid out in Section 2. In Section 3 we develop our calibration strategy. In Section 4 we describe the quantitative behavior of the model over the business cycle, both in the aggregate and for both groups of workers. We find that the model matches the cyclical volatility of labor market variables very well. Having verified that the model is a good quantitative laboratory, we conduct the analysis of the effects of policies in Section 5. The analysis is subdivided in two parts. First, we analyze the effects of policies theoretically to better understand how the model works and what features of the model are important for dampening the effects of policies. Next, we use the calibrated model to evaluate the effects of policies quantitatively. We find that the effects of policies are dampened substantially compared to the homogeneous agent version of the model. Section 6 concludes.

# 2 The Model

We consider a stochastic discrete time version of the Pissarides (1985, 2000) search and matching model with aggregate uncertainty and workers of two types $T \in \{L, H\}$, referring to low- and high-skilled workers, respectively.

## 2.1 Workers and Firms

There are measures $N^T$ of infinitely lived workers of each type and a continuum of infinitely lived firms. Workers maximize their expected lifetime utility:

$$\mathbb{E}\sum_{t=0}^{\infty}\delta^t y_t^T, \tag{1}$$

where $y_t^T$ represents income in period $t$ and $\delta \in (0,1)$ is workers' and firms' common discount factor.

There is a competitive final goods sector that combines 4 inputs to produce the final good - low-skilled labor $l_t$, high-skilled labor $h_t$, capital structures $k_{st}$ and capital equipment $k_{et}$ - through the following production function:

$$y_t = F(l_t, h_t, k_{st}, k_{et}) = A_t k_{st}^{\alpha}\left[\mu l_t^{\sigma} + (1-\mu)\left(\lambda k_{et}^{\rho} + (1-\lambda)h_t^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}}, \tag{2}$$

where $A_t$ is a neutral technology shock.

The resource constraint is

$$F(t) = C_t + i_{st} + \frac{i_{et}}{q_t}, \tag{3}$$

where $i_{st}$ is investment in capital structures, $i_{et}$ is investment in capital equipment, $C_t$ is consumption, and where the technology parameter $q_t$ determines the amount of equipment that can be produced by one unit of final output. In a perfectly competitive market, $q_t$ is also the relative price between consumption and equipment, a feature we exploit to measure $q$ in the calibration (as in Greenwood, Hercowitz, and Krusell (1997) and Krusell, Ohanian, Ríos-Rull, and Violante (2000)). The two stocks of capital evolve according to the following dynamic equations:

$$k_{s,t+1} = (1-d_s)k_{st} + i_{st} \tag{4}$$

$$k_{e,t+1} = (1-d_e)k_{et} + i_{et}, \tag{5}$$

where $d_e$ and $d_s$ are the depreciation rates of capital equipment and capital structures respectively.

Both $A_t$ and $q_t$ are assumed to follow AR(1) processes,

$$A_t = \kappa_A A_{t-1} + \epsilon_{A,t}, \tag{6}$$

$$q_t = \kappa_q q_{t-1} + \epsilon_{q,t}. \tag{7}$$

The two shocks, $\epsilon_{A,t}$ and $\epsilon_{q,t}$ are independent normal variables with respective standard deviations $\eta_A$ and $\eta_q$.

Each firm operating in the intermediate goods sector is either matched with an unskilled worker, matched with a skilled worker or posts a vacancy. If matched, it receives, from the competitive final sector, $p_{lt} = F_l(t)$ or $p_{ht} = F_h(t)$. There is free entry of firms. Firms attract unemployed workers by posting a vacancy at the flow cost $c^T$. Once matched, workers and firms separate exogenously with probability $s^T$ per period. Employed workers are paid a wage $w_t^T$, and firms make accounting profits of $p_t^T - w_t^T$ per worker each period in which they operate. Unemployed workers get flow utility $z^T$ from leisure/non-market activity.

## 2.2 Matching

Let $u_t^T$ denote the number of unemployed people and $n_t^T = N^T - u_t^T$ the number of employed people from group $T$ ($n^L = l$ and $n^H = h$). Let $v_t^T$ be the number of vacancies posted in period $t$. We refer to $\theta_t^T = v_t^T / u_t^T$ as the market tightness at time $t$ for type $T$. The aggregate market tightness is defined as $\theta_t = (v_t^H + v_t^L)/(u_t^H + u_t^L)$.

The number of new matches (starting to produce output at $t+1$) is given by a constant returns to scale matching function $m^T(u_t^T, v_t^T) \leq \min(u_t^T, v_t^T)$. Employment evolves according to the following law of motion:

$$n_{t+1}^T = (1 - s^T)n_t^T + m^T(u_t^T, v_t^T). \tag{8}$$

The probability that an unemployed worker will be matched with a vacancy next period equals $f^T(\theta_t^T) = m^T(u_t^T, v_t^T)/u_t^T = m^T(1, \theta_t^T)$. The probability that a vacancy will be filled next period equals $\phi^T(\theta_t^T) = m^T(u_t^T, v_t^T)/v_t^T = m^T(1/\theta_t^T, 1) = f^T(\theta_t^T)/\theta_t^T$.

## 2.3 Equilibrium

Denote the firm's value of a job (a filled vacancy) by $J^T$, the firm's value of an unfilled vacancy by $V^T$, the worker's value of having a job by $W^T$, and the worker's value of being unemployed by $U^T$. Bellman equations (9)-(12) describe an equilibrium of the model where $J^T$, $W^T$, $U^T$ and $V^T$ depend on the current shocks to productivity $A_t$ and $q_t$ and the stock of low-skilled $l_t$ and the stock of high-skilled $h_t$. Let $x_t = (A_t, q_t, l_t, h_t)$ be today's state vector and $x_{t+1} = (A_{t+1}, q_{t+1}, l_{t+1}, h_{t+1})$ be next period's state vector. The two capital stocks $k_e$ and $k_s$ do not have to be included in the state vector, since risk neutrality implies that they are already functions of $x$.[4]

$$\begin{aligned} J^T_{x_t} &= p^T_{x_t} - w^T_{x_t} + \delta(1 - s^T)E_{x_t}J^T_{x_{t+1}} && (9)\\ V^T_{x_t} &= -c^T_{x_t} + \delta\phi^T(\theta^T_{x_t})E_{x_t}J^T_{x_{t+1}} && (10)\\ U^T_{x_t} &= z^T_t + \delta\{f^T(\theta^T_{x_t})E_{x_t}W^T_{x_{t+1}} + (1 - f^T(\theta^T_{x_t}))E_{x_t}U^T_{x_{t+1}}\} && (11)\\ W^T_{x_t} &= w^T_{x_t} + \delta\{(1 - s^T)E_{x_t}W^T_{x_{t+1}} + s^T E_{x_t}U^T_{x_{t+1}}\}. && (12) \end{aligned}$$

The interpretation is straightforward. Operating firms earn profits $p^T_{x_t} - w^T_{x_t}$ and the matches are exogenously destroyed with probability $s^T$. A vacancy costs $c^T$ and is matched with a worker (becomes productive in period $t+1$) with probability $\phi^T(\theta^T_{x_t})$. An unemployed worker derives utility $z^T$ and finds a job next period with probability $f^T(\theta^T_{x_t})$. An employed worker earns wage $w^T_{x_t}$ but may lose her job with probability $s^T$ and become unemployed next period. Nash bargaining implies that a worker and a firm split the surplus $S^T_{x_t} = J^T_{x_t} + W^T_{x_t} - U^T_{x_t}$ such that

$$J^T_{x_t} = (1 - \beta^T)S^T_{x_t}, \tag{13}$$

$$W^T_{x_t} - U^T_{x_t} = \beta^T S^T_{x_t}. \tag{14}$$

Free entry implies that the value of posting a vacancy is zero: $V^T_{x_t} = 0$ for all $x_t$ and, therefore,

$$\begin{aligned} c^T &= \delta\phi^T(\theta^T_{x_t})E_{x_t}J^T_{x_{t+1}}\\ &= \delta\phi^T(\theta^T_{x_t})(1 - \beta^T)E_{x_t}S^T_{x_{t+1}}. && (15) \end{aligned}$$

The Bellman equation for the surplus is:

$$S^T_{x_t} = p^T_{x_t} - (z^T + \delta f^T(\theta^T_{x_t})\beta^T E_{x_t}S^T_{x_{t+1}}) + \delta(1 - s^T)E^T_{x_t}S^T_{x_{t+1}}. \tag{16}$$

To compute expectations, one has to know how the state variables evolve. The two productivity processes evolve according to the VAR(1) described above. The value of marginal

[4] The two first-order conditions for capital equipment and capital structures describe period $t$ capital stocks as functions of $x_t$ only because risk neutrality implies that the real interest rate is constant. Without risk neutrality this simplification would not be possible, since the period $t$ interest rate would depend on consumption in period $t$ and $t+1$.

productivity $p^T$ next period is endogenous and depends on how many workers are working today, how many vacancies are posted and how much capital is invested.

The market for capital equipment and structures is perfectly competitive and, each period, firms can rent capital to maximize profits. Households own the capital stock and invest to maximize their utility, which leads to the two first-order conditions for capital:[5]

$$E_t F_{k_s}(t+1) + (1-d_s) = \frac{1}{\delta}, \tag{17}$$

$$q_t E_t F_{k_e}(t+1) + (1-d_e) E_t \frac{q_t}{q_{t+1}} = \frac{1}{\delta}. \tag{18}$$

Note, that the decision on $k_{e,t+1}$ is taken in period $t$, but that the relative price of investment goods next period, $q_{t+1}$, matters for this decision as well.

We now derive the expressions for equilibrium wages and profits, which, except for being dependent on the type, take the usual form.[6] Because firms can buy and sell capital in a competitive market, the wage bargain is not affected as in Pissarides (2000). Using equation (13), it follows from the free-entry condition (15) and the flow equation (9) for $J^T$ that:

$$(1-\beta^T) S^T_{x_t} = p^T_{x_t} - w^T_{x_t} + (1-s^T) c^T / \phi^T(\theta^T_{x_t}). \tag{19}$$

Free entry and (16) imply that

$$S^T_{x_t} = p^T_{x_t} - z^T + (1 - s^T - f^T(\theta^T_{x_t})\beta) \frac{c^T}{\phi^T(\theta^T_{x_t})(1-\beta^T)}. \tag{20}$$

Thus, we have that

$$(1-\beta^T) S^T_{x_t} = (1-\beta^T)(p^T_{x_t} - z^T) + c^T \frac{1 - s^T - f^T(\theta^T_{x_t})\beta^T}{\phi^T(\theta^T_{x_t})}. \tag{21}$$

Rearranging (19) and substituting using (21), we find that wages are given by

$$\begin{aligned} w^T_{x_t} &= p^T_{x_t} - (1-\beta^T) S^T_{x_t} + (1-s^T) c^T / \phi^T(\theta^T_{x_t}) \\ &= \beta^T p^T_{x_t} + (1-\beta^T) z^T + c^T \beta^T \theta^T_{x_t}, \end{aligned} \tag{22}$$

[5] To see the second equation note that the $k_{e,t+1}$ is chosen to maximize $\ldots - \frac{k_{e,t+1}}{q_t} + \delta E_t (r_{t+1} k_{e,t+1} + \frac{(1-d_e) k_{e,t+1}}{q_{t+1}}) + \ldots$, where $r = F_{k_e}$ is the interest rate in the rental market.

[6] It is well known that only the present value of wages and not the specific sequence of wages matters. We adopt here the standard assumption of Nash bargaining to pin down this sequence. A popular alternative is to assume that wages are determined by contracts, an assumption that receives empirical support in the influential paper of Beaudry and DiNardo (1991). However, we show in Hagedorn and Manovskii (2008) that a selection effect and spot markets can explain the finding in Beaudry and DiNardo (1991).

and accounting profits are given by

$$\Pi^T_{x_t} = p^T_{x_t} - w^T_{x_t} = (1-\beta^T)(p^T_{x_t} - z^T) - c^T \beta^T \theta^T_{x_t}. \tag{23}$$

# 3 Calibration

In this section we calibrate the model to match U.S. labor market facts. We define the variables consistently with Krusell, Ohanian, Ríos-Rull, and Violante (2000) and conduct a measurement that ensures the comparability of our results to the large body of existing work on the cyclical behavior of unemployment and vacancies. In particular, we measure capital structures and equipment, output and employment in the non-farm business sector. As in Krusell, Ohanian, Ríos-Rull, and Violante (2000), the sample is restricted to individuals between 16 and 70 years old. The unskilled category includes individuals who have a high school diploma or less. The skilled category includes college-educated workers. Labor market data for the two subgroups comes from the monthly Current Population Surveys (CPS) from January 1976 to December 2006 and the CPS Outgoing Rotation Groups (ORG) covering the period January 1979 to December 2006. To aggregate individual observations we use CPS sample weights. On average over the sample period there are 2.6379 unskilled workers for each skilled worker. Whenever we are interested in cyclical properties of a variable observed at quarterly frequency, we use the HP-filter (Prescott (1986)) with a smoothing parameter of 1600. The data and variable construction procedures we use are detailed in Appendix I.

**Basics.** We choose the model period to be one week (one-twelfth of a quarter), which is lower than the frequency of the employment data we use, but necessary to deal with time aggregation. The data used to compute some of the targets have monthly, quarterly or annual frequency, and we aggregate the model appropriately when matching those targets. We set $\delta = 0.99^{1/12}$.

**Job-Finding and Separation Rates.** Using the CPS, we estimate, using the Shimer (2005b) two-state model described in Appendix I, the average monthly job-finding rate to be 0.3618 for skilled workers and 0.4185 for unskilled workers. The total separation rate (into unemployment, non-employment and job-to-job), not adjusted for time aggregation, for high-skilled equals 0.042 and for low-skilled 0.064 (Fallick and Fleischman (2004)). The separation rate into unemployment, also not adjusted for time aggregation, equals 0.0097 for the skilled and 0.0378 for the unskilled. We make this distinction between the rates of total separation and separation into unemployment, since what matters for firms' decisions is the expected duration of an employment spell, and this duration depends on the total rate of separation. We use this separation rate when modeling firms' decisions. To describe the average level and the evolution of unemployment for the two groups (Equation 8) we use the separation rate into unemployment only.

At a weekly frequency these estimates imply job-finding rates of $f^H = 0.1062$ and $f^L = 0.1268$, total job separation rates of $s^H = 0.0105$ and $s^L = 0.016$, rates of separation into unemployment $s_U^H = 0.0029$ and $s_U^L = 0.0117$, and steady state unemployment rates of $u^H = s_U^H/(s_U^H + f^H) = 0.0262$ and $u^L = s_U^L/(s_U^L + f^L) = 0.0846$.[7] These steady state unemployment rates are very similar to the average unemployment rate in the data of 0.0263 for skilled workers and 0.0838 for unskilled ones.

Petronglo and Pissarides (2001) survey the empirical evidence and conclude that the value of 0.5 for the elasticity of the aggregate job-finding rate with respect to aggregate labor market tightness is appropriate. By skill group, the elasticity of the job-finding probability with respect to overall labor market tightness is higher for high-skilled workers by a factor of 1.3345.

**Production Function Parameters.** We use the elasticity parameters of the production function $\alpha = 0.117$, $\sigma = 0.401$, and $\rho = -0.495$ and weekly depreciation rates of structures and equipment $d_s = 0.001068$ and $d_e = 0.002778$ estimated by Krusell, Ohanian, Ríos-Rull,

[7] We now illustrate this adjustment procedure in the case of skilled workers. The probability of not finding a job within a month is $1 - 0.3603 = 0.6382$. The probability of not finding a job within a week then equals $0.6382^{1/4} = 0.8938$ and the probability of finding a job equals $1 - 0.8938 = 0.1062$. The probability of observing someone not having a job who had a job one month ago equals (counting paths in a probability tree): $s\{(1-f)(fs+(1-f)^2)+f(s(1-f)+(1-s)s)\}+(1-s)\{s(fs+(1-f)^2)+(1-s)(s(1-f)+(1-s)s)\} = 0.0097$. Solving for $s$, we obtain $s = 0.0029$.

The *total* separation rate does not have to be adjusted for time aggregation, since it does not matter whether a worker switches employers once or multiple times between observation points. All we need to know is that the previous employment relationship ended.

and Violante (2000). Given the values of these parameters and the average employment levels of high- and low-skilled workers, we normalize the average stock of capital structures, $k_s = 399.7251$, capital equipment, $k_e = 389.8385$, and aggregate productivity $A = 0.4197$, and find the distribution parameters $\lambda = 0.9341$ and $\mu = .7445$ as solutions to a system of five equations. The system includes the first-order conditions (17) and (18) for structures and equipment, the normalization that the marginal product of low-skilled labor is equal to 1, the condition that the labor share is 2/3 of output, and the condition that the ratio of the marginal products of skilled and unskilled workers is equal to 1.9846, on average.[8]

The productivity of the two labor inputs is affected by the volatility of capital structures and equipment over the business cycle. In the data, the standard deviation of HP-filtered log capital structures is 0.0028 and the standard deviation of HP-filtered log quality-adjusted capital equipment is 0.0100 (see Appendix I). To ensure that the model matches the cyclical volatility of the capital series, we allow the depreciation rates for capital structures and equipment to depend on aggregate productivity. In particular, we introduce and calibrate a parameter $d_e^*$ and specify the depreciation of capital equipment at time $t$ to equal $d_e * (k_{e,t}/\overline{k}_e)^{d_e^*}$. Thus, if equilibrium capital equipment stock $k_{e,t}$ in period $t$ is equal to the average capital equipment $\overline{k}_e$, the depreciation rate is given by $d_e$. If capital in some periods deviates from its steady state value, the depreciation rate deviates in the same direction. The strength of the response of the depreciation rate is governed by parameter $d_e^*$. The depreciation rate for capital structures is defined symmetrically with parameters $d_s$ and $d_s^*$.

**Neutral and Capital Equipment-Specific Technologies.** We use the estimated production function parameters and compute the quarterly series for $A_t$ and $q_t$. We set $q_t$ equal to the NIPA price of consumption goods (non-durables and services), $p_t^c$, divided by the price of equipment investment goods, $p_t^e$. We use the $p_t^e$ series constructed by Schorfheide, Rios-Rull, Fuentes-Albero, Santaeulalia-Llopis, and Kryshko (2007). (They extend the annual series of Cummins and Violante (2002) to 2006 and convert the annual series to quarterly frequency similar to Fisher (2006)). We use the resulting price series to construct the quality-adjusted

[8]The last target is consistent with the competitive model but may not hold exactly in the model with search frictions. This theoretical inconsistency has a negligible impact on our findings because, in our calibration, the average wage is close to the marginal product.

stock of capital equipment using the perpetual inventory method.

We log and linearly de-trend the $A_t$ and $q_t$ series and use the resulting series to estimate the VAR in (6) and (7). To calibrate this process in the model, we consider quarterly averages of weekly productivity. We find that at weekly frequency we must set $\kappa_A = 0.9936$, $\kappa_q = 0.9988$, $\eta_A = 0.0035$, and $\eta_q = 0.0019$ to match the process in the data. We also normalize the average $\overline{q} = 1$ and $\overline{A} = .4197$, which yields the average productivity of an unskilled worker equal to one.

**Labor Market Tightness.** Hagedorn and Manovskii (2006) estimate an average value of labor market tightness of 0.634. This value lies between the estimates of 0.539 obtained by Hall (2005) and 0.72 obtained by Pissarides (2007).

In Hagedorn and Manovskii (2006) we used data on the time and costs involved in recruiting workers from the 1982 Employment Opportunity Pilot Project survey and the 1992 Small Business Administration survey reported in Barron, Berger, and Black (1997). These authors also estimate the vacancy duration equation $D = c_0 + c_1 X$ using the same datasets, where $D$ is the log of the duration time and $X$ is the set of controls including the log number of years of education, and report that the education coefficient is statistically significant in both datasets and equal to 0.886 and 2.432, respectively. Because the average years of education in our sample for high-skilled and low-skilled workers are equal to 16.54 and 10.83, respectively, these estimates imply that on average $\theta^L = 0.5858$ and $\theta^H = 1.0442$.

**Matching Functions.** We choose the Cobb-Douglas functional form of the matching functions of skilled and unskilled workers:

$$m(u^T, v^T) = \chi^T (u^T)^{\gamma^T} (v^T)^{1-\gamma^T}. \tag{24}$$

The two parameters, $\chi^T$, $\gamma^T$, that characterize the matching function differ for the two types. This allows us to match a different job-finding probability and a different elasticity of the job-finding probability with respect to labor market tightness.

**The Cyclicality of Wages.** Over the 1979:1-2006:4 period we find that a 1-percentage-point increase in labor productivity is associated with a 0.674-percentage-point increase in average real wages. Wages are measured as the non-farm business labor share constructed by the BLS times labor productivity defined as seasonally adjusted real non-farm business output constructed by the BLS from the NIPA divided by seasonally adjusted non-farm business employment form the monthly Current Population Survey. Both time series are in logs and HP-detrended. We also use CPS data to estimate the wage elasticity with respect to average output per person for each group separately. We find that wages for high-skilled workers are more cyclical than wages of low-skilled. The ratio of the two elasticities equals 1.771. To obtain the corresponding estimates in the model, we first aggregate the weekly model-generated data to replicate the quarterly frequency of the data. We then log and HP-filter the time series and estimate regressions identical to those estimated in the data.

**The Costs of Posting Vacancies.** In Hagedorn and Manovskii (2006) we found that the expected labor costs of posting vacancies equals 50.23% of average weekly labor productivity. The flow capital costs of posting vacancies equals 47.4% of average weekly labor productivity, which equals 1.2707, so that the capital costs equal 0.6023. The analysis from Hagedorn and Manovskii (2006) for these average numbers applies here as well. However, the presence of capital-skill complementarity and two types of capital implies that the numbers for the two groups are different.

For labor costs it is simple. We find that the skill premium in the data equals 1.9846. The expected costs of a vacancy in the model equals $\frac{c_W^T}{\phi^T}$, where $c_W^T$ is the flow cost and $\phi^T$ is the probability of filling a vacancy. The numbers we report above imply that $\phi^H = 0.1017$ and $\phi^L = 0.2165$. Solving $\frac{c_W^L}{\phi^L} = 0.5023$ and $\frac{c_W^H}{\phi^H} = 1.9846 \cdot 0.5023$, we find $c_W^H = 0.1014$ and $c_W^L = 0.1087$.

The specification of the production function in Krusell, Ohanian, Ríos-Rull, and Violante (2000) features capital-skill complementarity, so that more capital is bought when a high-skilled worker is hired than when a low-skilled worker is hired. The relative sizes of capital equipment and capital structures needed can be computed from the first-order conditions (17) and (18). For skilled workers, the implicit function theorem provides us with two functions $k_s(h)$ and $k_e(h)$ solving the two first-order conditions, keeping the number of unskilled workers fixed. Analogously for unskilled workers, we get two functions $k_s(l)$ and $k_e(l)$. The relative capital needs for capital equipment then equals $\frac{\frac{\partial k_e(h)}{\partial h}}{\frac{\partial k_e(l)}{\partial l}}$ and the relative capital needs for capital structures equals $\frac{\frac{\partial k_s(h)}{\partial h}}{\frac{\partial k_s(l)}{\partial l}}$. Evaluating these expressions at the steady state gives $\frac{\frac{\partial k_e(h)}{\partial h}}{\frac{\partial k_e(l)}{\partial l}} = 8.3384$ and $\frac{\frac{\partial k_s(h)}{\partial h}}{\frac{\partial k_s(l)}{\partial l}} = 1.9846$.

We can now compute the flow capital costs for high-skilled $c_e^H$ (for equipment) and $c_s^H$ (for structures) and for low skilled: $c_e^L$ (for equipment) and $c_s^L$ (for structures). The different capital needs imply that $c_e^H = 8.3384 c_e^L$ and $c_s^H = 1.9846 c_s^L$.

The average flow cost for equipment equals $c_e^H \frac{v^H}{v^H+v^L} + c_e^L \frac{v^L}{v^H+v^L}$ and that for structures equals $c_s^H \frac{v^H}{v^H+v^L} + c_s^L \frac{v^L}{v^H+v^L}$. Since the capital income share for structures equals 0.117 and that for equipment equals $(2/3 - 0.117)$ we solve

$$c_e^H \frac{v^H}{v^H+v^L} + c_e^L \frac{v^L}{v^H+v^L} = \frac{2/3 - 0.117}{2/3} 0.6023 \tag{25}$$

$$c_s^H \frac{v^H}{v^H+v^L} + c_s^L \frac{v^L}{v^H+v^L} = \frac{0.117}{2/3} 0.6023 \tag{26}$$

We find $c_e^H = 1.4359$, $c_s^H = 0.3585$, $c_e^L = 0.1722$ and $c_s^L = 0.1806$. Thus, overall, the flow costs of posting a vacancy for high-skilled workers equals $c^H = 1.4359 + 0.3585 + 0.1014 = 1.8958$ and for low-skilled workers it equals $c^L = 0.1722 + 0.1806 + 0.1087 = 0.4615$.

**Remaining Parameters.** Ten parameters remain to be determined: the values of non-market activity, $z^H$, $z^L$, worker's bargaining weights, $\beta^H$, $\beta^L$, the matching function parameters, $\chi^H$, $\chi^L$, $\gamma^H$, $\gamma^L$, and depreciation factors for capital structures and equipment, $d_s^*$, $d_e^*$. We choose the values for these parameters to match the data on the average value for labor market tightness for skilled and unskilled workers, the elasticity of wages with respect to aggregate productivity, the relative elasticity of wages with respect to aggregate productivity of skilled and unskilled workers, the average values for the job-finding rates of skilled and unskilled workers, the elasticity of the aggregate job-finding rate with respect to aggregate labor market tightness, the relative elasticity of the job-finding rate with respect to aggregate labor market tightness of skilled and unskilled workers, and the standard deviations of capital structures and equipment. Thus, there are ten targets, all described in the previous paragraphs, to pin down ten parameters.

To find the values of these parameters we solve the model numerically according to the computational algorithm described in Appendix II. The performance of the model in matching calibration targets is described in Table 1. We are able to match the targets almost exactly. Calibrated parameter values can be found in Table 2. To understand these results, it is useful to recall how the two key parameters - the bargaining power and the value of non-market activity - are determined in the homogeneous worker case (Hagedorn and Manovskii (2006)). Bargaining power is chosen to match the elasticity of wages, since a higher bargaining power of workers makes wages more responsive to changes in productivity. The level of non-market activity is then chosen to match the average level of wages. The average level of wages, holding fixed other parameters such as the separation rate and the interest rate, depends one-for-one on expected hiring costs $c/\phi$, since a higher level of expected costs requires higher profits and thus lower average wages. The same logic applies here. Since expected vacancy posting costs $c/\phi$ are about four times higher for high-skilled workers than for low-skilled workers (relative to productivity), $z^H$ is substantially lower than $z^L$ (relative to productivity). Bargaining power is again chosen to match the elasticity of wages with one modification. We match the elasticity of wages with respect to average productivity and not with respect to marginal productivity, since the latter is not directly observable. For the targeted elasticity it holds that $\epsilon_{w^T,p} = \epsilon_{w^T,p^T} \cdot \epsilon_{p^T,p}$, which makes a difference, since $\epsilon_{p^T,p}$ does not equal one ($\epsilon_{x,y}$ denotes the elasticity of $x$ with respect to $y$.). We find that $\epsilon_{p^H,p} = 1.316$ and $\epsilon_{p^L,p} = 0.935$, since changes in capital equipment mainly affect $p^H$. In equilibrium, the effect due to a higher volatility of productivity for high-skilled workers outweighs the effect due to a higher productivity elasticity of their wages, implying a lower bargaining power for them compared to low-skilled workers. A similar reasoning applies to measuring the elasticity of the matching function. It is identified by the elasticity of the job-finding rate with respect to the aggregate market tightness $\theta$ because $\theta^T$ is not observable. It holds that $\epsilon_{f^T,\theta} = \epsilon_{f^T,\theta^T} \cdot \epsilon_{\theta^T,\theta}$. We find that $\epsilon_{\theta^H,\theta} = 0.837$ and $\epsilon_{\theta^L,\theta} = 1.056$. The choice of the remaining parameters is simple. The matching function efficiency parameter $\chi^T$ determines the job finding rate and the depreciation factors are chosen to match the standard deviations of capital structures and equipment.

Table 1: Matching the Calibration Targets.

| | Target | Value | |
|---|---|---|---|
| | | Data | Model |
| 1. | Elasticity of wages wrt agg. productivity, $\epsilon_{w,p}$ | 0.674 | 0.671 |
| 2. | Relative elasticity of wages wrt agg. productivity, $\epsilon_{w^H,p}/\epsilon_{w^L,p}$ | 1.770 | 1.774 |
| 3. | Skilled job-finding rate, $f^H$ | 0.106 | 0.105 |
| 4. | Unskilled job-finding rate, $f^L$ | 0.127 | 0.126 |
| 5. | Skilled average market tightness, $\theta^H$ | 1.044 | 1.047 |
| 6. | Unskilled average market tightness, $\theta^L$ | 0.586 | 0.585 |
| 7. | Elasticity of agg. job-finding wrt agg. market tightness, $\epsilon_{f,\theta}$ | 0.500 | 0.513 |
| 8. | Relative elas. of job-finding wrt agg. mrkt tightness, $\epsilon_{f^H,\theta}/\epsilon_{f^L,\theta}$ | 1.335 | 1.332 |
| 9. | Standard deviation of capital structures | 0.003 | 0.003 |
| 10. | Standard deviation of capital equipment | 0.010 | 0.010 |

Note - The table describes the performance of the model in matching the calibration targets.

Table 2: Calibrated Parameter Values.

| Parameter | Definition | Value |
|---|---|---|
| $z^H$ | skilled value of non-market activity (share of their productivity) | 0.813 |
| $z^L$ | unskilled value of non-market activity (share of their productivity) | 0.929 |
| $\beta^H$ | skilled workers' bargaining power | 0.069 |
| $\beta^L$ | unskilled workers' bargaining power | 0.112 |
| $\gamma^H$ | skilled matching function elasticity | 0.199 |
| $\gamma^L$ | unskilled matching function elasticity | 0.529 |
| $\chi^H$ | skilled matching function efficiency | 0.102 |
| $\chi^L$ | unskilled matching function efficiency | 0.164 |
| $d_s^*$ | depreciation factor of capital structures | 11.200 |
| $d_e^*$ | depreciation factor of capital equipment | 1.399 |

Note - The table contains the calibrated parameter values in the benchmark calibration.

Table 3: Data and Results from the Calibrated Model.

| | Statistic | Value | |
|---|---|---|---|
| | | Data | Model |
| 1. | St. dev. of agg. productivity, $p$ | 0.013 | 0.013 |
| 2. | Autocorr. of agg. productivity, $p$ | 0.765 | 0.765 |
| 3. | St. dev. of agg. unemployment, $u$ | 0.090 | 0.086 |
| 4. | St. dev. of agg. vacancies, $v$ | 0.116 | 0.110 |
| 5. | St. dev. of agg. market tightness, $\theta$ | 0.202 | 0.196 |
| 6. | Corr. of agg. unemployment and vacancies | -0.910 | -0.777 |
| 1. | St. dev. of skilled productivity, $p^H$ | — | 0.018 |
| 2. | Autocorr. of skilled productivity, $p^H$ | — | 0.782 |
| 3. | St. dev. of skilled unemployment, $u^H$ | 0.111 | 0.114 |
| 4. | St. dev. of skilled vacancies, $v^H$ | — | 0.078 |
| 5. | St. dev. of skilled market tightness, $\theta^H$ | — | 0.162 |
| 1. | St. dev. of unskilled productivity, $p^L$ | — | 0.013 |
| 2. | Autocorr. of unskilled productivity, $p^L$ | — | 0.763 |
| 3. | St. dev. of unskilled unemployment, $u^L$ | 0.085 | 0.083 |
| 4. | St. dev. of unskilled vacancies, $v^L$ | — | 0.133 |
| 5. | St. dev. of unskilled market tightness, $\theta^L$ | — | 0.206 |

Note - Seasonally adjusted aggregate unemployment, $u$, is constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). Seasonally adjusted skill-group unemployment, $u^H$ and $u^L$, is constructed by the authors from the monthly Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index, $v$, is constructed by the Conference Board. $u$, $u^H$, $u^L$, and $v$ are quarterly averages of monthly series. Average labor productivity $p$ is seasonally adjusted quarterly real non-farm business output constructed by the BLS from the NIPA divided by non-farm business employment from the monthly Current Population Survey. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.

# 4 Business-Cycle Properties of the Model

The statistics of interest, computed from quarterly U.S. data from 1979:1-2006:4 and the results from the calibrated model are presented in Table 3.

**Aggregate Results.** A comparison between the corresponding statistics reveals that the model matches the key business-cycle facts quite well. In particular, the volatility of aggregate labor market tightness, unemployment, and vacancies is quite close to that in the data. Moreover, the model generates a strong negative correlation between unemployment and vacancies, i.e., the Beveridge curve.

**Results by Skill Group.** In the data the unemployment rate is 2.6% for skilled workers and 8.4% for unskilled ones. Both of these rates are highly volatile, with respective standard deviations of the HP-filtered logged unemployment rate of 0.111 and 0.085. Thus, while it is the low-skilled workers who account for most of the fluctuations in employment, the unemployment rate of skilled workers is even more volatile in percentage terms.

The model does an excellent job in matching these observations. It generates unemployment rates of 2.6% for skilled workers and 8.4% for unskilled ones, with respective standard deviations of the HP-filtered logged unemployment rate of 0.114 and 0.083. To understand these results we compute these statistics for the representative agent case twice, using the parameters for skilled and unskilled workers, respectively. We find that for skilled workers, market tightness is 9.2 times more volatile than their productivity. This high value, despite a low value of $z^H = 0.813$, is due to a substantially more persistent productivity process for high-skilled workers than for the representative agent. This high ratio translates into high volatility of market tightness, since the productivity process for high-skilled workers is also more volatile than for the representative agent.

For unskilled workers, the standard deviation of HP-filtered log market tightness $\theta^L$ is 0.206, which is 15.8 times higher than the volatility of their productivity. This higher volatility for low-skilled workers is due to a higher value of $z^L = 0.929$ (relative to their productivity). In the representative agent model of Hagedorn and Manovskii (2006), a value of $z = 0.94$ would be required to generate a volatility of market tightness of 0.206. A value of $z = 0.929$ would generate a volatility of only 0.177 in that model. The difference is due to a separation rate of low-skilled workers that is higher than the one used in the representative agent economy in Hagedorn and Manovskii (2006).[9]

The matching function translates the volatility of market tightness into volatile unemployment. For each group, the steady state elasticity of unemployment with respect to productivity can be expressed as

$$\epsilon_{u^T,p^T} = \epsilon_{u^T,f^T} \cdot \epsilon_{f^T,\theta^T} \cdot \epsilon_{\theta^T,p^T} = -(1 - \frac{u^T}{N^T})(1 - \gamma^T)\epsilon_{\theta^T,p^T} \tag{28}$$

Our finding that $1 - \gamma^H = 1 - 0.199$ is substantially larger than $1 - \gamma^L = 1 - 0.529$ explains why high-skilled unemployment is more volatile than low-skilled unemployment, although the opposite ordering between groups holds for market tightness.

The aggregate statistics targeted in this paper differ from those in Hagedorn and Manovskii (2006). We now calibrate the linear model with homogeneous workers in Hagedorn and Manovskii (2006) to match these same aggregate statistics. In particular, we target a wage elasticity of 0.67 (instead of 0.45 in Hagedorn and Manovskii (2006)) and also make the distinction between the total separation rate and the separation rate into unemployment. We find a standard deviation of market tightness of 0.11 and a standard deviation of unemployment of 0.049, which represent only about one-half of the corresponding numbers in the data. As we have shown above, only after we allow for heterogeneity, the model is able to replicate the observed volatilities. Two simple observations explain this finding. First, the volatility of unemployment is an increasing and convex function of $z$ (see equation (27)). Second, the calibrated value of $z$ in the homogeneous worker model lies between the two values $z^H$ and $z^L$ and is close to their weighted average. As a consequence, low-skilled unemployment and thus also overall unemployment are substantially more volatile than unemployment in the homogeneous worker model. To summarize, we find that the extended MP model calibrated using

[9] In Hagedorn and Manovskii (2006) we derive, in the model without aggregate uncertainty, the elasticity of labor market tightness with respect to aggregate productivity to be:

$$\epsilon_{\theta,p} = \frac{p}{p-z} \frac{\beta f(\theta) + (1 - \delta(1-s))/\delta}{\beta f(\theta) + (1-\eta)(1 - \delta(1-s))/\delta}, \tag{27}$$

where $\eta$ is the elasticity of $f(\theta)$ with respect to $\theta$. This equation may be used to quite accurately evaluate the impact of various parameter values, such as the separation rate, on the volatility of market tightness.

the strategy proposed in Hagedorn and Manovskii (2006) is consistent with labor market volatilities in the aggregate, once we allow for heterogeneity, and in subgroups.

**Robustness.** The only target in our benchmark calibration that is not standard is the elasticity of wages with respect to aggregate productivity. Recall that we define productivity as non-farm business output divided by employment from the monthly Current Population Survey. Shimer (2005a) used the same measure of output but divided it by employment measured in the Current Employment Statistics. The estimated elasticity of wages with respect to aggregate productivity is affected by this choice. Our measure of productivity implies an elasticity of 0.67, while Shimer's measure implies an elasticity of only 0.5.[10] We now recalibrate the model to match the same calibration targets but target a low wage elasticity of 0.5.

The performance of the model in matching the calibration targets with a low wage elasticity, the calibrated parameter values, and the results are described in Appendix Tables A-1, A-2, and A-3, respectively. The changes in the calibrated values of the bargaining power $\beta$ and the value of non-market activity $z$ are as expected. A lower value for the targeted wage elasticity for both groups leads to lower values for the bargaining power of both types, $\beta^H$ and $\beta^L$. Since the expected costs of posting vacancies remain unchanged, per period profits and thus average wages do not change either. To generate the same level of wages with a lower bargaining power requires then a higher value of non-market activity for both types, $z^H$ and $z^L$. Higher values of non-market activity result in more volatile labor market variables in the aggregate and for each worker type as compared to the benchmark calibration.

An additional benefit of this experiment is that it (coincidentally) targets virtually the same aggregate statistics computed over the 1951-2004 period as in Shimer (2005a) and Hagedorn and Manovskii (2006). For comparison, we reproduce these statistics in Column (1) of Appendix Table A-3 and the results from the calibration of the linear MP model with homogeneous workers in Column (3) (targeting the same aggregate statistics as in the model with heterogeneity). A comparison of the results based on the model with worker

[10]The differences between the cyclical properties of these series are documented in Hagedorn and Manovskii (2007). There we argue why a productivity measure based on CPS employment might be preferred.

heterogeneity with the results from the linear model implies that the model with worker heterogeneity again generates higher volatility of aggregate labor market statistics and is closer to the data than the homogeneous worker model.

A new feature of our calibration is that we make a distinction between the total separation rate and the separation rate into unemployment. We now recalibrate the model to match the same calibration targets but without making this distinction. The performance of the model in matching the calibration targets, calibrated parameter values, and results are described in Appendix Tables A-4, A-5, and A-6, respectively. A lower total separation rate increases expected profits from a filled vacancy. Since vacancy posting costs are unchanged, a higher value of non-market activity $z$ is required to keep profits unchanged. A higher value of $z$ leads to more volatility in market tightness and in wages. Thus a lower value of the bargaining weight is chosen to match a wage elasticity of 0.67. Again the model with heterogeneity is closer to the data, since the linear model with homogeneous workers generates too little volatility.

Finally, we have assumed throughout the paper that the two shocks, $\epsilon_{A,t}$ and $\epsilon_{q,t}$ are independent. Estimating their correlation in the data, we obtain a correlation of 0.2644. We have recalibrated the model with this correlation and found that our results are not affected. Introducing this correlation makes capital slightly more volatile because the price of capital equipment is lower when TFP is higher. However, the depreciation factors adjust to match capital volatility in the data, and all of the other statistics remain unchanged.

# 5 Policy Effects

## 5.1 Empirical Evidence on the Effects of Changes in Policy on Unemployment

In the previous section, we established that our model accounts quite well for the data. Thus, it seems to provide a good laboratory for computational policy experiments in the tradition of Prescott (1986). Costain and Reiter (2008), instead, use the outcome of such experiments as a further test of the model. They present evidence on the effects of policies such as levels of taxes or unemployment benefits based on cross-country regressions and compare it to the effects of changes in $z$ on unemployment in the theoretical model. In other words, they assume that a change in, e.g., the unemployment insurance replacement rate changes the value of $z$ one-for-one. They find the semi-elasticity of unemployment with respect to the UI replacement rate to be around two. When they exclude Scandinavian countries, a value of about 5.6% cannot be rejected with 95% confidence (6.5% cannot be rejected with 99% confidence).

The analysis in Costain and Reiter (2008) is without any doubt very careful but suffers - as most cross-country regressions do - from serious endogeneity problems. One example is the exclusion of Scandinavian countries that have a high replacement rate and a low unemployment rate but also have other effective labor market policies, such as job training and job search enforcement, which are not included in the regression. For example, Heckman, LaLonde, and Smith (1999) report that in 1994/1995 Sweden spent 2.18% of GDP on training programs for unemployed workers and 2.54% of GDP on unemployment compensation and early retirement benefits for labor market reasons. The U.S. in the same years spent 0.16% of GDP on training programs for the unemployed and 0.35% of GDP on unemployment benefits. Another example is that changes in unemployment insurance lead to important substitution effects. For instance, Gruber and Cullen (2000) find that for each dollar of a husband's unemployment insurance received, wives earn 73 cents less. Moreover, a higher replacement rate crowds out private (precautionary) savings (Gruber and Engen (2001)). Taking into account the latter two effects is important to quantitatively assess the effect of changes in unemployment insurance in a cross-country comparison, since the degree of women's labor force participation and the development of financial markets differ between countries. But all these important determinants of labor supply are not held constant in cross-country regressions of the unemployment rate on unemployment benefits. Thus, we think that results from such regressions come with a degree of uncertainty. Nevertheless, keeping this uncertainty in mind, we will compare the performance of our model to the empirical results of Costain and Reiter (2008).

The finding that policy effects are sizable in a model where shocks to productivity are strongly amplified is expected.[11] The argument is simple. Any sequence of productivity shocks

[11]One may even expect that policy amplifies productivity (and other) shocks as it does, for example, in

can be replicated through a sequence of sales taxes. For example, in a basic (one-sector, representative agent) real business-cycle model, productivity and tax changes have identical effects both on first-order conditions and on household's budget constraint - the conditions that characterize equilibrium. In other words, the MP model and its close cousin the basic RBC model belong to a class of models where short-run implications (in a business-cycle analysis) are similar to lower frequency implications (in a policy experiment). In this paper, we propose one way to break this linkage: productivity responds to policy changes. In the next section, we analyze this possibility theoretically before assessing its quantitative performance. We study the effects of changes in unemployment insurance, but all of our results fully apply to the effects of changes in taxes.

## 5.2 Theoretical Analysis

In this section, we show that changes in unemployment insurance (changes in $z$) change not only employment but also productivity, which can mitigate or amplify the changes in employment. If, for example, an increase in $z$ increases productivity, the drop in employment is smaller than it would be with a constant level of productivity. To show this we consider a simplified (relative to (2)) constant return to scale (CRS) production technology

$$y_t = G(l, h, k), \tag{29}$$

where $k$ is capital and $l$ and $h$ are two different labor types, and $G$ is increasing and concave in each argument.[12]

A drop in $l$ (due to an increase in $z^l$) increases the productivity $G_l$ of low-skilled workers if the levels of $h$ and $k$ are unchanged. But $h$ and $k$ adjust as well, and this adjustment can overturn this conclusion, depending on the properties of $G$. The following sections investigate these properties.

Cole and Ohanian (2004).

[12]The technology in (2) takes this form for $\alpha = 0$. Since (2) combines capital structures and $G$ through a Cobb-Douglas aggregator, assuming (2) would not change the conclusions of this section. The Cobb-Douglas specification implies that capital structures change one-for-one with $G$.

### 5.2.1 Equilibrium Conditions for Capital, Employment and Market Tightness

Given the production function $G$, we now consider how the productivities $G_l, G_h$ and $G_k$, labor inputs $l$ and $h$, capital $k$ and the policy parameter $z$ are related.

Capital solves the first-order condition ($d$ is the depreciation rate)

$$G_k = \frac{1}{\delta} - (1-d), \tag{30}$$

which defines capital implicitly as a function of $l$ and $h$: $k(l,h)$.

For the two labor inputs, we can derive in the case of no aggregate uncertainty (see Hagedorn and Manovskii (2006)) the following two relationships between market tightness and productivity for each group (we suppress the dependence on type $T$).

$$\frac{1-\delta(1-s)}{\delta q(\theta)} + \beta\theta = \frac{p-z}{c}(1-\beta). \tag{31}$$

The steady state conditions for employment $l$ and $h$ are

$$l = \frac{f^L(\theta^L)}{s^L + f^L(\theta^L)} \quad \text{and} \quad h = \frac{f^H(\theta^H)}{s^H + f^H(\theta^H)}. \tag{32}$$

The last two equations together imply two functions that relate the level of employment to $p$ and $z$:

$$l = L(p^l, z^l), \tag{33}$$

$$h = H(p^h, z^h). \tag{34}$$

Denote the marginal productivity of group $l$:

$$p^l = G_l(l,h,k), \tag{35}$$

and the marginal productivity of group $h$:

$$p^h = G_h(l,h,k). \tag{36}$$

Taking into account that capital $k$ is a function of $l$ and $h$, $k(l,h)$, allows us to express productivities as functions of $l$ and $h$ only

$$p^l = G_l(l,h,k(l,h)) = \pi^l(l,h), \tag{37}$$

$$p^h = G_h(l,h,k(l,h)) = \pi^h(l,h). \tag{38}$$

Plugging the expression for $L(p^l, z^l)$ and $H(p^h, z^h)$ into the functions $\pi$, results in two functions $A$ and $B$:

$$p^l = A(p^l, z^l, p^h, z^h) = \pi^l(L(p^l, z^l), H(p^h, z^h)), \tag{39}$$

$$p^h = B(p^l, z^l, p^h, z^h) = \pi^h(L(p^l, z^l), H(p^h, z^h)). \tag{40}$$

which jointly describe the two productivity levels $(p^l, p^h)$ as a fixed point, depending on the two parameters $(z^l, z^h)$. We now want to investigate how changing $(z^l, z^h)$ affects the fixed point $(p^l, p^h)$.

### 5.2.2 Productivity changes

To characterize how productivity $(p^l, p^h)$ depends on $(z^l, z^h)$ requires knowing how the functions $A$ and $B$ depend on productivities $(p^l, p^h)$. The next proposition accomplishes this.

**Proposition 1**

$$\epsilon_{A,p^l} = \epsilon_{\pi^l,l}\epsilon_{L,p^l} = \{-\epsilon_{G_l,h} + \frac{\epsilon_{G_k,h} \cdot \epsilon_{G_l,k}}{\epsilon_{G_k,k}}\}\epsilon_{L,p^l}, \tag{41}$$

$$\epsilon_{A,p^h} = \epsilon_{\pi^l,h}\epsilon_{H,p^l} = \{\epsilon_{G_l,h} - \frac{\epsilon_{G_k,h} \cdot \epsilon_{G_l,k}}{\epsilon_{G_k,k}}\}\epsilon_{H,p^l}, \tag{42}$$

$$\epsilon_{B,p^l} = \epsilon_{\pi^h,l}\epsilon_{L,p^h} = \{\epsilon_{G_H,L} - \frac{\epsilon_{G_K,L} \cdot \epsilon_{G_H,K}}{\epsilon_{G_K,K}}\}\epsilon_{L,p^h}, \tag{43}$$

$$\epsilon_{B,p^h} = \epsilon_{\pi^h,h}\epsilon_{H,p^h} = \{-\epsilon_{G_H,L} + \frac{\epsilon_{G_K,L} \cdot \epsilon_{G_H,K}}{\epsilon_{G_K,K}}\}\epsilon_{H,p^h}, \tag{44}$$

*where $\epsilon_{x,y}$ is the elasticity of $x$ w.r.t. $y$.*

We can consider two special cases in which productivity is invariant when policy is changed. The first case arises if the two types of workers are perfect substitutes, so that the production part of the model is equivalent to a model with homogeneous workers. In this case the invariance of productivity is not very surprising. Any drop in labor leads to a drop in capital, such that the capital-labor ratio remains unchanged. Since labor productivity is a function of the capital-labor ratio, it also does not change.

The assumption that the two labor inputs are perfect substitutes implies that $G_{ll} = G_{hh} = G_{lh}$ and that $G_{kl} = G_{kh}$ and it implies the following proposition:

**Proposition 2 (Special Case: L and H are Perfect Substitutes)** *If the two labor inputs $l$ and $h$ are perfect substitutes, then the labor productivities do not change with changes in labor inputs:* $\epsilon_{\pi^l,l} = \epsilon_{\pi^h,h} = \epsilon_{\pi^l,h} = \epsilon_{\pi^l,h} = 0$.

A similar logic applies when one of the two labor inputs is unrelated to the other labor input and capital, that is, either $G_{lh} = 0$ and $G_{kl} = 0$ or $G_{lh} = 0$ and $G_{kh} = 0$. In each of these two cases, the economy consists of two unrelated economies, each of which has only one type of worker. Since "both economies" have a CRS production function with a representative agent, the previous proposition applies.

**Proposition 3 (Special Case: L and H are Unrelated Inputs)** *If either $G_{lh} = 0$ and $G_{kl} = 0$ or $G_{lh} = 0$ and $G_{kh} = 0$, then productivity remains unchanged:* $\epsilon_{\pi^l,l} = \epsilon_{\pi^h,h} = \epsilon_{\pi^l,h} = \epsilon_{\pi^l,h} = 0$.

The production function we use in this paper does not fall into one of the two special cases. Instead it implies the following assumption:

**Assumption 1** *$G_{lh} \geq 0$ and $G_{kl}G_{kh} \geq 0$, where at least one inequality is strict.*

With this assumption, we can show that productivity indeed changes when the policy parameter $z$ is changed and we know the sign of this change. The key step is to show that labor productivity changes if the amount of labor input is changed. The reason why these changes are not zero is that the above logic does not fully apply anymore. With a representative agent, a fully flexible capital stock adjusts to keep the capital-labor ratio and thus labor productivity constant. If, instead, capital was fixed or not fully flexible, labor productivity would increase in response to a decrease in labor. With two types of labor a similar effect obtains. Capital cannot fully adjust to keep the two capital-labor ratios constant. Instead, there is only partial adjustment, as would be the case with a representative agent if the capital stock is a fixed factor. As a consequence, labor productivity is not constant. The next proposition states this and also establishes how the functions $A$ and $B$ respond to changes in $p^l$ and $p^h$.

**Proposition 4** *If assumption 1 holds, then*

$$\epsilon_{\pi^l,L}, \epsilon_{\pi^h,H}, A_{p^l}, B_{p^h} < 0, \tag{45}$$

$$\epsilon_{\pi^l,H}, \epsilon_{\pi^h,L}, A_{p^h}, B_{p^l} > 0, \tag{46}$$

Once the signs of the derivatives of the functions $A$ and $B$ are known, the last step is easy:

**Proposition 5**

$$\frac{\partial p^l}{\partial z^l} = \frac{-A_{p^l}}{1 - A_{p^l} - B_{p^h}} > 0, \tag{47}$$

$$\frac{\partial p^h}{\partial z^l} = \frac{-B_{p^l}}{1 - A_{p^l} - B_{p^h}} < 0, \tag{48}$$

$$\frac{\partial p^l}{\partial z^h} = \frac{-A_{p^h}}{1 - A_{p^l} - B_{p^h}} < 0, \tag{49}$$

$$\frac{\partial p^h}{\partial z^h} = \frac{-B_{p^h}}{1 - A_{p^l} - B_{p^h}} > 0. \tag{50}$$

*It also holds that* $\frac{\partial p^l}{\partial z^l} < 1$ *and* $\frac{\partial p^h}{\partial z^h} < 1$*, so that* $\frac{\partial (p^l - z^l)}{\partial z^l} < 0$ *and* $\frac{\partial (p^h - z^h)}{\partial z^h} < 0$*.*

### 5.2.3 What does this mean for employment changes?

Once the change in productivity is known, it is sufficient to look at equations (33) and (34) to figure out the change in employment. For example, if $p - z$ increases, employment increases, and if $p - z$ decreases, employment decreases.

More generally, the change in total employment $l + h$ in response to a change in $z^l$ is:

$$\begin{aligned} \epsilon_{l+h,z^l} &= ((L_{p^l}\frac{\partial p^l}{\partial z^l} + L_{z^l}) + H_{p^h}\frac{\partial p^h}{\partial z^l})\frac{z^l}{l+h} \\ &= (\epsilon_{L,p^l}\epsilon_{p^l,z^l} + \epsilon_{L,z^l})\frac{l}{l+h} + \epsilon_{H,p^h}\epsilon_{p^h,z^l}\frac{h}{l+h}, \end{aligned} \tag{51}$$

which means that the total employment change is a weighted sum of the change in $l$ and in $h$. Similarly, the change in total employment in response to a change in $z^h$ is:

$$\begin{aligned} \epsilon_{l+h,z^h} &= ((H_{p^h}\frac{\partial p^h}{\partial z^h} + H_{z^h}) + L_{p^l}\frac{\partial p^l}{\partial z^h})\frac{z^h}{l+h} \\ &= (\epsilon_{H,p^h}\epsilon_{p^h,z^h} + \epsilon_{H,z^h})\frac{h}{l+h} + \epsilon_{L,p^l}\epsilon_{p^l,z^h}\frac{l}{l+h}. \end{aligned} \tag{52}$$

The total change, if $z^l$ goes up by 1% and $z^h$ increases by $\chi$%,[13] equals

$$\epsilon_{l+h,z^l} + \chi\epsilon_{l+h,z^h} \tag{53}$$

[13] $\chi < 1$ corresponds to a situation where high-skilled workers (with higher wages) have a lower replacement rate on average.

For $\chi = 1$ (the case we consider in the quantitative analysis) this expression equals

$$\begin{aligned}
&\epsilon_{l+h,z^l} + \epsilon_{l+h,z^h} = \qquad (54)\\
&\frac{l}{l+h}(\epsilon_{L,p^l}(\epsilon_{p^l,z^h} + \epsilon_{p^l,z^l}) + \epsilon_{L,z^l}) + \frac{h}{l+h}(\epsilon_{H,p^h}(\epsilon_{p^h,z^l} + \epsilon_{p^h,z^h}) + \epsilon_{H,z^h})\\
&\frac{l}{l+h}(\frac{\epsilon_{L,p^l}}{\vartheta}(\epsilon_{\pi^l,h}\epsilon_{H,z^h} + \epsilon_{\pi^l,l}\epsilon_{L,z^l}) + \epsilon_{L,z^l}) + \frac{h}{l+h}(\frac{\epsilon_{H,p^h}}{\vartheta}(\epsilon_{\pi^h,l}\epsilon_{L,z^l} + \epsilon_{\pi^h,h}\epsilon_{H,z^h}) + \epsilon_{H,z^h})\\
&\frac{l}{l+h}(\epsilon_{L,p^l}(\frac{\epsilon_{\pi^l,l}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h}) + \epsilon_{L,z^l}) + \frac{h}{l+h}(\epsilon_{H,p^h}(\frac{\epsilon_{\pi^h,h}}{\vartheta}(\epsilon_{H,z^h} - \epsilon_{L,z^l}) + \epsilon_{H,z^h}),
\end{aligned}$$

where $\vartheta = 1 - A_{p^l} - B_{p^h}$, which is positive under assumption 1. This expression is quite insightful. The change in $l-$productivity $p^l$ due to a change in $z$ equals $\frac{1}{\vartheta}(\epsilon_{\pi^l,h}\epsilon_{H,z^h} + \epsilon_{\pi^l,l}\epsilon_{L,z^l})$ and similarly the change of the $h-$productivity $p^h$ equals $\frac{1}{\vartheta}(\epsilon_{\pi^h,l}\epsilon_{L,z^l} + \epsilon_{\pi^h,h}\epsilon_{H,z^h})$. If these changes are zero, this means productivity is constant, and the change in employment would equal

$$\frac{l}{l+h}\epsilon_{L,z^l} + \frac{h}{l+h}\epsilon_{H,z^h}, \qquad (55)$$

which is a weighted sum of the changes in $l$ and in $h$. This *composition effect* strictly dampens the change in employment (and thus unemployment) relative to the group effects, whenever one group is more responsive to policy than the other group, for example, if $|\epsilon_{L,z^l}| > |\epsilon_{H,z^h}|$.

If, however, productivity responds to changes in $z$, the response of group employment changes. If productivity increases in response to an increase in $z$, the employment effect is mitigated ($p - z$ decreases by less); if productivity decreases in response to an increase in $z$, the employment effect is amplified ($p - z$ decreases by more).

Whether productivity increases or decreases for group $l$ and group $h$ is described by the signs of $\frac{\epsilon_{\pi^h,h}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h})$ and of $\frac{\epsilon_{\pi^h,h}}{\vartheta}(\epsilon_{H,z^h} - \epsilon_{L,z^l})$. Multiplying these expressions with $\epsilon_{L,p^l}$ and $\epsilon_{H,p^h}$, respectively, translates the productivity changes into employment changes (higher productivity leads to higher employment).

One implication of the above expression is that the change in employment is equal to that with constant productivity if $\epsilon_{L,z^l} - \epsilon_{H,z^h} = 0$ (both types of labor respond in the same way to changes in unemployment insurance), namely,

**Proposition 6** *If $\epsilon_{L,z^l} - \epsilon_{H,z^h} = 0$, then productivity does not change and the change in total employment equals*

$$\epsilon_{l+h,z^l} + \epsilon_{l+h,z^h} = \frac{l}{l+h}\epsilon_{L,z^l} + \frac{h}{l+h}\epsilon_{H,z^h}, \qquad (56)$$

because productivity would in this case not move (endogenously).

Furthermore, it follows that if one group has a stronger labor demand elasticity, for example, group $L$ ($\epsilon_{L,z^l} - \epsilon_{H,z^h} < 0$), then the productivity of this group increases and the drop in employment is mitigated, whereas the productivity of the other group decreases (since $\epsilon_{L,p^l} > 0, \epsilon_{p^l,L} < 0, \epsilon_{H,p^h} > 0, \epsilon_{p^h,H} < 0$).

**Proposition 7** *If $\epsilon_{L,z^l} - \epsilon_{H,z^h} < 0$, then $p^l$ increases and $p^h$ decreases. As a consequence the employment response of group $l$ is mitigated (relative to constant productivity) and the employment response of group $h$ is amplified (relative to constant productivity).*

The overall effect on employment due to the change in productivity would be (since $\epsilon_{p^h,H} = \frac{lp^l}{hp^h}\epsilon_{p^l,L}$)

$$\begin{aligned} &\frac{l}{l+h}(\epsilon_{L,p^l}(\frac{\epsilon_{\pi^l,l}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h})) + \frac{h}{l+h}(\epsilon_{H,p^h}(\frac{\epsilon_{\pi^h,h}}{\vartheta}(\epsilon_{H,z^h} - \epsilon_{L,z^l})) \\ &= \frac{l}{l+h}\frac{\epsilon_{\pi^l,l}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h})(\epsilon_{L,p^l} - \frac{p^l}{p^h}\epsilon_{H,p^h}), \end{aligned} \tag{57}$$

which is positive if $\frac{p^l}{p^h}$ is not substantially larger than one (if group $h$ are high-skilled workers with lower relative $z$ and higher productivity, this conclusion obviously holds).

**Proposition 8** *The overall effect on employment due to the change in productivity equals*

$$\frac{l}{l+h}\frac{\epsilon_{\pi^l,l}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h})(\epsilon_{L,p^l} - \frac{p^l}{p^h}\epsilon_{H,p^h}), \tag{58}$$

*which is positive if $\frac{p^l}{p^h}$ is not substantially larger than one.*

### 5.2.4 Comparative statics

Consider the impact of different parameter values on the overall effect on employment in equation (58).

**Proposition 9** *Consider the employment effect due to productivity changes:*

- **Skill premium:** *A decrease in $\frac{p^l}{p^h}$ increases the effect if $\epsilon_{L,z^l} - \epsilon_{H,z^h} > 0$.*
- **Preferences:** *An increase in $\epsilon_{L,z^l} - \epsilon_{H,z^h} (> 0)$ (for example if $z^l - z^h$ increases) increases the effect.*
- **Production:** *Any change in the production function that lowers $\epsilon_{\pi^l,L} < 0$ increases the effect. This would happen if one of the positive values $G_{lh}$, $G_{lk}$, $G_{hk}$ increases.*

## 5.3 Quantitative Evaluation

Costain and Reiter (2008) proposed to evaluate a change in taxes or in the generosity of the unemployment insurance system by a change in $z$. In other words, they assume that a change in taxes or in the unemployment insurance replacement rate changes the value of $z$ one-for-one. In the linear MP model with homogeneous workers the elasticity of unemployment with respect to the replacement rate is larger than the relative volatility of market tightness to output. As discussed above, Costain and Reiter (2008) show that the opposite holds in the data. In this section we investigate quantitatively whether our modified MP model has the same problem.

Consider an experiment where we keep all the parameter values the same as in our benchmark calibration except for increasing the value on non-market activity by 1% for both skilled and unskilled workers (i.e., setting $z^H = 1.6332$ and $z^L = 0.9383$). Performing this experiment we find that the unemployment rate increases by 6.6% from 0.070 to 0.074.[14] The elasticity of unemployment with respect to $z$ of 6.6 is close to the set of plausible values reported by Costain and Reiter (2008) when they exclude Scandinavian countries.[15]

Section 5.2 implies that the change in unemployment can be decomposed into two effects: the change due to productivity changes and a composition effect. With a constant level of productivity, the response of low-skilled unemployment would be to increase by 8.5% and high-skilled unemployment would increase by 7.1%. Thus, the overall increase in unemployment with unchanged productivity would be 8.2%. The endogenous change in productivity leads to a further reduction. Productivity of low-skilled workers increases by 0.2% and productivity of high skilled decreases by 0.02%. This implies that low-skilled unemployment increases by 6.5% (instead of 8.5% with constant productivity) and high-skilled unemployment increases

[14] To put this number into perspective, suppose that the unemployment insurance replacement rate is 20% of average productivity (a plausible number in the case of the U.S.). Increasing the value of non-market activity by 1% then amounts to expanding unemployment insurance by 5%. The model predicts that this will raise the unemployment rate by less than half of a percentage point.

[15] The elasticity of employment with respect to $z$ equals $-0.5$ in the model. Hagedorn and Manovskii (2006) show that the increase in $z$ by 1% is equivalent to an increase of the labor tax rate from 40% to 40.6%. Prescott (2004) finds that in an RBC model such an increase in taxes results in a similar, although somewhat larger, drop in employment of $-0.75$%.

by 7.4% (instead of 7.1% with constant productivity) and overall unemployment increases by 6.6% (instead of 8.2% with constant productivity).

These policy effects are much lower than those implied by the standard linear MP model with homogeneous workers. A meaningful comparison of the size of policy effects between the two models requires that they both generate the same amount of volatility in market tightness. Otherwise, one model could generate small policy effects just because it does not generate much volatility (an arbitrarily low value of $z$ would ensure this). To generate a volatility of 0.2 in the linear model requires that $z = 0.929$ (all other parameters except for vacancy posting costs are chosen to match the same aggregate statistics as in our benchmark calibration). For this value of $z$ we find an elasticity of 9.5.

The results are even stronger if one considers the low wage elasticity calibration. In that case we have to set $z = 0.943$ in the standard model to generate a volatility of 0.255, which implies an elasticity of 16.1, whereas our model with heterogeneity implies an elasticity of only 8.7.

## 5.4 Additional Mechanisms

In the previous section we have shown how curvature in productivity arises and that it substantially dampens policy effects. Two related modifications of the basic MP model help generate this result: two skill groups and capital skill complementarity. Capital equipment, since it is complementary to skilled labor, serves as a fixed factor when it comes to changes in unskilled labor. As a result, productivity of unskilled workers increases if the generosity of unemployment insurance is increased. In a representative agent economy in contrast, capital fully adjusts to policy changes and productivity remains unchanged (there is no curvature in a p/z diagram). These modifications brought us close to a value consistent with Costain and Reiter (2008). There are additional mechanisms that can further reduce the effects of labor market policies in the MP model without affecting its ability to generate a volatile labor market over the business cycle. In this section we briefly review them.

### 5.4.1 Induced technological change

In our model, productivity of low-skilled workers increases in response to an increase in $z$ because capital equipment serves as a "fixed" factor in aggregate production. The literature on induced technical change, notably, Acemoglu (2002, 2007), provides an additional mechanism based on the Le Chatelier-Samuelson principle. If unemployed low-skilled labor becomes more abundant, technologies that are biased toward low-skilled labor and thus increase its productivity are more likely to be developed in the long run. Modeling and measuring this effect is beyond the scope of this paper but is likely to further dampen the effects of policies. However, taking this approach would also require adding some heterogeneity to the basic MP model, as we propose in this paper.

### 5.4.2 Modeling curvature in $z$

Endogenous productivity is only one candidate for adding curvature. In the model, $z$ does not depend on the length of the unemployment spell. This is a strong assumption. The long-term unemployed are definitely worse off, since they face problems replacing their durable consumption goods (a broken TV, dishwasher, microwave, etc.). Furthermore, having a month off to enjoy leisure has a high value, but the enjoyment of a year of unemployment is questionable. In our calibration we (implicitly) estimate the average $z$ of all unemployed. Since the job-finding rate equals 45% per month on average, the short-term unemployed make up the bulk of the observations. Thus, our estimate of $z$ represents the value of unemployment for the average worker, who finds employment quickly. It is not informative about the value of long-term unemployment, since this is a low probability event.

Allowing $z$ to decrease with the length of the unemployment spell makes $z$ endogenous. When productivity declines, the average duration of unemployment increases and thus the average $z$ of the unemployment pool declines as well. This is an interesting and, we believe, productive way to add curvature to the model on the worker side. Modeling curvature in this way is unlikely to dampen the model's ability to replicate business-cycle facts. It creates some procyclicality in $z$, but our calibration strategy would then reduce bargaining power to match the cyclicality of wages.[16] Since the effects of productivity shocks are relatively short-lived,

[16]The value of non-market activity can be cyclical for several other reasons, such as a correlation between

the average duration of unemployment and, thus, the average $z$ are unlikely to change much over the business cycle.[17]

Adding curvature in the value of non-market activity also dampens the effects of policies. The value of $z$ is likely to be decreasing with the length of the unemployment spell, for example, as in the top panel of Figure 1. Consider a change in policy, such as an increase in tax rates or unemployment benefits, that increases $z$ relative to $p$. A stylized illustration of the effects is shown in the bottom panel of Figure 1. In response to such a policy, firms post fewer vacancies. This leads to an increase in the average duration of unemployment accompanied by a decline in the average $z$ of the unemployment pool. This works against the direct effect of the policy and moves the economy closer to the equilibrium prior to the change in the policy. As discussed above, this is unlikely to dampen the model's ability to replicate business-cycle facts because cyclical fluctuations in productivity are relatively short-lived compared to more permanent changes in policy. Depending on the curvature of $z$, however, the policy's effect may be entirely canceled out.

### 5.4.3 Dependence of unemployment benefits on employment histories

Finally, Faig and Zhang (2008) note that in most unemployment insurance systems the entitlement to unemployment insurance benefits must be earned through prior employment. Thus, unemployment benefits are not only the opportunity cost of employment but also an indirect benefit of employment. With this insight they establish the following result: If UI rules can prevent the moral hazard behavior of becoming or remaining unemployed, each employed worker is charged a fair UI fee, and utilities are linear, then the generosity of UI benefits, the duration of these benefits, and the time it takes to become eligible for UI are all irrelevant to the determination of output, vacancies, and unemployment. Not all of these conditions are likely to be satisfied by the actual unemployment insurance systems, but this

market and home technology. Unfortunately, the correlation between $p$ and $z$ and the bargaining power $\beta$ are not separately identified from the observation of wages $w_p = \beta p + (1-\beta)z + c\beta\theta_p$. A related problem plagues the estimation of real business-cycle models with home production (e.g., McGrattan, Rogerson, and Wright (1997)). They cannot simultaneously identify the elasticity of substitution between market and non-market activities and the correlation between market and non-market productivity.

[17] Our discussion here is reduced form. This is intentional. The structural model of the evolution of $z$ and its identification are too complex to be adequately developed in this paper.

effect undoubtedly helps to further lower the effects of changes in unemployment insurance policies. This mechanism does not apply to changes in taxes, however.

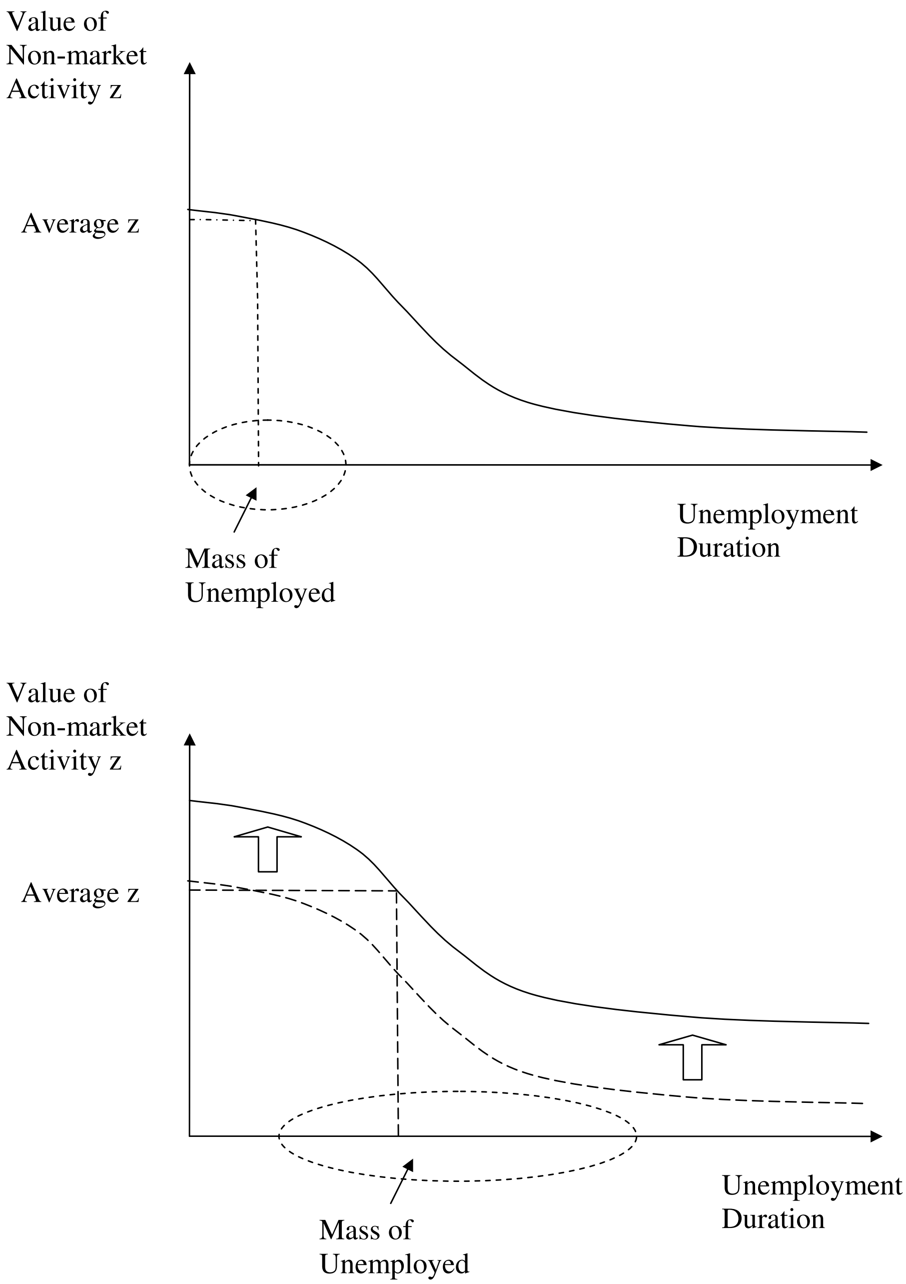
# 6 Conclusion

We extended the basic Mortensen-Pissarides search and matching model along two dimensions. First, we allowed for ex-ante heterogeneity between workers, low and high skilled. Second, we allowed two technology shocks, neutral and investment-specific, to be the driving forces of the economy. Specifically, we integrated the framework of Krusell, Ohanian, Ríos-Rull, and Violante (2000) - a production function with capital-skill complementarity and two skill-groups - into a business-cycle search and matching model. We calibrated the model using the approach in Hagedorn and Manovskii (2006) and found that the model accounts well for the cyclical behavior of labor market variables in the aggregate and for each demographic group.

Our calibration implies that the flow value of non-market activity of high-skilled workers is considerably lower than the corresponding value for a representative worker in the model with homogeneous workers. For low-skilled workers the flow value of non-market activity is slightly higher than the value for a representative worker. Nevertheless, in the model, as in the data, the unemployment rates for these two groups of workers are highly and roughly equally volatile over the business cycle. The fact that the unemployment rate of low-skilled workers is highly volatile is not surprising given the results in Hagedorn and Manovskii (2006). The accounting profits that firms make on these workers are small and thus respond strongly in percentage terms to fluctuations in the marginal product of these workers. The fact that the unemployment rate of highly skilled workers is also highly volatile, despite the fact that the accounting profits firms make on them are relatively large, is due to the higher volatility and the higher persistence of their marginal product relative to the representative worker case.

We find that the response of unemployment to changes in taxes or unemployment insurance benefits is substantially lower in the model with worker heterogeneity than in the model with homogeneous workers if both models generate the same volatility of market tightness. We show that this difference in policy effects is due to an endogenous response of productivity. Consider, for example, an increase in unemployment insurance. Because the flow utility of unemployment for high-skilled workers is relatively low, a change in the unemployment benefit replacement rate does not substantially affect the decisions of firms to post vacancies in a hope of hiring these workers. Thus, they serve as a fixed factor in the aggregate production. Because capital equipment is complementary with these workers and since the stock of high-skilled workers is little changed, the stock of capital equipment is little changed as well, even in the long run. In turn, if the productivity of low-skilled workers remained unchanged, a change in policy that squeezes the profits that firms make on them would induce firms to post fewer vacancies and the employment of low-skilled workers would fall. However, as their employment falls, their productivity increases because capital equipment and high-skilled workers remain in place. This increase in productivity of low-skilled workers acts to restore the profits that firms make on these workers and counteracts the effect of the change in the policy. Thus, the endogenous response of productivity significantly dampens the effect of a change in the unemployment insurance replacement rate on unemployment. The same effects obtain in response to a change in tax policies. Note that these effects are driven by the presence of worker heterogeneity and not by the curvature in the production *per se*. With a one-sector Cobb Douglas production function, capital would adjust after a change in policy to keep the capital-labor ratio and thus productivity constant.

Figure 1: Effects of an Increase in Unemployment Insurance: An Illustration.



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**APPENDICES**

# I Data and Variable Construction Procedures

## I.1 Aggregate Data

**Output.** Output is BLS non-farm business output.[18]

**Employment.** Aggregate employment is computed using monthly Current Population Surveys (MCPS) from January 1976 to December 2006. MCPS data are available at *http://www.nber.org/data/cps_basic.html*. To make this measure of employment consistent with the aggregate measure of output we exclude government, private households and unpaid family workers. We keep government agriculture workers because the CPS did not distinguish between private and government workers in agriculture before July 1985. Since there are only a few government agriculture workers in each sample after June 1985, they do not affect the results. The only inconsistency is that BLS business output does not include the output of non-profit institutions but our measure of employment includes employees of those institutions (because we cannot identify these people in the data). The resulting monthly employment series is seasonally adjusted using the ratio to moving average method and averaged into quarterly series.

**Productivity.** Aggregate productivity is defined as a ratio of output and employment.

**Wages.** Aggregate wage series is constructed as BLS labor share in non-farm business sector times productivity.

**Capital Structures.** We construct quarterly quality-adjusted stock of capital structures using the perpetual inventory method, $k_{s,t+1} = (1-d_{st})k_{st}+i_{st}$. Annual series for depreciation of capital structures, $d_{st}$, for the period from 1947 to 2000 comes from Cummins and Violante (2002). To compute the quarterly series we assume constant depreciation during a year. For the years 2001 through 2006 we assume that $d_{st}$ is constant and equal to its value in the

[18] BLS data used are available at *http://data.bls.gov/cgi-bin/dsrv?pr*.

year 2000. Quality-adjusted investment in structures, $i_{st}$, is constructed using private fixed investment in structures (BEA-NIPA Table 5.3.5) deflated by the price index of non-durables consumption and services, $PCONS_t$.[19] $PCONS_t$ is calculated using a Tornqvist procedure. If we have $N$ goods, the change in the price index is

$$\Delta PCONS_t = \sum_{i=1}^{N} \log\left(\frac{p_t^i}{p_{t-1}^i}\right) \frac{s_t^i + s_{t-1}^i}{2},$$

and the price index is calculated then recursively

$$PCONS_t = PCONS_{t-1} \exp(\Delta PCONS_t),$$

where the initial value for the price index is set equal to 1. The price index for good $i$, $p_t^i$, $i =$ non-durables consumption, services, is taken from BEA-NIPA Table 2.3.4 and the nominal share for good $i$, $s_t^i$, is calculated using BEA-NIPA Table 2.3.5. The initial value (year 1947) for the stock of capital structures comes from BEA-FAT Table 2.1. The obtained series is then truncated for the years before 1976.

**Capital Equipment.** Quarterly quality-adjusted stock of capital equipment is also constructed using the perpetual inventory method, $k_{e,t+1} = (1 - d_{et})k_{et} + i_{et}$. Annual series for depreciation of capital equipment, $d_{et}$, is also taken from Cummins and Violante (2002), assuming that $d_{et}$ is constant during a year and equal to its value in the year 2000 during the period from 2001 to 2006. We construct the series for nominal investment in equipment as the sum of private fixed investment in equipment (BEA-NIPA Table 5.3.5), changes in inventories (BEA-NIPA Table 1.1.5) and consumer durables (BEA-NIPA Table 1.1.5) and deflate it by the price index for equipment investment, $PEQ_t$, to get $i_{et}$. We use $PEQ_t$ series constructed by Schorfheide, Rios-Rull, Fuentes-Albero, Santaeulalia-Llopis, and Kryshko (2007). It is constructed using the annual price index of equipment investment computed by Cummins and Violante (2002) and imputing the quarterly movements of the official NIPA price index of equipment investment.[20] The initial value (year = 1947) for the stock of capital equipment

[19] As a robustness check we computed the price index of non-durables consumption and services excluding energy and housing and did not get any significant changes in the results.

[20] See Schorfheide, Rios-Rull, Fuentes-Albero, Santaeulalia-Llopis, and Kryshko (2007) for details.

comes from BEA-FAT Table 2.1.The obtained series is then truncated for the years before 1976.[21]

## I.2 Skill-Group Employment and Wages

The sources of employment and wage data by skill group are monthly Current Population Surveys (MCPS) from January 1976 to December 2006 and CPS Outgoing Rotation Groups(ORG) covering the period January 1979 to December 2006. MCPS data are available at *http://www.nber.org/data/cps_basic.html* and CPS ORG data are available at *http://www.ceprdata.org/cps/org_index.php*. To compute the employment series by skill group we use the same procedure as for aggregate employment.

To compute wage series for skilled and unskilled categories we use data constructed by Schmitt (2003) from CPS ORG. Following the approach adopted in Krusell, Ohanian, Ríos-Rull, and Violante (2000) we divide our workers into 198 groups based on their demographic characteristics. There are 11 five-year age groups, 3 race groups (white, black and others), 2 gender groups and 3 education groups (less than high school diploma, high school diploma and college degree and more). Each group, $g$, is defined by age, race, gender and education. The set of groups is denoted by $G$. The measure of the group hourly wage is defined as

$$w_{gt} = \frac{\sum_{i \in g} w_{it} h_{it} \mu_{it}}{\sum_{i \in g} \mu_{it}},$$

where $t = 01.1979, ..., 12.2006$, $\mu_{it}$ - individual's $i$ earnings weight, $h_{it}$ - individual's $i$ usual weekly hours,$w_{it}$ - the measure of individual $i$ hourly wage constructed by Schmitt (2003) from CPS ORG. This measure uses a log-normal imputation to adjust for top-coding, trims data below US\$1 and US\$100 per hour (in constant 2002 dollars), includes overtime, tips and commissions for hourly paid workers and imputes usual weekly hours for those who report "hours vary" starting from 1994.

The measure of wages for skilled and unskilled workers in period $t$ is constructed as follows

$$W_t^j = \sum_{g \in G_t^j} w_{gt} \bar{\mu}_g^j,$$

[21] As a robustness check we computed the series for the stocks of capital structures and equipment for the period from 1976 to 2006 using 1976 stock as an initial value. There were no important changes in the results.

where $j \in \{u, s\}$ indicates unskilled and skilled type, respectively, $\bar{\mu}_g^j = \frac{\sum_{t=1}^T \mu_{gt}^j}{T}$ - temporal average proportion of group $g$ workers in $G^j$, $T$ - number of time periods, $\mu_{gt}^j = \frac{\sum_{i \in g} \mu_{i,t}}{\sum_{i \in G_t^j} \mu_{i,t}}$. The resulting monthly series are deflated using monthly CPI-U, seasonally adjusted using the ratio to moving average method and averaged into quarterly series.

## I.3 Technology Shocks

The series of investment-specific technology change is calculated as

$$q_t = \frac{PCONS_t}{PEQ_t}.$$

To measure neutral technology shocks we use the production function parameters calibrated in Section 3. The monthly skill-group employment series constructed above are seasonally adjusted using the ratio to moving average method and averaged into annual series denoted by $L_t$ and $H_t$, respectively. Low-skilled labor $l_t$ and high-skilled labor $h_t$ are normalized as follows

$$l_t = 2.6379 * 0.9163 \frac{L_t}{\sum_{t=1}^T L_t/T}, t = 1976, .., 2006$$

and

$$h_t = 0.9706 \frac{H_t}{\sum_{t=1}^T H_t/T}, t = 1976, .., 2006,$$

where 2.6379 is the measure of low-skilled workers,[22] and 0.9163 and 0.9706 are employment rates for low-skilled and high-skilled workers, respectively.

The series of neutral technology change is calculated as

$$A_t = \frac{Output}{k_{st}^\alpha \Big[\mu l_t^\sigma + (1-\mu)\big(\lambda k_{et}^\rho + (1-\lambda) h_t^\rho\big)^{\frac{\sigma}{\rho}}\Big]^{\frac{1-\alpha}{\sigma}}}.$$

## I.4 Job-Finding and Job Separation Probabilities

To calculate job-finding and job separation probabilities we employ Shimer (2005b) two state approach. Assuming constant labor force,

$$u_{t+1} = u_t(1 - f_t) + u_{t+1}^s,$$

[22]This number is calculated as the average of $\left\{\frac{L_t}{H_t}\right\}_{t=01.1976,..,12.2006}$. The measure of high-skilled workers is normalized to 1

where $u_{t+1}$ the number of unemployed individuals in month $t$, $u^s_{t+1}$ the number of individuals unemployed for less than one month in month $t$, and $f_t \equiv \frac{m(u,v)}{u}$ is a probability that an unemployed individual finds a job. The measure of job separation probability is[23]

$$s_t = \frac{u_{t+1} - (1 - f_t)u_t}{e_t}.$$

We use basic monthly CPS data for the number of unemployed individuals and number of people unemployed for less than 4 weeks to construct $f_t$ and $s_f$ for skilled and unskilled categories.

Until 1994, all unemployed workers were asked about the duration of unemployment. Starting from 1994, the BLS adds the intervening time for unemployed individuals who have been asked about the duration of unemployment in the previous month. To account for this change in methodology we follow the procedure in Shimer (2005b) and multiply all computed series for short-term unemployment by 1.1 after 1994. The resulting monthly series are seasonally adjusted using the ratio to moving average method.

# II Computation

We use the free-entry condition (15) and flow equation for the surplus (16) to derive the following difference equations in $\theta^T$:

$$\frac{c^T_{x_t}}{\delta\phi^T(\theta^T_{x_t})} = E_{x_t}\{(1-\beta^T)(p^T_{x_{t+1}} - z^T) - c^T_{x_t}\beta^T\theta^T_{x_t} + \frac{(1-s^T)c^T_{x_{t+1}}}{\phi^T(\theta^T_{x_{t+1}})}\}. \qquad \text{(A1)}$$

We solve this system of difference equations to find $\theta^T$ as a function of $x$. Next, we simulate the model to generate artificial time series for neutral and investment-specific shocks, stocks of capital structures and equipment, unemployment, vacancies, and wages for each of the two worker types and the aggregate economy. To do so, we start with an initial value for unemployment of the two groups of workers, as well as neutral and investment-specific productivity shocks. Using the law of motion for employment, we compute next period's employment level $n^l_{t+1} = l_{t+1}$ and $n^h_{t+1} = h_{t+1}$. Using these numbers compute capital $k_{e,t+1}$ and $k_{s,t+1}$ from the corresponding first-order conditions. Next, we draw a new pair of shocks to productivity and

[23]Note that this formula does not take time aggregation into account, since in our model inputs are measured at weekly frequencies.

the price of capital equipment according to the stochastic process describing their evolution. We then know $\theta^T$ and, thus, the job-finding rate and the new unemployment rate. Iterating this procedure generates the time series of interest.

# III Proofs

**Implicit differentiation**

We show how productivity changes in response to changes in $z$, where $p^l$ and $p^h$ are the fixed point of

$$A(p^l, z^l, p^h, z^h) - p^l = 0, \tag{A2}$$

$$B(p^l, z^l, p^h, z^h) - p^h = 0. \tag{A3}$$

It holds that

$$\begin{pmatrix} A_{p^l} - 1 & A_{p^h} \\ B_{p^l} & B_{p^h} - 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial p^l}{\partial z^l} \\ \frac{\partial p^h}{\partial z^l} \end{pmatrix} = \begin{pmatrix} -A_{z^l} \\ -B_{z^l} \end{pmatrix}.$$

This implies that

$$\begin{pmatrix} \frac{\partial p^l}{\partial z^l} \\ \frac{\partial p^h}{\partial z^l} \end{pmatrix} = -1/DD \begin{pmatrix} B_{p^h} - 1 & -A_{p^h} \\ -B_{p^l} & A_{p^l} - 1 \end{pmatrix} \cdot \begin{pmatrix} A_{z^l} \\ B_{z^l} \end{pmatrix},$$

where $DD := (1 - A_{p^l})(1 - B_{p^h}) - A_{p^h} B_{p^l}$. For the derivatives it holds (because equation (31) depends only on $p - z$) that

$$A_{p^l} = -A_{z^l}, \tag{A4}$$

$$A_{p^h} = -A_{z^h}, \tag{A5}$$

$$B_{p^l} = -B_{z^l}, \tag{A6}$$

$$B_{p^l} = -B_{z^l}. \tag{A7}$$

This means that

$$\frac{\partial p^l}{\partial z^l} = \frac{-A_{p^l}(1 - B_{p^h}) - A_{p^h} B_{p^l}}{(1 - A_{p^l})(1 - B_{p^h}) - A_{p^h} B_{p^l}}, \tag{A8}$$

$$\frac{\partial p^h}{\partial z^l} = \frac{-B_{p^l}(1 - A_{p^l}) - A_{p^l} B_{p^l}}{(1 - A_{p^l})(1 - B_{p^h}) - A_{p^h} B_{p^l}}. \tag{A9}$$

To simplify this expression, we have to compute $\frac{\partial \pi^l}{\partial l}$, $\frac{\partial \pi^l}{\partial h}$, $\frac{\partial \pi^h}{\partial l}$ and $\frac{\partial \pi^h}{\partial h}$.
First compute $\frac{\partial \pi^l}{\partial l}$:

$$\begin{aligned} \frac{\partial \pi^l}{\partial l} &= G_{lk}k_L + G_{ll} \quad \text{(A10)} \\ &= G_{lk}\frac{-G_{lk}}{G_{kk}} + G_{ll} \\ &= G_{lk}\frac{k}{l} + G_{lk}\frac{G_{kh}h}{G_{kk}l} + G_{ll} \\ &= \frac{h}{l}(-G_{lh} + G_{kh}\frac{G_{kl}}{G_{kk}}), \end{aligned}$$

where the first equality follows from implicit differentiation of (30) and the second and third equalities are a consequence of constant returns to scale (which implies that $G_k$ and $G_l$ are homogeneous of degree zero):

$$G_{kk}k + G_{kh}h + G_{kl}l = 0, \quad \text{(A11)}$$

$$G_{lk}k + G_{lh}h + G_{ll}l = 0. \quad \text{(A12)}$$

Now compute $\frac{\partial \pi^l}{\partial h}$:

$$\begin{aligned} \frac{\partial \pi^l}{\partial h} &= -G_{lk}\frac{G_{kh}}{G_{kk}} + G_{lh} \quad \text{(A13)} \\ &= -\frac{l}{h}\frac{\partial p^l}{\partial l}. \end{aligned}$$

Making use of similar arguments, it also holds that

$$\begin{aligned} \frac{\partial \pi^h}{\partial h} &= G_{hk}k_H + G_{hh} \quad \text{(A14)} \\ &= G_{hk}\frac{-G_{hk}}{G_{kk}} + G_{hh} \\ &= G_{hk}\frac{k}{h} + G_{hk}\frac{G_{kl}l}{G_{kk}h} + G_{hh} \\ &= \frac{l}{h}(-G_{lh} + G_{kh}\frac{G_{kl}}{G_{kk}}), \end{aligned}$$

and

$$\begin{aligned} \frac{\partial p^h}{\partial l} &= -G_{hk}\frac{G_{kl}}{G_{kk}} + G_{lh} \quad \text{(A15)} \\ &= -\frac{h}{l}\frac{\partial p^h}{\partial h}, \end{aligned}$$

and

$$\frac{\partial \pi^l}{\partial l} = \frac{h^2}{l^2}\frac{\partial p^h}{\partial h}. \tag{A16}$$

We can now simplify $\frac{\partial \pi^l}{\partial z^l}$ and $\frac{\partial \pi^h}{\partial z^l}$:

$$\begin{aligned} A_{p^l}B_{p^h} - A_{p^h}B_{p^l} &= (\frac{\partial \pi^l}{\partial l}L_{p^l})(\frac{\partial \pi^h}{\partial h}H_{p^H}) - (\frac{\partial \pi^l}{\partial h}H_{p^H})(\frac{\partial \pi^h}{\partial l}L_{p^l}) \\ &= L_{p^l}H_{p^H}(\frac{\partial \pi^l}{\partial l}\frac{\partial \pi^h}{\partial h} - (\frac{-l}{h}\frac{\partial \pi^l}{\partial l}\frac{-h}{l}\frac{\partial \pi^h}{\partial h}) = 0, \end{aligned} \tag{A17}$$

and, thus,

$$\frac{\partial p^l}{\partial z^l} = \frac{-A_{p^l}}{1 - A_{p^l} - B_{p^h}}, \tag{A18}$$

$$\frac{\partial p^h}{\partial z^l} = \frac{-B_{p^l}}{1 - A_{p^l} - B_{p^h}}. \tag{A19}$$

By the same arguments it follows that

$$\frac{\partial p^l}{\partial z^h} = \frac{-A_{p^h}}{1 - A_{p^l} - B_{p^h}}, \tag{A20}$$

$$\frac{\partial p^h}{\partial z^h} = \frac{-B_{p^h}}{1 - A_{p^l} - B_{p^h}}. \tag{A21}$$

**Proof of Proposition 1**

Using the above expressions for $\frac{\partial \pi^l}{\partial l}$, $\frac{\partial \pi^l}{\partial h}$, $\frac{\partial \pi^h}{\partial l}$ and $\frac{\partial \pi^h}{\partial h}$, we find that

$$\begin{aligned} \epsilon_{\pi^l,l} &= \frac{h}{p^l}(-G_{lh} + G_{kh}\frac{G_{kl}}{G_{kk}}) \\ &= -\epsilon_{G_l,h} + \frac{\epsilon_{G_k,h}\cdot\epsilon_{G_l,k}}{\epsilon_{G_k,k}} \end{aligned} \tag{A22}$$

$$\begin{aligned} \epsilon_{\pi^l,h} &= \frac{h}{p^l}(-G_{LH} + G_{KH}\frac{G_{KL}}{G_{KK}}) \\ &= -\epsilon_{p^l,L} \end{aligned} \tag{A23}$$

$$\begin{aligned} \epsilon_{\pi^h,h} &= \frac{l}{p^h}(-G_{LH} + G_{KH}\frac{G_{KL}}{G_{KK}}) \\ &= -\epsilon_{G_H,L} + \frac{\epsilon_{G_K,L}\cdot\epsilon_{G_H,K}}{\epsilon_{G_K,K}} \\ &= \frac{lp^l}{hp^h}\epsilon_{p^l,L} \end{aligned} \tag{A24}$$

$$\begin{aligned} \epsilon_{\pi^h,l} &= -G_{hk}\frac{G_{kl}}{G_{kk}} + G_{lh} \\ &= -\epsilon_{\pi^h,h} \end{aligned} \tag{A25}$$

From the definitions of the functions $A, B, \pi^l$ and $\pi^h$ it follows that

$$\epsilon_{A,p^l} = \epsilon_{\pi^l,l}\epsilon_{L,p^l}, \tag{A26}$$

$$\epsilon_{A,p^h} = \epsilon_{\pi^l,h}\epsilon_{H,p^l}, \tag{A27}$$

$$\epsilon_{B,p^l} = \epsilon_{\pi^h,l}\epsilon_{L,p^h}, \tag{A28}$$

$$\epsilon_{B,p^h} = \epsilon_{\pi^h,h}\epsilon_{H,p^h}, \tag{A29}$$

which proves the proposition.

**Proof of Proposition 2**

CRS and perfect substitutes imply that

$$G_{kk}k + G_{kh}h + G_{kl}l = G_{kk}k + G_{kl}(h+l) = 0, \tag{A30}$$

$$G_{lk}k + G_{lh}h + G_{kk}l = G_{lk}k + G_{ll}(h+l) = 0. \tag{A31}$$

The first equation implies that

$$(h+l) = -\frac{G_{kl}}{G_{kk}}k. \tag{A32}$$

Plugging this into the second equation implies that

$$G_{lk}k + G_{ll} - \frac{G_{kl}}{G_{kk}}k = 0. \tag{A33}$$

This implies that

$$\begin{aligned} \epsilon_{\pi^l,l} &= \frac{h}{p^l}(-G_{lh} + G_{kh}\frac{G_{kl}}{G_{kk}}) \\ &= \frac{h}{p^l}(-G_{ll} + G_{kl}\frac{G_{kl}}{G_{kk}}) = 0. \end{aligned} \tag{A34}$$

Noting that all of the four elasticities are just a multiple of each other concludes the proof.

**Proof of Proposition 3**

Follows directly from inspection of $\frac{\partial \pi^l}{\partial l}$.

**Proof of Proposition 4**

Follows directly from assumption 1 and proposition 1.

**Proof of Proposition 5**

How the derivatives of $p$ with respect to $z$ are related to the derivatives of $A$ and $B$ was shown above. The sign of these derivatives then follows immediately from proposition 3.

**Proof of Propositions 6, 7, 8 and 9**

The derivation of these results are discussed in the main text, which is based on equation (54).

# IV Appendix Tables

Table A-1: Matching the Calibration Targets with Low Wage Elasticity.

| | Target | Value | |
|---|---|---|---|
| | | Data | Model |
| 1. | Elasticity of wages wrt agg. productivity, $\epsilon_{w,p}$ | 0.500 | 0.498 |
| 2. | Relative elasticity of wages wrt agg. productivity, $\epsilon_{w^H,p}/\epsilon_{w^L,p}$ | 1.770 | 1.775 |
| 3. | Skilled job-finding rate, $f^H$ | 0.106 | 0.105 |
| 4. | Unskilled job-finding rate, $f^L$ | 0.127 | 0.126 |
| 5. | Skilled average market tightness, $\theta^H$ | 1.044 | 1.039 |
| 6. | Unskilled average market tightness, $\theta^L$ | 0.586 | 0.584 |
| 7. | Elasticity of agg. job-finding wrt agg. market tightness, $\epsilon_{f,\theta}$ | 0.500 | 0.497 |
| 8. | Relative elas. of job-finding wrt agg. mrkt tightness, $\epsilon_{f^H,\theta}/\epsilon_{f^L,\theta}$ | 1.335 | 1.335 |
| 9. | Standard deviation of capital structures | 0.003 | 0.003 |
| 10. | Standard deviation of capital equipment | 0.010 | 0.010 |

Note - The table describes the model's performance in matching the calibration targets, including low wage elasticity.

Table A-2: Calibrated Parameter Values with Low Wage Elasticity.

| Parameter | Definition | Value |
|---|---|---|
| $z^H$ | skilled value of non-market activity (share of their productivity) | 0.848 |
| $z^L$ | unskilled value of non-market activity (share of their productivity) | 0.945 |
| $\beta^H$ | skilled workers' bargaining power | 0.043 |
| $\beta^L$ | unskilled workers' bargaining power | 0.072 |
| $\gamma^H$ | skilled matching function elasticity | 0.238 |
| $\gamma^L$ | unskilled matching function elasticity | 0.544 |
| $\chi^H$ | skilled matching function efficiency | 0.104 |
| $\chi^L$ | unskilled matching function efficiency | 0.165 |
| $d_s^*$ | depreciation factor of capital structures | 11.800 |
| $d_e^*$ | depreciation factor of capital equipment | 1.460 |

Note - The table contains the calibrated parameter values in the low wage elasticity calibration.

Table A-3: Results from the Calibrated Model with Low Wage Elasticity.

| | Statistic | Value | | |
|---|---|---|---|---|
| | | Data, 1951-2004 (1) | Model (2) | LM (3) |
| 1. | St. dev. of agg. productivity, $p$ | 0.013 | 0.013 | 0.013 |
| 2. | Autocorr. of agg. productivity, $p$ | 0.765 | 0.765 | 0.765 |
| 3. | St. dev. of agg. unemployment, $u$ | 0.125 | 0.104 | 0.071 |
| 4. | St. dev. of agg. vacancies, $v$ | 0.139 | 0.142 | 0.101 |
| 5. | St. dev. of agg. market tightness, $\theta$ | 0.259 | 0.246 | 0.163 |
| 6. | Corr. of agg. unemployment and vacancies | -0.919 | -0.782 | -0.780 |
| 1. | St. dev. of skilled productivity, $p^H$ | — | 0.018 | — |
| 2. | Autocorr. of skilled productivity, $p^H$ | — | 0.779 | — |
| 3. | St. dev. of skilled unemployment, $u^H$ | — | 0.138 | — |
| 4. | St. dev. of skilled vacancies, $v^H$ | — | 0.103 | — |
| 5. | St. dev. of skilled market tightness, $\theta^H$ | — | 0.207 | — |
| 1. | St. dev. of unskilled productivity, $p^L$ | — | 0.013 | — |
| 2. | Autocorr. of unskilled productivity, $p^L$ | — | 0.754 | — |
| 3. | St. dev. of unskilled unemployment, $u^L$ | — | 0.100 | — |
| 4. | St. dev. of unskilled vacancies, $v^L$ | — | 0.170 | — |
| 5. | St. dev. of unskilled market tightness, $\theta^L$ | — | 0.258 | — |

Note - Column (1) contains aggregate statistics computed over the 1951:1 to 2004:4 period as in Shimer (2005a). Hornstein, Krusell, and Violante (2005) report virtually identical numbers. In Column (1) seasonally adjusted unemployment, $u$, is constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index, $v$, is constructed by the Conference Board. Both $u$ and $v$ are quarterly averages of monthly series. Average labor productivity $p$ is seasonally adjusted real average output per person in the non-farm business sector, constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics. Column (2) contains the results from the model calibrated with low wage elasticity. Column (3) reproduces the results from the linear model with homogeneous workers for the same aggregate calibration targets. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.

Table A-4: Matching the Calibration Targets with $s = s_U$.

| | Target | Value | |
|---|---|---|---|
| | | Data | Model |
| 1. | Elasticity of wages wrt agg. productivity, $\epsilon_{w,p}$ | 0.670 | 0.669 |
| 2. | Relative elasticity of wages wrt agg. productivity, $\epsilon_{w^H,p}/\epsilon_{w^L,p}$ | 1.770 | 1.801 |
| 3. | Skilled job-finding rate, $f^H$ | 0.106 | 0.105 |
| 4. | Unskilled job-finding rate, $f^L$ | 0.127 | 0.127 |
| 5. | Skilled average market tightness, $\theta^H$ | 1.044 | 1.059 |
| 6. | Unskilled average market tightness, $\theta^L$ | 0.586 | 0.604 |
| 7. | Elasticity of agg. job-finding wrt agg. market tightness, $\epsilon_{f,\theta}$ | 0.500 | 0.501 |
| 8. | Relative elas. of job-finding wrt agg. mrkt tightness, $\epsilon_{f^H,\theta}/\epsilon_{f^L,\theta}$ | 1.335 | 1.346 |
| 9. | Standard deviation of capital structures | 0.003 | 0.003 |
| 10. | Standard deviation of capital equipment | 0.010 | 0.010 |

Note - The table describes the model's performance in matching the calibration targets without distinguishing between the total separation rate and the separation rate into unemployment.

Table A-5: Calibrated Parameter Values with $s = s_U$.

| Parameter | Definition | Value |
|---|---|---|
| $z^H$ | skilled value of non-market activity (share of their productivity) | 0.897 |
| $z^L$ | unskilled value of non-market activity (share of their productivity) | 0.943 |
| $\beta^H$ | skilled workers' bargaining power | 0.064 |
| $\beta^L$ | unskilled workers' bargaining power | 0.098 |
| $\gamma^H$ | skilled matching function elasticity | 0.230 |
| $\gamma^L$ | unskilled matching function elasticity | 0.540 |
| $\chi^H$ | skilled matching function efficiency | 0.102 |
| $\chi^L$ | unskilled matching function efficiency | 0.164 |
| $d_s^*$ | depreciation factor of capital structures | 11.500 |
| $d_e^*$ | depreciation factor of capital equipment | 1.420 |

Note - The table contains the calibrated parameter values in the calibration without distinguishing between the total separation rate and the separation rate into unemployment.

Table A-6: Results from the Calibrated Model with $s = s_U$.

| | Statistic | Value | | |
|---|---|---|---|---|
| | | Data (1) | Model (2) | LM (3) |
| 1. | St. dev. of agg. productivity, $p$ | 0.013 | 0.013 | 0.013 |
| 2. | Autocorr. of agg. productivity, $p$ | 0.765 | 0.765 | 0.765 |
| 3. | St. dev. of agg. unemployment, $u$ | 0.090 | 0.096 | 0.061 |
| 4. | St. dev. of agg. vacancies, $v$ | 0.116 | 0.130 | 0.086 |
| 5. | St. dev. of agg. market tightness, $\theta$ | 0.202 | 0.227 | 0.139 |
| 6. | Corr. of agg. unemployment and vacancies | -0.910 | -0.780 | -0.780 |
| 1. | St. dev. of skilled productivity, $p^H$ | — | 0.018 | — |
| 2. | Autocorr. of skilled productivity, $p^H$ | — | 0.778 | — |
| 3. | St. dev. of skilled unemployment, $u^H$ | 0.111 | 0.129 | — |
| 4. | St. dev. of skilled vacancies, $v^H$ | — | 0.096 | — |
| 5. | St. dev. of skilled market tightness, $\theta^H$ | — | 0.192 | — |
| 1. | St. dev. of unskilled productivity, $p^L$ | — | 0.013 | — |
| 2. | Autocorr. of unskilled productivity, $p^L$ | — | 0.758 | — |
| 3. | St. dev. of unskilled unemployment, $u^L$ | 0.085 | 0.093 | — |
| 4. | St. dev. of unskilled vacancies, $v^L$ | — | 0.156 | — |
| 5. | St. dev. of unskilled market tightness, $\theta^L$ | — | 0.238 | — |

Note - Column (1) reproduces Column (1) of Table 3. See notes to that table for details. Column (2) contains the results from the model calibrated without distinguishing between the total separation rate and the separation rate into unemployment. Column (3) shows the results from the linear model with homogeneous workers for the same aggregate calibration targets. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.