

# Optimal Pilot Waveform Assisted Modulation for Ultra-Wideband Communications\*

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## Abstract

*Performance of Ultra-Wideband (UWB) communication systems can be enhanced by collecting multipath diversity gains, once the channels are acquired at the receiver. In this paper, we develop a novel pilot waveform assisted modulation (PWAM) scheme that is tailored for power-limited UWB communications, and can be implemented in analog form. The PWAM parameters are designed to jointly optimize performance, and information rate. The resulting transmitter design also minimizes the mean-square error (MSE) of channel estimation, and thereby achieves the Cramer-Rao Lower Bound (CRLB).*

## 1 Introduction

UWB systems receive increasing interest for short range high data rate indoor wireless communications [3, 9]. Conveying information with ultra-short pulses, UWB transmissions can resolve many paths, and are thus rich in multipath diversity. This has motivated research towards designing Rake receivers to collect the available diversity, and thus enhance the performance of UWB communication systems [2, 10]. Since the received waveform contains many delayed and scaled replicas of the transmitted pulses, a large number of fingers is needed. Moreover, each of the resolvable waveforms undergoes a different channel, which causes distortion in the received pulse shape, and renders usage of an ideal line-of-sight path signal as a template sub-optimal.

To improve template matching performance, an old (see e.g., [8]) so called transmitted reference (TR) spread spectrum scheme was advocated recently in [4], and its performance was analyzed in [1]. The TR signalling scheme couples each transmitted information conveying pulse with an unmodulated (reference a.k.a. pilot) pulse. The received waveform corresponding to the pilot pulse is used as the correlator template to decode the received information bearing pulse. Conceptually, this is analogous to a Rake receiver with one finger, and a composite correlator template. As half of transmitted waveforms are used as pilots in TR, regardless of the channel, the rate drops by 50%.

In this paper, we introduce a general pilot waveform assisted modulation (PWAM) scheme, which subsumes TR as a special case. To account for both performance and bandwidth efficiency, we design our PWAM to minimize

the channel's MSE, and maximize the average capacity. Tailoring our optimal PWAM for UWB-specific needs, we also develop an optimal (so termed ES-PWAM) scheme, which features pilot- and information- pulses with identical signal-to-noise ratios (SNR). PWAM is applicable to both pulse amplitude modulation (PAM), and pulse position modulation (PPM) [5, 9, 11]. In this paper, we focus on PAM for simplicity, though the analysis carries over to PPM as well.

## 2 System Model

Consider a peer-to-peer UWB communication system, where binary information symbols are conveyed by a stream of ultra-short pulses. The transmit-pulse  $w(t)$  has typical duration  $T_w$  between  $0.2ns$  to  $2ns$ . Every binary  $\pm 1$  symbol is shaped by  $w(t)$ , and is transmitted repeatedly over  $N_f$  consecutive frames, each of duration  $T_f$ .

The overall channel  $h(t)$  comprises the convolution of the pulse shaper  $w(t)$  with the physical multipath channel  $g(t)$ , as shown in Fig. 1. With  $T_g$  denoting the maximum delay spread of the dense multipath channel, we avoid ISI by simply choosing  $T_f > T_g + T_w$ .

We model the channel in an indoor environment as quasi-static. More precisely, we assume that  $h(t)$  remains invariant over a burst of duration  $\bar{N}T_f$  seconds, but may change from burst to burst. Each burst includes up to  $N := \bar{N}/N_f$  symbols that are either training or information bearing. During each burst,  $N_s$  distinct information symbols are transmitted. In other words,  $\bar{N}_s := N_s N_f$  out of the total of  $\bar{N}$  transmitted waveforms of each burst are information conveying. Consequently, the number of training (pilot) waveforms is given by  $\bar{N}_p = \bar{N} - \bar{N}_s$ . Clearly, the number of symbols (information and pilot) per burst satisfies  $N = N_s + N_p$ , where  $N_p := \bar{N}_p/N_f$  can be interpreted as the number of pilots per burst.

Supposing that timing has been acquired, an estimate of the composite channel  $h(t)$  can be formed based on the received pilot waveforms. The estimate  $\hat{h}(t)$  is then used as the correlator template to decode the received information conveying waveforms, as illustrated in Fig. 1.

Our *objective* is to select the PWAM parameters which optimize not only the channel estimation performance, but also the information rate. Due to lack of space, we will present our results without proof. For details, please refer to [11].

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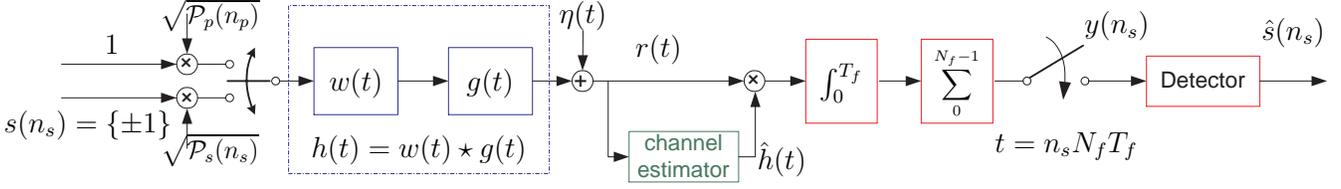


Figure 1: System block diagram.

### 3 Design Criteria

In this section, we will derive the criteria of our optimal PWAM design. More precisely, we will form the channel estimator using the  $\bar{N}_p$  pilot waveforms, and give explicit expression of both the channel MMSE and the average capacity of the overall underlying system.

#### 3.1 Channel Estimator and Channel MMSE

Let  $\mathcal{P}_p$  denote the total power assigned to pilot waveforms. With  $\mathcal{P}_p(n_p)$  denoting the power of the  $n_p$ -th pilot waveform, we have  $\mathcal{P}_p = \sum_{n_p=0}^{\bar{N}_p-1} \mathcal{P}_p(n_p)$ . The received waveform corresponding to the  $n_p$ -th pilot waveform is

$$r_{n_p}(t) = \sqrt{\mathcal{P}_p(n_p)}h(t) + \eta_{n_p}(t), \quad \forall n_p \in [0, \bar{N}_p - 1], \quad (1)$$

where  $\eta_{n_p}(t)$  is the additive white Gaussian noise (AWGN) during the frame which contains the  $n_p$ -th pilot waveform.

A total of  $\bar{N}_p$  received pilot waveforms are summed up to form the channel estimate:

$$\hat{h}(t) = \beta \sum_{n_p=0}^{\bar{N}_p-1} r_{n_p}(t), \quad (2)$$

where the sum is pre-multiplied by a constant  $\beta = \left(\sum_{n_p=0}^{\bar{N}_p-1} \sqrt{\mathcal{P}_p(n_p)}\right)^{-1}$  to guarantee the unbiasedness of  $\hat{h}(t)$ . It can be readily shown that this simple estimator achieves the CRLB with appropriate power distribution among pilot waveforms  $\{\mathcal{P}_p(n_p)\}_{n_p=0}^{\bar{N}_p-1}$ . Defining the channel estimation error  $\tilde{h}(t) := \hat{h}(t) - h(t)$ , we have the following result.

**Proposition 1** *Given the total number of pilot waveforms per burst  $\bar{N}_p$ , and the total power  $\mathcal{P}_p$  assigned to them, equi-powered pilot waveforms minimize the MSE in channel estimation. The resulting  $\beta = 1/\sqrt{\bar{N}_p \mathcal{P}_p}$  yields the channel MMSE:*

$$\sigma_{\tilde{h}}^2 = \sigma^2 / \mathcal{P}_p, \quad (3)$$

which achieves also the CRLB that benchmarks all unbiased channel estimators.

As  $\mathcal{P}_p$  increases, the channel MMSE decreases monotonically. On the other hand, for a fixed total transmission power per burst  $\mathcal{P} = \mathcal{P}_s + \mathcal{P}_p$ , the power assigned to information symbols,  $\mathcal{P}_s$ , decreases with increasing  $\mathcal{P}_p$ . The optimal  $\mathcal{P}_p$  is not yet obvious from the preceding analysis.

Furthermore, the channel MMSE depends on  $\mathcal{P}_p$ , but not on the number of pilot waveforms  $\bar{N}_p$ . To find the optimal  $\bar{N}_p$  and the optimal  $\mathcal{P}_p$  subject to a fixed burst size  $N$ , for a total power  $\mathcal{P}$ , we need a criterion capturing information rate aspects.

#### 3.2 Average Capacity

Towards this objective, we will use the average capacity  $C$  conditioned on our overall system model depicted in Fig. 1. Notice that  $C$  depends on the modulation size, receiver structure, and provides a metric of both performance and information rate achievable by our UWB system with channel estimation, Rake reception, and decoding.

As mentioned earlier, the placement of pilot waveforms does not affect either the performance of our channel estimator, or the decoding stage that depends on it. In the analysis hereafter, we will assume that all pilot waveforms are gathered at the end of each burst, just for the simplicity of notation.

We start from the received waveform during the  $n$ -th frame,  $n \in [0, \bar{N}_s - 1]$ :

$$r_n(t) = \sqrt{\frac{\mathcal{P}_s(n_s)}{N_f}} s(n_s)h(t) + \eta_n(t), \quad t \in [0, T_f], \quad (4)$$

where  $n_s := \lfloor n/N_f \rfloor$  takes the integer part of  $n/N_f$ , and denotes the index of the information symbol transmitted during the  $n$ -th frame.

Using  $\hat{h}(t)$  as a template, the correlator output is

$$x(n) = \sqrt{\frac{\mathcal{P}_s(n_s)}{N_f}} \mathcal{P}_h s(n_s) + \zeta(n), \quad (5)$$

where  $\mathcal{P}_h := \int_0^{T_f} h^2(t)dt$  captures the gain provided by the channel, and  $\zeta(n) := \zeta_1(n) + \zeta_2(n) + \zeta_3(n)$  is the filtered and sampled noise induced by AWGN. The three noise terms can be shown to be independent Gaussian with zero mean, and variances given by  $\mathcal{P}_h \sigma^2$ ,  $\mathcal{P}_h \mathcal{P}_s(n_s) \sigma_{\tilde{h}}^2 / N_f$ , and  $T_f \sigma^2 \sigma_{\tilde{h}}^2$ , respectively.

Recalling that each symbol is transmitted over  $N_f$  frames, the decision statistic for the  $n_s$ -th symbol  $s(n_s)$  is then calculated by summing up  $N_f$  correlator output samples:

$$y(n_s) = \sqrt{N_f \mathcal{P}_s(n_s)} \mathcal{P}_h s(n_s) + \xi(n_s), \quad (6)$$

where  $\xi(n_s) := \sum_{n=n_s N_f}^{(n_s+1)N_f-1} \zeta(n)$  is zero mean Gaussian with variance  $\sigma_{\xi(n_s)}^2 := \mathbb{E}[\xi(n_s)^2] = N_f \mathcal{P}_h \sigma^2 +$

$(\mathcal{P}_h \mathcal{P}_s(n_s) + T_f \sigma^2) N_f \sigma_h^2$ . Consequently, the effective SNR for the  $n_s$ -th received symbol is given by:

$$\rho_e(n_s) = \frac{\mathcal{P}_h^2 \mathcal{P}_s(n_s)}{\mathcal{P}_h \sigma^2 + (\mathcal{P}_h \mathcal{P}_s(n_s) + T_f \sigma^2) \sigma_h^2}. \quad (7)$$

The system in Fig. 1 has binary input  $s(n_s)$  and binary output  $\hat{s}(n_s)$ . The probability that an input symbol  $s(n_s)$  is erroneously decoded is determined by its corresponding effective SNR,  $\rho_e(n_s)$ , and is given by  $p(n_s) := Q\left(\sqrt{\rho_e(n_s)}\right)$ , where  $Q(x) := (1/\sqrt{2\pi}) \int_x^\infty \exp(-y^2/2) dy$ . Accordingly, the overall system can be viewed as a binary symmetric channel (BSC) with transition probability  $p(n_s)$ , which varies from symbol to symbol. For such channels, it is well known that the mutual information is maximized with equi-probable input (see e.g. [6, Chapter 7]). Recalling that for each transmitted burst,  $N_s$  out of  $N$  symbols are information conveying, the resulting average capacity is given by:

$$C = \frac{1}{N} \mathbb{E} \left[ \sum_{n_s=0}^{N_s-1} p(n_s) \log_2 p(n_s) + (1 - p(n_s)) \log_2 (1 - p(n_s)) + 1 \right]. \quad (8)$$

In the following section, we will maximize the average capacity over our design parameters, which will be shown equivalent to minimizing the MSE.

## 4 Optimal PWAM Parameters

In this section, we will determine how to allocate power between pilot and information waveforms, how to distribute power among information symbols, and how many pilot waveforms to transmit per burst.

### 4.1 Optimizing over information symbol power

Defining the ‘‘instantaneous’’ (per channel realization) capacity as

$$C_i := \sum_{n_s=0}^{N_s-1} p(n_s) \log_2 p(n_s) + (1 - p(n_s)) \log_2 (1 - p(n_s)) + 1,$$

the average capacity in (8) is the averaged  $C_i/N$  over the channel pdf. To maximize  $C$  for any given burst size  $N$ , it suffices to maximize  $C_i$  for every channel realization. Maximizing  $C_i$  over the power distribution among information symbols, we obtain the following result.

**Proposition 2** *For any given powers  $\mathcal{P}_s$ ,  $\mathcal{P}_p$ , and burst size  $N$ , equi-powered information symbols maximize  $C_i$ , and thus, the average capacity  $C$ .*

Substituting  $\mathcal{P}_s(n_s) = \mathcal{P}_s/N_s$  into (7), we have

$$\rho_e = \frac{\mathcal{P}_h^2 \mathcal{P}_s}{N_s \mathcal{P}_h \sigma^2 + (\mathcal{P}_h \mathcal{P}_s + N_s T_f \sigma^2) \sigma_h^2}, \quad \forall n_s. \quad (9)$$

The average capacity in (8) becomes:

$$C = \frac{N_s}{N} \mathbb{E} [p \log_2 p + (1 - p) \log_2 (1 - p) + 1], \quad (10)$$

with  $p := Q\left(\sqrt{\rho_e}\right)$  the same  $\forall n_s$ .

In Proposition 1, we showed that equi-powered pilot waveforms minimize the channel MSE for any given pilot power  $\mathcal{P}_p$ . We will show next that maximizing  $C$  is equivalent to minimizing MSE. Differentiating  $C$  with respect to  $\rho_e$ , and treating  $N$  and  $N_s$  as constants, we have

$$\frac{\partial C}{\partial \rho_e} = \frac{1}{2\sqrt{2\pi}} \frac{N_s}{N} \mathbb{E} \left[ \frac{1}{\sqrt{\rho_e}} e^{-\rho_e/2} \log_2 \frac{1-p}{p} \right] > 0, \quad (11)$$

because  $1 - p > 0.5 > p, \forall \rho_e$ . Furthermore,  $\rho_e$  increases monotonically with decreasing  $\sigma_h^2$  (c.f. (9)) for fixed  $\mathcal{P}_s$ ,  $\mathcal{P}_p$ , and  $\sigma^2$ . Therefore, the equi-powered pilot waveforms not only minimize channel MSE, but also maximize the average capacity. With the minimum MSE given in (3), the effective SNR is now given by:

$$\rho_e = \frac{\mathcal{P}_h^2 \frac{\mathcal{P}_p}{\sigma^2} \frac{\mathcal{P}_s}{N_s \sigma^2}}{\mathcal{P}_h \left( \frac{\mathcal{P}_p}{\sigma^2} + \frac{\mathcal{P}_s}{N_s \sigma^2} \right) + T_f}. \quad (12)$$

Recalling that  $T_f$  is in the order of  $10^{-9}$ , and UWB enjoys dense multipath, with moderate SNR, (12) becomes

$$\rho_e = \frac{\mathcal{P}_h^2 \frac{\mathcal{P}_p}{\sigma^2} \frac{\mathcal{P}_s}{N_s \sigma^2}}{\mathcal{P}_h \frac{\mathcal{P}_p}{\sigma^2} + \frac{\mathcal{P}_s}{N_s \sigma^2}} = \frac{\mathcal{P}_p \mathcal{P}_s}{N_s \mathcal{P}_p + \mathcal{P}_s} \cdot \frac{\mathcal{P}_h}{\sigma^2}. \quad (13)$$

### 4.2 Optimizing over the number of pilot waveforms

As is evident in (10), increasing  $N_s$  boosts  $C$  for any given burst size  $N$ . Although not as evident,  $C$  also depends on  $N_s$  through  $p$ , which depends on  $\rho_e$ . Referring to (9), we observe that with other parameters fixed,  $\rho_e$  decreases as  $N_s$  increases. The resulting increase in  $p$  causes  $C$  to decrease. In short, increasing the number of pilot waveforms  $\bar{N}_p$  enhances  $C$  through increasing  $\rho_e$ , but reduces  $C$  through decreasing  $N_s/N$ .

To find out the optimal  $\bar{N}_p$ , and thus  $N_s$ , that results in the maximum  $C$ , we first show that:

**Lemma 1** *For any given powers  $\mathcal{P}_s$  and  $\mathcal{P}_p$ , and burst size  $N$ , the average capacity  $C$  decreases monotonically as the number of pilot waveforms  $\bar{N}_p$  increases beyond  $\bar{N}_p^* := (N - N_s^*) N_f$ , where  $N_s^*$  is given by:*

$$N_s^* = \begin{cases} N - 1, & \text{if } N \text{ is an integer} \\ \lfloor N \rfloor, & \text{otherwise} \end{cases}. \quad (14)$$

Over each burst containing  $\bar{N}$  waveforms, Lemma 1 asserts that we should use  $N_s^* N_f$  as information bearing waveforms, and concentrate the available power for training,  $\mathcal{P}_p$ , to the rest  $\bar{N}_p^* \leq N_f$  pilot waveforms. As a consequence of Lemma 1, the optimal number of pilot waveforms is chosen according to the following proposition.

**Proposition 3** *For any given powers  $\mathcal{P}_s$  and  $\mathcal{P}_p$ , and burst size  $N$ , the number of pilot waveforms  $\bar{N}_p$  that maximizes  $C$  is given by  $\bar{N}_p^*$ , which is defined in Lemma 1.*

Notice that the optimal number of pilot waveforms is no more than  $N_f$  for any given power and burst size. When  $\bar{N}_p = \bar{N}_p^*$ , the maximum average capacity is given by:

$$C = \frac{N_s^*}{N} \mathbb{E}[p \log_2 p + (1-p) \log_2(1-p) + 1]. \quad (15)$$

To complete our optimal PWAM design, we need to determine how to allocate the total transmission power per burst  $\mathcal{P}$  to information and pilot waveforms.

### 4.3 Optimizing over the power allocation

As shown earlier (see (11)), for fixed  $N$  and  $N_s$ , maximizing  $C$  is equivalent to maximizing  $\rho_e$  in (9). From (3), we observe that as  $\mathcal{P}_p$  increases,  $\sigma_h^2$  decreases and tends to enhance  $\rho_e$ . But at the same time,  $\mathcal{P}_s$  also decreases and tends to reduce  $\rho_e$ . The maximization then amounts to optimally allocating the fixed transmission power per burst  $\mathcal{P}$  to information and pilot waveforms.

Defining the power allocation factor  $\alpha := \mathcal{P}_s/\mathcal{P} \in (0, 1)$  as the fraction of the total transmission power per burst that is allocated to information waveforms, we have accordingly  $\mathcal{P}_p = (1-\alpha)\mathcal{P}$ . Also defining  $\rho := \mathcal{P}/(N\sigma^2)$  as the nominal SNR per received symbol, (13) becomes

$$\rho_e = \frac{\alpha(1-\alpha)\rho N}{\alpha + (1-\alpha)N_s} \mathcal{P}_h. \quad (16)$$

Differentiating  $\rho_e$  with respect to  $\alpha$ , we have

**Proposition 4** *With fixed burst size  $N$ , number of information symbols per burst  $N_s$ , and total transmission power per burst  $\mathcal{P}$ , the optimal power allocation factor  $\alpha = \mathcal{P}_s/\mathcal{P}$  which maximizes  $C$  is given by*

$$\alpha = \frac{\sqrt{N_s}}{\sqrt{N_s} + 1}, \quad (17)$$

which results in the maximum effective SNR

$$\rho_e = \frac{\rho N}{(\sqrt{N_s} + 1)^2} \mathcal{P}_h. \quad (18)$$

The optimization over the power allocation factor  $\alpha$  is carried out for any given total transmission power  $\mathcal{P}$ , burst size  $N$ , and number of information symbols per burst  $N_s$ . Therefore, this optimization step does not affect any of the preceding optimal parameter designs. In fact, all of the preceding optimization steps are decoupled. Consequently, they provide together an overall optimal PWAM design.

## 5 Further Considerations

In the preceding section, we successfully designed an optimal PWAM. The design minimizes channel estimation MSE, and maximizes the average capacity of the overall system simultaneously. Nevertheless, besides the channel MMSE and average capacity, there might be other concerns when implementing power-limited UWB communication systems. In this section, we will modify our optimal PWAM to fit some of these concerns.

In UWB transmissions, each information symbol is repeated over  $N_f$  frames. Our optimal PWAM can be modified so that pilot waveforms are also transmitted in groups of size  $N_f$ .

It is evident that under such a constraint, Propositions 1 and 2 still hold true without modification. As to Proposition 3, we will always take  $\bar{N}_p = N_f$ ; i.e., the optimal number of pilot symbols is  $N_p = 1$ . Now with  $N_s^* = N - 1, \forall N$ , the optimal power allocation factor turns out to be  $\alpha = \sqrt{N-1}/(\sqrt{N-1} + 1)$ .

Similar to the definition of nominal SNR  $\rho$ , we define the information SNR and the pilot SNR as  $\rho_s := \mathcal{P}_s/(N_s\sigma^2) = \alpha\rho N/N_s$ , and  $\rho_p := \mathcal{P}_p/(N_p\sigma^2) = (1-\alpha)\rho N/N_p$ , respectively. Notice that generally  $\rho_s \neq \rho_p$ . In order to maintain constant modulus transmissions,  $\rho_s = \rho_p$  is desirable. It can be shown that, subject to such a constraint, the effective SNR is given by:  $\rho_e = (1-\alpha)\rho N \mathcal{P}_h / ((1-\alpha)N + 1)$ , and the average is given by [c.f. (10)]:  $C = \alpha \mathbb{E}[p \log_2 p + (1-p) \log_2(1-p) + 1]$ .

Once again, we observe the opposing trends of  $C$  as  $N_p$  increases. But this time, the powers  $\mathcal{P}_s$  and  $\mathcal{P}_p$  not only change with  $N_p$ , but are also uniquely determined by  $N_p$  for a fixed burst size  $N$ . Therefore, this optimization problem differs from the one in the previous section. We will resort to numerical search to find this optimal  $N_p$  subject to  $\rho_s = \rho_p$ . The resulting PWAM maximizes  $C$  under the aforementioned equi-SNR constraint, and we abbreviate it as optimal ES-PWAM.

As we mentioned in the introduction, the recently proposed TR transmission for UWB communications shares some features with our PWAM design. They both average previously received pilot waveforms (so called transmitted reference in [1, 4]) to form the correlator template of the Rake receiver. The difference is that PWAM is optimal with respect to the number of pilot waveforms and the power allocation, while half of the transmitted waveforms are always used as pilots in TR [1, 4]. Interestingly, when  $N = 2$ , we have  $N_p = N_s = 1$ , and  $\alpha = 1/2$  for optimal PWAM. In this case only, half of the equi-powered transmitted waveforms are used as pilots. The resulting UWB system turns out to be equivalent to the TR signalling scheme, which reveals the optimality of TR in this special case. In fact, in optimal PWAM, the number of pilots satisfies  $N_s \geq N/2$ , and is thus always no less than that in TR. Moreover,  $\rho_e$  is maximized in optimal PWAM for any  $N_s$ , and is also always no less than that in TR. The equality only occurs when  $N = 2$ . As a result, TR is not optimal for all  $N \neq 2$ , but is optimal when  $N = 2$ . Our proposed PWAM and TR will be compared by simulations on various aspects.

## 6 Simulations

In this section, we present simulations and comparisons to validate our analyses and designs. In all cases, the random channels are generated according to [7], where rays

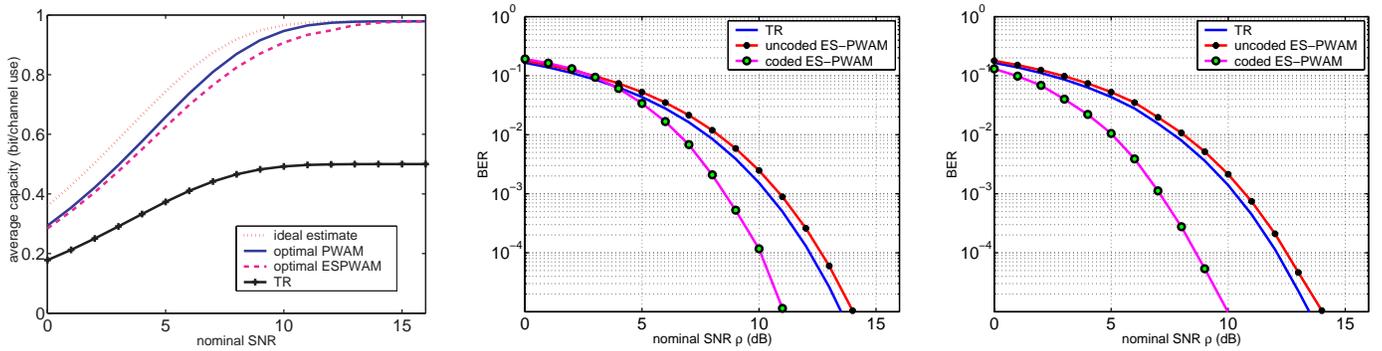


Figure 2: (a) average capacity vs. nominal SNR  $\rho$  ( $N = 100$ ); (b) BER performance ( $N = 48$ ). Info. rates:  $87.5Kbps$  (uncoded ES-PWAM),  $5Mbps$  (coded ES-PWAM & TR); (c) BER performance ( $N = 100$ ). Info. rates:  $92Kbps$  (uncoded ES-PWAM),  $46Kbps$  (coded ES-PWAM), and  $46Kbps$  (TR).

arrive in several clusters within an observation window. The cluster arrival times are modeled as Poisson variables with cluster arrival rate  $\Lambda$ . Rays within each cluster also arrive according to a Poisson process with ray arrival rate  $\lambda$ . The amplitude of each arriving ray is a Rayleigh distributed random variable having exponentially decaying mean square value with parameters  $\Gamma$  and  $\gamma$ . Parameters of this channel model are chosen as:  $\Gamma = 33ns$ ,  $\gamma = 5ns$ ,  $1/\Lambda = 2ns$ , and  $1/\lambda = 0.5ns$ . We select the pulse shaper to be the second derivative of the Gaussian function with unit energy, and  $0.7ns$  pulse width. The frame duration is chosen to be  $T_f = 100ns$ , which is also the maximum delay spread. For ease of comparisons, the optimal PWAM is designed with integer  $N_p$ 's, unless otherwise specified.

We first compare the average capacity of our optimal PWAM with both ES-PWAM, and with the TR signalling scheme [1]. Fig. 2(a) depicts the  $C$  associated with both our optimal PWAM's and TR [1]. The gap is evident, and is increasing as SNR increases.

We also carry out performance comparisons between our optimal ES-PWAM and TR scheme, since the latter also requires  $\rho_s = \rho_p$ . With burst size  $N = 48$ , TR has information rate of  $50Kbps$ , while our uncoded ES-PWAM yields  $87.5Kbps$ . Despite of the large discrepancy of their supporting rates, their performance is very close. To equalize the information rate, we use (2,3,2) convolutional coding together with our ES-PWAM. As shown in Figs. 2(b), our coded ES-PWAM outperforms TR by  $3dB$  at  $BER = 10^{-4}$ . At  $N = 100$ , we use a (1, 2, 3) convolutional code that yields a slightly lower information rate than TR. The resulting BER performance is shown in Fig. 2(c).

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