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PUBLIC DEBT STRATEGIC PLANNING AND BENCHMARK COMPOSITION

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Abstract

In the past decades, public debt theories were centered on the contribution of debt management to the macroeconomic stabilization, in particular, whether the size and structure of the debt placed constraints to the monetary policy. However debt management neutrality relies on strong assumptions, which are unlikely to hold in practice. Alternatively, if we make more realistic assumption, we conclude that the level and composition of the debt are relevant. Based on that, recent models have focused more on the relationship between debt management and fiscal policy and the secondary market development.

Most of the current literature is focused on issues of risk. In particular, the optimal taxation approach tries to find the path for taxes that minimizes the total welfare loss. The public debt management could contribute to smooth the tax burden through the changing return on debt. This argument moves us towards the issuance of debt contingent on the outcome for government spending. However, in practice governments do not issue state-contingent debt, and a number of papers have looked at whether the optimal fiscal policy could be supported by conventional debt instruments. In spite of this discussion, the optimal debt structure would depend on the interaction between changes in inflation and changes in government spending and revenue, and will vary from country to country, depending on the structure of the tax system, the nature of the government's spending commitments and the different types of shocks the economy is subject to.

The primary objective of Brazilian Debt Management is to minimize long-term financing costs, while maintaining risks at prudent levels and contributing to the favorable performance of the public securities market. One could say that this objective is contradictory with the tax smoothing approach. However, we can consider that “maintaining the risk at prudent levels” means to monitor the financial and refinancing risks in order to not reach, under some stress scenarios, a ceiling that would force the government to reduce the expenses or increase the taxes, compromising the tax smoothing rule.

The Brazilian Federal Government Assets and Liabilities Management approach shows net liability mismatches in exchange rate, floating rate and fixed rate, and net asset mismatch in inflation index rate. Considering the particularities of the country, the federal government revenues are expected being correlated with fixed rate (on the short term) and with inflation index (on the medium and long term). The Brazilian Treasury has been developing an analytical approach to address the optimum composition of public debt, as well as its convergence from the current state to this desired one. The model is basically an efficient frontier analysis on the risk and return of net debt to GDP ratios in a steady-state situation.

In a more simplified model, focused on the external debt portfolio, it were simulated bonds with different characteristics, including the new Brazilian domestic currency bond, and maturities. Besides its simplicity, it could illustrate some important trade-offs frequent faced by debt managers, such as the currency and the refinancing risk, and was useful to analyze the entire portfolio in terms of risk/return efficiency. While the costs are very similar, the “short” term bond has a wider probability distribution. As expected, floating rate bonds in US dollars are, on average, less expensive and more volatile, comparatively with the fixed rate bonds in US dollars. A last point to highlight refers to the volatility of the “reais” interest rate coupons, lower than all other instruments. The efficient frontier for the whole portfolio, considering the stylized bonds, shows that the current Brazilian External Debt composition is not on the efficient frontier, which means there is room to improve it.

Introduction

The objective of this paper is, in the light of the more recent theories and technical tolls related to Public Debt Management, to highlight relevant trade-offs that should be included in the analytical framework to help debt managers to discuss the best debt composition, considering the objectives and guidelines previously defined by the government.

For such, in the first chapter we analyze the evolution of the macroeconomic theories that in some way influenced the definition of the objectives and guidelines for the public debt management. In the second, we discuss the public debt objectives and guidelines, specially the trade-off between costs and risks, a traditional challenge faced by the Debt Management Offices – DMOs, and the development of secondary market. In the sequence, the third chapter tells us how the Brazilian National Treasury deals with these challenges, the public debt objectives, guidelines and recent evolution.

The forth chapter describes a model developed by the Treasury to capture more accurately the trade-off between risks and costs related to each debt composition, considering the feature of the Brazilian domestic and external bonds. In the fifth and last chapter, it was presented a more simplified model focused on the external debt portfolio, to simulated several debt compositions, using bonds with different characteristics and maturities. Besides its simplicity, the model could illustrate some important trade-offs frequently faced by debt managers, such as the currency and refinancing risk, and was useful to analyze the entire portfolio in terms of risk/return efficiency.

1. Theoretical Framework

In the past decades, public debt theories were centered on the contribution of debt management to monetary policy[1], in particular, whether the size and structure of the debt place any constraints on the ability of the government to control monetary policy. These theories were centered in macroeconomic models based on the representative agent, infinite horizon, and complete markets assumptions.

In these models, the government debt level does not have any impact on economic activity. Households recognize that, for any given path of spending, a higher debt level today implies higher taxes in the future, and save accordingly. As a result, consumption remains unchanged. This is known as the Ricardian Equivalence Theorem. Under similar assumptions, it can be shown that the debt composition does not have real effects. Essentially, for any change in the government debt composition, households are able to choose an appropriate portfolio of assets that preserves their original consumption plan.

However debt management neutrality relies on strong assumptions, which are unlikely to hold in practice, such as: (i) households recognize that changes in debt financing costs lead to changes in future tax liabilities, being able to adjust their consumptions and savings to absorb the impacts of government's budget constraint; (ii) taxes are non-distortionary (so that changes in taxes do not affect economic behavior); (iii) taxpayers and bondholders are effectively the same agents (so that distributional concerns are not relevant); and (iv) the investment portfolio choices available to the private sector must include the same risk-return trade-offs as those

[1] This view is mainly centered on the seminar James Tobin paper of 1963, named "An Essay on Principles of Debt Management".

provided by government securities (so that government borrowing does not provide new investment opportunities which are otherwise unavailable).

Alternatively, if we make more realistic assumptions, such as (i) agents have finite lives, and they will not necessarily care about the welfare of future generations; (ii) taxes are distortionary; and (iii) the private sector has restrictions to replicate the state-contingent structure of the government security (either government bonds don't have a private sector substitute or the private sector can't deal unlimited short sales or purchases of government securities), we conclude that the level and composition of the debt are relevant.

Because the Ricardian Equivalence assumptions appear to be unrealistic, more recent theories consider its conclusions not relevant to debt management. Recent developments have focused more on the relationship between debt management and fiscal policy and the secondary market development[2].

2. Public Debt Objectives

2.1 Trade-off between cost and risk

The trade-off between cost and risk is a traditional challenge faced by Debt Management Offices – DMOs. As this concept is very common to private sector portfolio managers, it suggests that the government might be able to apply the standard portfolio theory in determining its debt strategy. However, there are a number of factors peculiar to the government, which may complicate the use of standard theory, such as:

- the government can have more complex objectives than simply reducing its overall cost, subject to a certain maximum level of risk;

- the government may also be sensitive to different types of risks than those explored in the corporate finance literature. Governments have traditionally focused on the cash flow and the annual budgetary impacts of debt servicing costs, and have tended to not consider as relevant changes in the market value of the debt portfolio. This has potentially quite different implications for the optimal portfolio[3];

- because of the size and nature of government debt issuance, the government may find it difficult to enter and leave the market without affecting prices. It means that the government can influence the cost and risk of its borrowing through its own financing strategies;

- the government has the power to tax both current and future generations. Government securities are therefore subject to a lower degree of risk than similar private sector securities;

- not all government assets and liabilities should be fully accounted. Even where there is a comprehensive list of assets, it may be difficult to value them or to judge the impact of

[2] See Missale (1999), for a comprehensive review of the Public Debt Management theories.

[3] For example, a portfolio composed by floating rate bonds might have a relatively stable market value, although it would have potentially great variability in annual debt servicing costs. The opposite can be seen with the nominal bonds.

changes in their values on the government's finances. A debt management strategy based on assets that can, without such restrictions as legal, institutional or even operational, generate flows to reduce the borrowing requirement, sounds to be the more correct one;

- because the government is usually a significant player (particularly in the domestic market), a more discretionary approach, attempting to exploit possible arbitrage opportunities, can create uncertainty over the true price of debt. Conversely, policies which enhance the predictability and transparency of government debt policy may help to lower costs in the long-term by reducing the perceptions that the government might be tempted to act opportunistically and, consequently, lower the risk premium attached to uncertainty over issuance policy.

This does not imply that the tools of standard portfolio theory are irrelevant, but careful thought need to be given, for example, to the definition of risk, and to the wider implications of the debt management strategy. This suggests an approach that explicitly incorporates the objectives and constraints peculiar to the government.

Despite the fact that several government debt managers focus on minimizing cost, most of the current literature is focused on risk. This may reflect in part the nature of the specific models used. In the public debt traditional literature, for example, taxpayers and bondholders are basically the same agents, so that distributional issues do not matter. In reality, however, taxpayers and bondholders are perceived to be different groups and governments should be concerned with the distributional implications of debt servicing costs, monitoring the cost of insurance and comparing the gains from cheaper financing (lower taxes) in the short run against the possible gains to taxpayers from a smoother path for the government's finances.

This brings about some expensive forms of insurance, such as state-contingent bonds. The insight of the optimal taxation approach[4] is to show that, because of the nature of the loss function from taxes, the government should follow the risk preferences of the median taxpayer, who is expected to be risk averse. This means that the government should be willing to accept higher debt servicing costs, if it reduces unexpected fluctuations in tax rates.

The optimal taxation approach begins by assuming that the main reason for changes in taxes is to meet the government's long-run financing constraint[5]. This could be met by a variety of deficit paths – the government could choose to balance the budget at every period[6], or it could run deficits for a while, followed by surpluses. However, taxation lowers welfare, because it creates a wedge between the value to the individual and the value to society.

Optimal taxation models try to find the path for taxes that minimizes the total welfare loss. Because the welfare losses from taxation increase more than linearly with changes in taxes (convex function), the total loss from raising taxes in one period and then lowering them in the next period will tend to be higher than if taxes were the same over both periods. So, the optimal

[4] The optimal taxation literature is centered on the seminar Robert Barro paper of 1979, named "On the Determination of the Public Debt".

[5] To ensure the government remains solvent, the present value of primary surpluses (taxes less expenditures, excluding debt service) needs to be sufficient to pay the total existing debt held by the public. This is called the government's intertemporal budget constraint.

[6] If the government decided to balance the budget at every point in time, taxes would rise and fall with changes in spending.

policy for the government is to define the tax rate so that it remains the same over all future periods (then the term "tax smoothing").

In this case, the government would expect to run a deficit when spending requirements are expected to be high, and surpluses when spending requirements were relatively low. However, if an unanticipated shock hit the economy, the tax rate would need to change in order to ensure that the government continues to meet its intertemporal budget constraint. If however the debt cost ex-post could be made negatively correlated to such shocks (or state-contingent), then the government could smooth its spending requirements through the changing return on debt, rather than by changing the tax rate.

In Lucas and Stokey (1983), the government can issue debt instruments of any maturity, contingent on the outcome for government spending. Because markets are complete, it can issue a full set of instruments to insure against any possible shock. This means that whatever the shock to government spending in future periods, the government can always meet its intertemporal budget constraint. If state-contingent debt were available, then debt (and taxes) wouldn't need to change significantly in response to fiscal shocks. It means that debt management could provide full fiscal insurance.

In practice, however, governments do not issue state-contingent debt. First, new state-contingent debt instruments may, at least initially, be relatively costly to introduce because of illiquidity. In addition, there might be considerable data problems involved in ensuring that any index used in such bonds would be acceptable to all investors, easily measured and not subject to revision.

Therefore, a number of papers have looked at whether the optimal fiscal policy could be supported by conventional debt instruments. The precise structure would depend on the composition of shocks that hit the economy. Aggregate supply shocks lead to a negative correlation between prices and real output. This leads to a positive correlation between the return on nominal debt and tax revenues, what means that, as inflation rises, the cost of nominal debt falls, offsetting the impact of falling tax revenues, in real terms. Aggregate demand shocks on the other hand lead to a positive correlation between prices and output. In this case, inflation indexed debt adds value to the total portfolio, because its cost rises when output and tax revenues go up. This means that if output and inflation are negatively correlated (supply shock), it increases the optimal share of nominal bonds. If however inflation and output are positively correlated (demand shock), it increases the optimal share of inflation-linked bonds.

Index-linked bonds have an advantage. They not only can reduce financial risk for investors, but also reduce the variability of the government's real borrowing costs, so that they may benefit both borrower and lender. However if index-linked bonds are not sufficiently liquid, then this is likely to reduce their value to the private sector, which may make them a more expensive form of financing. Additionally, if they are too expensive (partially because of the low liquidity) governments may not find it worthy to issue them.

One could say that this line of reasoning could also be extended to foreign-currency debt. In this sense, if the real exchange rate tends to appreciate when output fall and depreciate when output was unexpectedly high, then foreign-currency debt could potentially be used to hedge against output shocks. However, because the foreign-currency debt cost is affected by unexpected variations in the real exchange rate, its return (in domestic currency) is more volatile than domestic debt.

For Latin American countries, whose trade balance is closer than its peers in Eastern Europe and Asia, foreign currency debt, cultivated by the "original sin", has historically brought dangerous risks to fiscal sustainability. First, because in a supply shock generated by a negative external scenario, the government revenues are expected to decrease, in real terms (lower real output) and the debt cost tends to be higher (due to higher real exchange rate). As the country is closed, the nominal output (and consequently the government revenues) is weakly affected by the exchange rate. Second, to adjust faster the balance of payments to the new environment, a closed country needs to produce an overshooting on the exchange rate. To avoid a high pass-through to inflation, the Central Bank will increase the interest rate, additionally contributing to a lower output. In these situations, the correlation between government revenues and exchange rate debt tends to be very low, if not negative.

This means that the optimal debt structure will depend on the interaction between changes in inflation and changes in government spending and revenue, and will vary from country to country, depending on the structure of the tax system, the nature of the government's spending commitments and the different types of shocks the economy is subject to. But this also suggests that it is difficult to generalize, picking any particular type of debt as being more or less risky than another, no matter the country, since this will depend on assumptions as to what sort of shock is most likely to occur and how the country reacts to it. The incidence of any particular type of shock is difficult to predict. Historical correlations among variables can change over time, particularly if the economy is subject to structural changes. This makes it difficult to define the optimal risk-minimizing portfolio.

2.2 Secondary Market Development

Another observed objective defined by the DMOs is the development of the secondary domestic market for public debt bonds. The microeconomic theory suggests that markets are more efficient when highly competitive, with a large number of participants. Governments have introduced a number of policies for the improvement of both primary and secondary markets aimed at enhancing liquidity and efficiency of the market for debt.

These can include the establishment of a system of primary dealers, who have an obligation to promote a liquid secondary market, the issue of benchmark bonds (large fungible bond issues with the same coupon and maturity date); and the introduction of strips (where a bond is separated into its individual cash flows).

One motivation for such policies has been the desire to increase the market capacity to absorb larger amounts of debt. Another is because liquid secondary markets may reduce transaction costs and risks. The main argument to support these motivations is that investors are likely to prefer assets with easily observable prices and which can be easily traded. This may reflect the existence of trading externalities, which make it less costly to price more standard debt instruments.

An alternative argument is that cost savings could be made by tailoring bonds closer to investor preferences. This would move towards issuing higher numbers of bonds, with perhaps more complicated characteristics. However the trend has been for governments to move towards greater standardization, and fewer types of securities offered, suggesting that the advantages of liquidity offset the potential gains from more adjusted liability matching. In addition, investor demands for more specialized bonds may have been met at least partially by the introduction of strips and mix of different market instruments.

An alternative argument for the government to issue particular types of debt is that the government can facilitate risk sharing in the private sector. This could also lower the cost of borrowing, because investors, valuing the insurance, would be willing to pay a premium for such bonds. However, if the government provides such an asset, it is not clear that it necessarily leads to an improvement in overall welfare. Returning to the social-welfare convex function previously mentioned, if the government takes risk, this is reflected in greater uncertainty in the tax burden. So, the policy-maker should be focused on trying to answer if a specific risk-sharing scheme must generate sufficient welfare gains to offset the loss from a possible increase in distortionary taxation.

3. The Brazilian Case

3.1 Institutional Advances

The National Treasury, within the Ministry of Finance, is the institution responsible for the Federal Public Debt Management, considering the domestic and external, bonded and contractual debts. At the institutional level, 2004 was an important year for the Public Debt Management due to the decision of transferring to the National Treasury the responsibility for issuing the external debt, previously under the responsibility of the Central Bank.

The centralized management of domestic and external debt put the Brazilian Treasury in line with the best recommended practices for public debt management and is yet another step in the strengthening of public institutions in Brazil. It also freed the Central Bank to concentrate on its core responsibility of ensuring price stability. The management of both debts helped the development of a consistent and broad view about the relevant borrowing requirement for the government, and their consolidation in just one piece of information to develop the short, medium and long term strategic planning and the benchmark composition.

3.2 Transparency

In order to enhance predictability and transparency related to public debt, the National Treasury took some important decisions along the recent years. First, in 2001, it decided to publish at the beginning of each fiscal year an Annual Borrowing Plan - ABP, a document that defines the objectives, guidelines and consolidates the strategic planning for the whole year, increasing year by year the amount of information publicly available. The document also discuses risk management for the public debt and brings improvements in the Asset and Liability Management Model.

Second, in 2004, the National Treasury decided to publish a Public Debt Annual Report - PDAR. This report has been praised by market participants and independent experts as a suitable complement to information published in the ABP. Publication of the ABP and the PDAR is not a legal requirement, although the government has renewed its commitment towards transparency with the strengthening of these information tools.

Third, the National Treasury publishes, since 1999 and on a monthly basis, a report detailing information on the level and composition of the Federal Government Domestic Debt, as well as indicators of average maturity and percentage of the debt issued by the National Treasury outstanding in the short run, among other relevant statistics.

Fourth, after the decisions taken during the monthly Public Debt Committee meetings, the National Treasury releases a Domestic Debt Auctions Calendar, defining the maximum amount to be issued and describing the auctions dates, types and bonds characteristics. The Debt Management Department retains some discretion over the amount of bond to be sold at each auction.

And last, a presentation for investors covering the main fiscal, external and monetary indicators, comparing some statistics with other peer countries and showing the results for the recent auctions on the domestic and external markets is updated every week. This document presents the public debt statistics released on the ABP as targets and their current position. ABP, PDAR,

monthly domestic debt report and presentation for investors are available in Portuguese and English version, through internet.[7]

3.3 Objectives, Guidelines, Strategic Planning and Risk Management

The primary objective of Brazilian Debt Management is to "minimize long-term financing costs, while maintaining risks at prudent levels and contributing to the favorable performance of the public securities market"[8]. One could say that this objective is contradictory with the tax smoothing approach. However, we can consider that "maintaining the risk at prudent levels" means to monitor the financial and refinancing risks in order to not reach, under some stress scenarios, a ceiling[9] that would force the government to reduce the expenses or increase the taxes, compromising the tax smoothing rule.

To define the prudent risk levels to be taken, the Debt Management Office takes into account all assets and liabilities which direct and indirectly affect the debt strategy. These assets and liabilities are consolidated in one spreadsheet by main risk factors, volume and cash flow. Additionally, they are presented by accrual basis and mark-to-market basis. Below, we can see a more recent Assets and Liabilities Management - ALM figure:

Table I - Assets and Liabilities Management (August 2005)

INDEX		ASSETS	R$ bi	INDEX		LIABILITIES	R$ bi	MISMATCHES
TOTAL			**500,73**	**TOTAL**			**1.124,39**	**(-623,66)**
Price Index			**340,20**	**Price Index**			**131,91**	**208,29**
WPI			340,20	WPI			93,98	
	NTS	Law nº 9.496/97	248,73		NTS	NTN-C	75,41	
	NTS	Provisory Measure nº 2.185	37,63		NTS	Other WPI	18,58	
	NTS	Loan to CB / BANERJ - MP 2.179	9,59	CPI	NTS	NTN-B	37,91	
	NTS	Others WPI	44,25	CPI	NTS		0,02	
Exchange Rate			**68,06**	**Exchange Rate**			**209,99**	**(-141,93)**
	NTS	ROYALTIES - from states - Law nº 7.990/89	16,85	Denominated Debt				
	NTS	MLTD - Resolution nº 98/92	14,32	US$	NTS	Bradies - fixed rate	6,53	
	NTS	Law nº 7.976/89 - MF 030	5,16		NTS	Bradies - floating rate	13,94	
	NTS	Others	31,73		NTS	Globais - fixed rate	94,22	
					NTS	Globais - floating rate	1,72	
					NTS	Paris Club	6,01	
					NTS	Contractual Debt	27,88	
				Euro	NTS	Euro Bonds	16,39	
					NTS	Other Debts	2,53	
				Yen	NTS	Samurai Bonds	2,90	
				FX-linked Debt				
					NTS	Fixed rate	19,94	
					NTS	Floating rate	0,29	
					CB	Fixed rate	12,50	
					CB	Swaps operations	5,15	
Floating Rate			**79,00**	**Floating Rate**			**562,70**	**(-483,70)**
	NTS	SELIC (overnight rate)	79,00		NTS	SELIC (overnight rate)	**519,44**	
	CB	Swaps operations	5,15		NTS	LFT	513,82	
	NTS	TR	44,19		NTS	Others	5,63	
	NTS	TJLP	3,20					
					NTS	TR	44,19	
					NTS	TJLP	1,39	
					TNS	Contingent Liabilities	79,00	
					CB	Open market operations (one day)	(2,32)	
Fixed Rate	NTS	Several Programs	**6,35**	**Fixed Rate**			**219,79**	**(-213,43)**
					NTS	LTN	212,43	
					NTS	Others	7,36	
					CB	Open market operations (more than one month)	46,07	
Shares	NTS	FAD and FND	**7,12**					7,12

[7] www.tesouro.fazenda.gov.br/divida_publica/index.asp.
[8] Annual Borrowing Plan 2005.
[9] In terms of GDP percentage points, for example.

Some inclusions or exclusions are very controversial, but the Brazilian DMO tries to use the more technical approach, avoid being contaminated by some political arguments. This is the reason why all contingent liabilities which where already assumed by the government, according to a law or judiciary decision, are taken into account. By the same token, we do not consider the monetary base and some assets that do not generate revenues to reduce the Federal Public Debt borrowing requirement and we do consider the dividends received from government companies.

As we can see, the federal government ALM shows net liability mismatches in exchange rate, floating rate (overnight rate) and fixed rate, and net asset mismatch in inflation index. Considering the particularities of the country, the federal government revenues[10] are expected being correlated with fixed rate (on the short term) and with inflation index (on the medium and long term). However, both the objective of "contributing to the favorable performance of the public securities market", that is better achieved using fixed rate bonds, and the not so high demand for inflation index bonds[11] that push the cost up, can make the optimal composition move towards a not so efficient solution.

Considering the objectives defined and the ALM feature, the main guidelines for the Brazilian Federal Public Debt – DPF management are:

- Lengthening of the DPF average maturity, mainly by increasing the average term of securities issued in public offerings (auctions);

- Reduction of the share of debt due in 12 months;

- Gradual replacement of floating-rate (Selic-linked) securities, as well as of exchange-linked securities, by fixed-rate and price-indexed securities; and

- Support to the development of the yield curve for public securities.

The financing strategy for a specific year, taking into account the DPF objective and guidelines, is translated into a range of possible issuances of specific debt instruments, based on the analysis of alternative scenarios.

Upper and lower targets are defined by each main index chosen (stock, composition by risk factor, average maturity and percentage of the debt outstanding on the short term). As we can see on the following tables, the targets try to express the guidelines previously defined, despite some market restrictions.

[10] The revenues earmarked to pay the Federal Public Debt are basically the federal government primary surplus and the revenues from states and municipalities (amounting roughly R$ 250 billion), generated by the last debt renegotiation. These revenues are remunerated by wholesale price index plus 6% and are expected to be paid during the following 30 years, in monthly basis.

[11] The pension funds are moving fast from the defined benefit system to the defined contribution one, reducing the manager commitment with certain target in terms of purchase power.

Table II - Results and Targets for the Federal Domestic Public Debt – DPMFi

Indicators	Dec/02	Dec/03	Dec/04	PAF-2005	
				Minimum	Maximum
Stock of DPMFi held by the public (R$ billion)	623.2	731.4	810.3	940	1000
Average maturity of DPMFi (months)	33.2	31.3	28.1	28	34
% Due in 12 months	38.9	35.3	46.1	40	45
Share of DPMFi					
Fixed-rate (%)	2.2	12.5	20.1	20	30
Floating-rate (%)	60.8	61.4	57.1	47	57
Price-Index (%)	12.5	13.6	14.9	15	20
Exchange-rate (%)	22.4	10.8	5.2	3	5
Others (%)	2.1	1.8	2.7	2	4

Table III - Results and Targets for the Federal Public Debt – DPF

Indicators	Dec/02	Dec/03	Dec/04	PAF-2005	
				Minimum	Maximum
Stock of DPF held by the public (R$ billion)	893.3	965.8	1013.9	1160	1240
Average maturity - Federal Outstanding Debt (months)	42.6	39.0	35.3	36	41
% Due in 12 months	32.7	30.7	39.3	34	40
Share of DPF					
Fixed-rate (%)	1.5	9.5	16.1	16	25
Floating-rate (%)	41.8	46.5	45.7	39	47
Price-Index (%)	9.2	10.3	11.9	12	16
Exchange-rate (%)	45.8	32.4	24.2	18	23
Others (%)	1.6	1.4	2.1	1	3

The strategy followed along the last three to four years reduced sharply the exchange rate risk and the floating rate risk, with great focus on the first one. The high exchange rate share in the DPF was responsible in the recent past by the strong volatility in the Net Public Sector Debt, which increased from 40% in 1999 to 51% in 2005, peaking 62% in 2002. This is the reason why the government decided so firmly to focus on reducing its share in the DPF, with great success, as we can see from table above.

The impact from this measure on risk reducing can be seen below. Stress tests highlight the strides made in reducing the vulnerability of the public debt. In particular, the impact on the outstanding debt of shocks in the exchange and domestic interest rates has been considerably reduced since 2002, dropping by more than 60.0% since then[12].

Table IV - Interest and Exchange Rate Stress Test

Index	Share of DPMFi (%)				Stress Impact (R$ Billion)			
	2002	2003	2004	2005*	2002	2003	2004	2005*
Floating-Rate	60,8	61,4	57,2	52,0	33,59	33,91	31,62	28,73
Exchange-Rate	22,4	10,8	5,1	4,0	98,37	47,25	22,57	17,57
Total	83,2	72,2	62,4	56,0	131,96	81,17	54,19	46,29

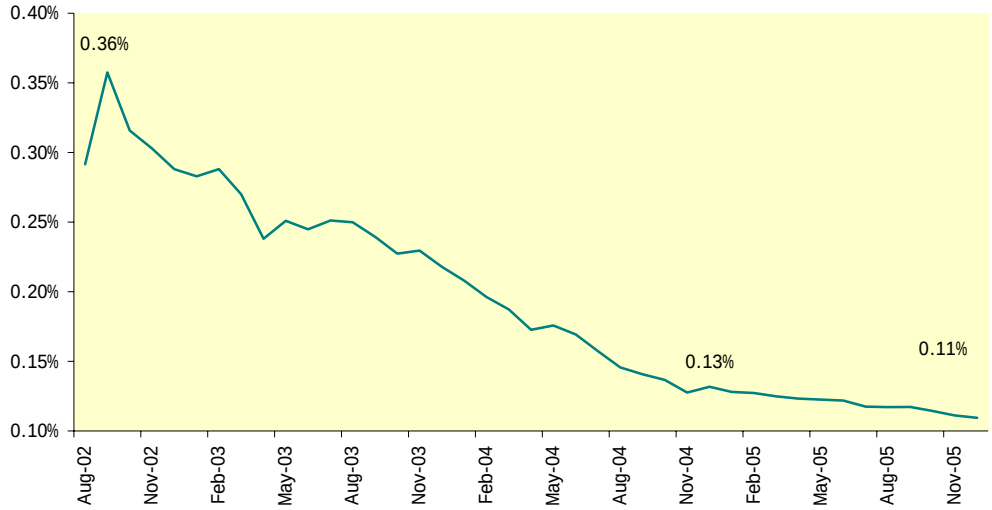
* The 2005 DPMFi composition corresponds to the median point of the indicative PAF-2005 targets.

Source: National Treasury

[12] This exercise considers the impact of shocks to these variables sustained for a 12-month period and size corresponding to three standard deviations of the 12-month rolling average of the interest and exchange rates observed in the period between January 2000 and December 2004 (monthly observations).

Similarly, the impact on the Public Sector Net Debt/GDP of a 1% variation in the exchange rate ratio has dropped from 0.36% in September 2002 to 0.13% by end of 2004, being projected to drop to 0.11% by the end of 2005.

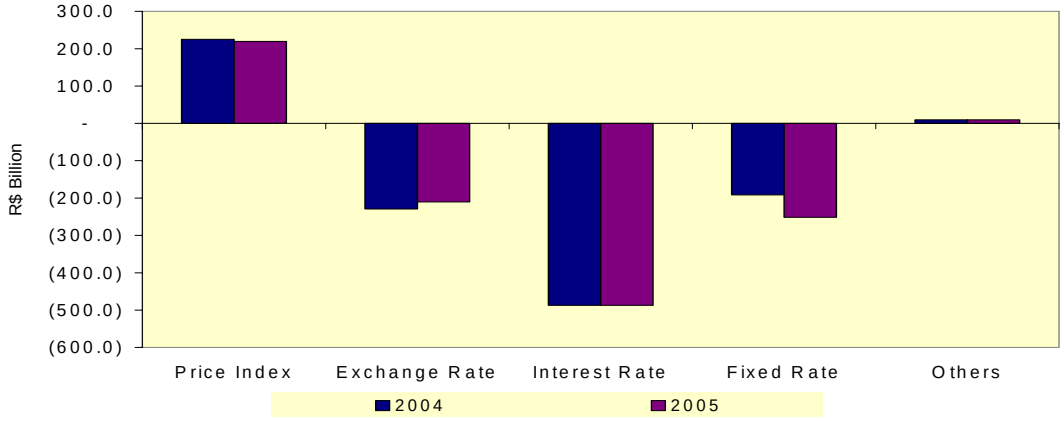
Graph I - Impact of 1% FX Devaluation on Public Sector Net Debt / GDP



As mentioned, the reduction in the sensitivity of the debt was more intense in the case of shocks to the exchange rate, although it also declined in the case of short-term interest rates. Specifically, the sensitivity of the debt to these variables has dropped to about 1/3 of its value in 2002. Indeed, had the debt structure existing in 2002 been maintained, pressures such as those simulated in the exercise would increase the amount of the debt over a period of 12 months by R$ 131 billion (i.e., more than 16% of the Federal Domestic Public Debt - DPMFi or 7.3% of GDP). However, the structure of the debt projected for the end of 2005, points to an impact of R$ 46 billion, (i.e., less than 6% of the DPMFi or 2,6% of GDP).

Another improvement in risk management refers to mismatches between types of assets and liabilities. In particular indexation mismatches are also projected to decline in 2005. The chart below shows that, considering mid-range values for the issuance of each instrument, liabilities mismatch of foreign exchange rates will continue to be reduced, the positive mismatching of price-indexed assets is projected to be roughly stable, while the exposure of the federal government to fixed-rate net liabilities is projected to increase, changes that are favorable to the Treasury.

Graph II - Mismatch Between Assets and Liabilities



Considering the guideline of reduction in the foreign exchange debt, both denominated and linked, to minimize the spillover effect from BP adjustments to the fiscal side, the Federal Government issued a Brazilian R$ 3.4 billion global bond due in 2016. The bond is denominated in reais but payable in dollars, and the yield at issuance on this 10-year bond is 12.75%. The foreign exchange risk is transferred to the investor.

Brazil's domestic nominal yield curve does not extend to 10 years. The average maturity of its domestic debt is around 28 months, being the fixed rate debt around 10 months. Nonresidents do hold some domestic debt, but they represent less than 5% of the total domestic debt. The yields along the curve for Brazilian government debt are inverted and range from 19% for three-month paper to 15.5% for a seven-year bond. Therefore, this debt issue not only provides a tenor that is not available otherwise to the government, at least in this amount, but also provides a lower coupon than can be achieved at any other point in the curve. So the bond provides a fiscal savings compared with domestic debt and avoids the balance-sheet mismatch that foreign currency debt entails.

We can observe the movement in direction of trying to finance its borrowing requirements in the domestic market, which means, in last instance, to match the liabilities characteristics with the assets ones, in several Latin American countries. Some of them have made impressive efforts to deepen their local markets. They have done so by taking steps to develop a natural long-term domestic investor base, improving corporate and market governance, lengthening the government yield curve, and increasing the mix of local currency government debt to total debt. The table below shows important Latin American sovereigns that have increased the share of local currency debt in their debt profiles, some of them additionally expanding the use of inflation-indexed local currency bond.

Table V - Local Currency Government Bonds versus Total Government Bonds (in %)

Year ended Dec. 31

Country	**2000**	**2004**
Brazil	60.7	73.6
Chile	9.1	16.0
Colombia	47.4	54.2
Mexico	48.3	59.4
Venezuela	28.6	36.5

Source: S&P

4. A Model to Assess Optimal Public Debt Portfolios

The Brazilian Treasury has been developing an analytical framework to address the optimum composition of public debt (called Benchmark), as well as its convergence from the current state to this desired one. The idea is to have an ALM based model, considering the relevant assets and liabilities of the federal government.

The model is basically an efficient frontier analysis on the risk and return of net debt to GDP ratios in a steady-state situation. All different compositions of the Treasury's portfolio are considered and an efficient frontier can be drawn. It's out of the scope of this paper to detail this model as it has so many particularities. The aim is, however, to highlight some key features that we believe a good model should address and the benefits of having this kind of analytic tools.

First, all assets and liabilities that, in some way, can affect the Federal Public Debt management should be considered. The ALM feature is the starting point, because it not only maps the stock of assets and liabilities but the flows (revenues and expenses) generated by them. On the liability side, all debt, external and domestic[13], should be considered.

Second, the evolution of the main macroeconomic variables should be simulated[14]. In our model, we use two different and complementary approaches. In the first one, we simulate all variables with correlated financial stochastic models, such as the CIR or the CKLS model. In the second, we use a small macro-structural model to represent the economy.

Third, from a given portfolio composition, we are able to calculate the risk and return for the debt-to-GDP ratio (considering a tax smoothing approach, for example) or other debt indexes over a number of periods and for many simulations. In our model, we end up with debt-to-GDP empirical distributions from which we can extract the main statistics and develop mean-variance analysis. Finally, once chosen an acceptable level of risk, the optimum composition can be pointed out.

Related to the statistics generated by the model, to capture the main financial and rollover risks there are some variables that we believe must be seriously taken into account, such as: average maturity, percentage of the debt outstanding on the short term, composition (fixed-rate, floating-rate, inflation-indexed, among others) and currency composition (in the case of exchange rate linked or denominated debt).

However, to illustrate better some of these features and the trade-offs often faced by the public debt managers we decided to present a more simplified model. The model is focused only on the external debt portfolio and so it's not a complete guide to address optimum debt portfolio composition.

We simulate bonds with maturities from five up to thirty years, fixed and floating, denominated in dollars, euros and Brazilian real. The assumption is that a fixed amount by instrument is issued at the beginning of the period under analysis and when the bond matures it is refinanced

[13] In the Brazilian case, domestic debt includes fixed-rate bonds, floating-rate bonds and inflation-linked and exchange-linked instruments.

[14] The relevant macroeconomic variables to be simulated are different from country to country, depending on their macroeconomics traditional risks and the characteristics of the debt compositions. In the Brazilian case, we simulate inflation, exchange rate, domestic and foreign interest rates, and GDP growth.

by another bond with similar characteristics. The whole portfolio is evaluated through a horizon of forty years.

All yields are simulated through CIR models. It's useful to remember that in these models the short-term interest rate has a diffusion process given by:

$$dr_t = \beta(r^* - r_t)dt + \sigma r_t^{1/2} dz$$

where:
r: instantaneous interest rate
beta: mean reversion velocity
r*: long-run equilibrium interest-rate
sigma: interest-rate volatility
dz: wiener process

The price, and consequently the yield, of a fixed-rate bond in the CIR model can be calculated by:

$$P(t,T) = A(t,T)e^{-B(t,T)r_t}$$

where: $B(t,T) = \dfrac{2\left(e^{\gamma(T-t)} - 1\right)}{(\gamma+\alpha)\left(e^{\gamma(T-t)} - 1\right) + 2\gamma}$

$$A(t,T) = \left[\frac{2\gamma e^{(\alpha+\gamma)(T-t)/2}}{(\gamma+\alpha)\left(e^{\gamma(T-t)} - 1\right) + 2\gamma}\right]^{2\alpha J^*_1/\sigma_1^2}$$

$$\gamma = \sqrt{\alpha^2 + 2\sigma_1^2}$$

As well known in the literature, the CIR model exhibits the desired property of mean reversion, doesn't allow for negative interest rates and has an explicit formula for pricing bonds. It's not a no-arbitrage model, what isn't a problem for our goals.

Domestic and external inflation indexes are simulated through an ordinary Brownian motion, whose diffusion process is given by:

$$dI_t = \mu I_t dt + \sigma\ I_t dz_t$$

where:
I: price index
mi: growth rate of inflation
sigma: inflation volatility
dz: wiener process

It implies that the inflation index has a constant drift rate and volatility around it:

$$\frac{dI_t}{I_t} = \mu dt + \sigma_3 dz$$

We use a particular CKLS model as the diffusion process for the real exchange rate:

$$\frac{dC_t}{C_t} = \beta\left(\frac{C^*}{C_t} - 1\right)dt + \sigma_2 dz_t$$

Where:
C: real exchange rate
Beta: mean reversion velocity
C*: long-run equilibrium real exchange-rate
Sigma: real exchange-rate volatility
dz: wiener process

This process is used for both R$/US$ and R$/Euro rates, with different parameters for each one. Nominal exchange-rates are obtainded from the diference between domestic and external inflations and the real exchange-rate processes above described.

As said before, an initial amount (the same amount in Brazilian currency) of each bond is issued and all yieds are calculated through the models, not only the inicial ones, but also the yields at those moments when each bond is refinanced. Once we have the yields and the nominal exchange rates, we are able to build the cash-flow for all bonds in different simulations. Finally, we can calculate, for each simulation, the internal rate of return (IRR) of each bond, endind up with an empirical distribution of IRR's. From these distributions we extract the two first moments and develop a mean-variance portfolio analysis.

What is really interesting about the model is that, besides its simplicity, it can illustrate some important trade-offs frequent faced by debt managers, such as the currency (local *vs.* external) and the refinancing risk, and it's also useful to analyse the entire portfolio in terms of risk/return efficiency. In the next section, we try to point out and exemplify some of these questions.

5. The Simulation

The aim of this section is to test the model presented earlier. We will run three exercises. The first one has the objective of evaluating the refinancing effect over the expected cost of the debt and its volatility. In the second, it will be shown the relevance of thinking the debt in terms of local currency. To do that, we will run an exercise showing the costs and risks of a local and a foreign currency debt. Finally, we will calculate the efficient frontier with stylized bonds (USD linked fixed and floating-rate, Eurobonds, and Real bonds) as well as the relative position of the current Brazilian external portfolio.

5.1 Parameters for the Estimation Process

The estimation of the parameters for the stochastic models is the first step in the process of running computational simulations. In order to perform this task, it was used traditional ordinary least squares technique (OLS). The data used to run the estimations was basically the historical yields for some specific maturities of the sovereign Brazilian yield curve.

In addition, it was necessary to make some simplifications and assumptions in order to calibrate the Brazilian Real linked bonds. Considering that it was issued at the first time recently, there wasn't data to perform a better estimation, so we used local and foreign market information to build the time series. The last part of this process is to estimate the correlation matrix, used in the model to maintain macroeconomic coherency among the main variables.

5.2 Assessing the Refinancing Issue

In order to evaluate the impact of the refinancing process in a specific portfolio, we will simulate the cost and the volatility of two bonds with different maturities (10 and 20 years). Considering the refinancing process, it is expected a higher volatility in the ten years bond compared to the twenty years one. This uncertainty could make the "short" term bond riskier, although it is, on average, less expensive. Depending on the Government risk aversion, it will be better to choose the longer bond, in order to reduce risk exposition.

In fact, most public debt models despise this very important feature of the debt management process. This handicap may lead to wrong conclusions, specially considering that some Latin America countries are moving towards an increase on the domestic debt share, traditionally with shorter maturity than the external one. It means that, for a better decision of the public debt manager, the refinancing risk should be taken into account as a relevant item. In the Brazilian case, sometimes[15] it was more relevant than the debt composition in terms of risk factors.

The mechanism behind the problem is directly related to the uncertainty of new debt costs, at the time of the refinancing. The table below shows the yield probability distribution.

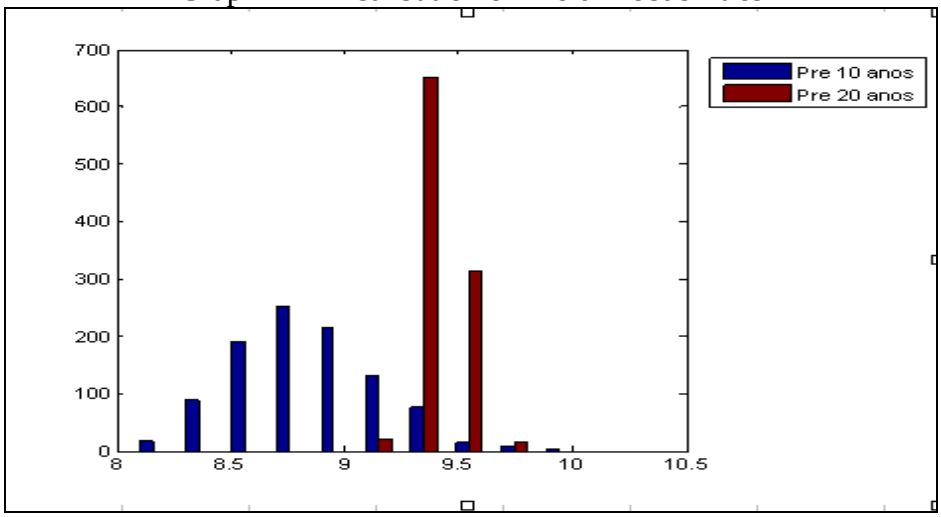
[15] In 1998, during the Russia Crisis, the Brazilian National Treasury decided to issue 24 and 36 months floating rate bonds to substitute the outstanding 6 and 12 moths fixed rate bonds. The objective was to avoid an increase in the percentage of the domestic debt outstanding in 12 months to more that 50%, according to estimates. In that time, the government didn't have yet the good practice of maintaining a "public debt reserve" (budgetary resources earmarked exclusive to pay the debt) equivalent to three months of debt service.

Table VI - Distribution of Yields

	Pre 10 US$	Pre 20 US$
99° Percentile	9,68	9,67
95° Percentile	9,38	9,58
75° Percentile	9,03	9,48
50° Percentile	8,81	9,41
25° Percentile	8,61	9,35
5° Percentile	8,36	9,27
Volatility	0,32	0,09
Mean	8,83	9,41
Mean + 2 STD	9,46	9,60
Mean - 2 STD	8,20	9,23

The following graph[16] shows the distribution of yield probabilities during an interval of 40 years, for each bond. It is important to remember that a ten years bond will be refinanced four times in this period, while the twenty years bond will be only twice. As it can be seen, while the costs are very similar, the "short" term bond has a wider probability distribution. This feature is a result of its higher uncertainty since it will be refinanced two times more than the twenty years bond.

Graph III - Distribution of Yield Probabilities



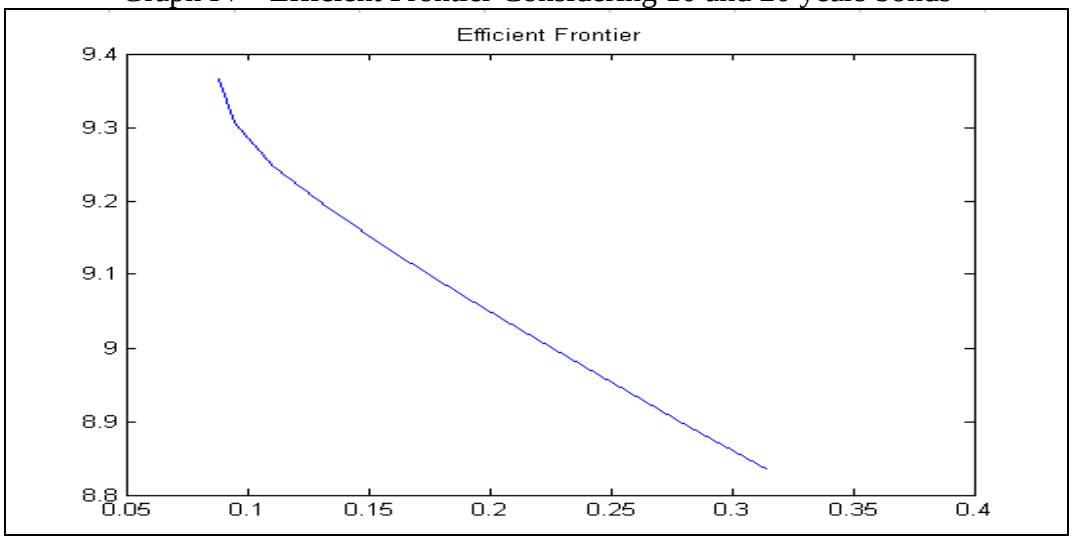
The table below shows the data used to draw the efficient frontier presented in the graph. As expected, in the case of public debt, efficient frontier is drawn in the opposite direction of a "traditional" efficient frontier since the main issue here is to minimize the costs.

Table VII - Costs and Risks for each Portfolio

	Pre 10 US$	Pre 20 US$	Risk	Cost
Portfolio 1	8%	92%	0,09%	9,37%
Portfolio 2	19%	81%	0,09%	9,31%
Portfolio 3	29%	71%	0,11%	9,25%
Portfolio 4	39%	61%	0,13%	9,19%
Portfolio 5	49%	51%	0,16%	9,13%
Portfolio 6	59%	41%	0,19%	9,07%
Portfolio 7	69%	31%	0,22%	9,01%
Portfolio 8	80%	20%	0,25%	8,95%
Portfolio 9	90%	10%	0,28%	8,89%
Portfolio 10	100%	0%	0,31%	8,83%

[16] The parameters used to run the simulations will be shown in the appendix.

Graph IV - Efficient Frontier Considering 10 and 20 years bonds



As it could be seen previously, to ignore the refinancing process in the assessment of optimal indebtedness portfolio could be interpreted as an important handicap since this issue is relevant over the costs and volatilities.

5.3 Evaluating the Currency Risk

This second exercise shows the effect of debt evaluation in terms of local or foreign currency (in special, US dollar). Although it's common to see public debt researches using the dollar as the currency to measure the portfolio costs and risks, we believe the correct way to assess this subject should be in terms of local currency, since it is the government revenues currency. Any possible correlation between the government revenues and a specific foreign currency should be captured when estimating the revenues by the same method used to estimate the debt service. Therefore, the final outcomes will show lower risks in debt compositions whose characteristics move towards the revenues composition.

To better understand the intuition behind the concept, imagine that one specific country has a borrowing requirement and need to choose between two different fixed rate bonds with similar characteristics. The first one is denominated in a foreign currency while the other is in local currency. In order to simplify the exercise, other features were adjusted to isolate the currency effect from the others.

From the Government's point of view, local currency bond has a well-known cash flow, in advance. By other hand, if the country issue a foreign currency bond, the cash flow, in terms of that currency, will be known in advance either, but not in terms of its local currency. If we adopt an approach based, e.g., on a US dollar currency, probably one could recommend the issuance of US dollars fixed rate bond. Besides she/he could recommend some hedging strategies to swap Euro and Yen debt to dollar, in order to "reduce" the exposition[17]. From the government point of view, this strategy not only does not reduce the risk exposition but also damage a possible diversification effect.

[17] Although one could consider this kind of proposal very weird, a lot of financial institutions visit the Brazilian National Treasury to present suggestions of "hedging" foreign debt with swaps from Euro and Yen denominated debt to US dollars, to reduce financial "risk", without any consideration related to the federal government revenues, or the Brazilian international reserves or, even, the current account cash flow.

This is a standard recommendation, considering a country that has no option other than issue in foreign currency and which government revenues are unless moderately affected by the exchange rate, but in no way can be seen as an interesting option for a relatively closed country, with a sufficiently developed domestic market.

Returning to our exercise, it will be compared two ten years bonds with same general features, but one of them denominated in US dollars and the other in Brazilian reais. Considering a "domestic currency" point of view, US dollar denominated debt is a "post-fixed"[18] debt. Thus, it is expected that the US dollar denominated debt have a broader probability distribution (that implies more volatility). As it may be seen on the following table, thinking in terms of Brazilian reais US dollar indebtedness is much more volatile.

Table VIII - Distribution of Yields

	Pre 10 US$	Pre 10 Real$
99° Percentile	16,18	14,84
95° Percentile	15,22	14,23
75° Percentile	14,00	13,43
50° Percentile	13,20	13,15
25° Percentile	12,46	12,85
5° Percentile	11,63	11,95
Volatility	1,13	0,71
Mean	13,28	13,12
Mean + 2 STD	15,55	14,55
Mean - 2 STD	11,02	11,69

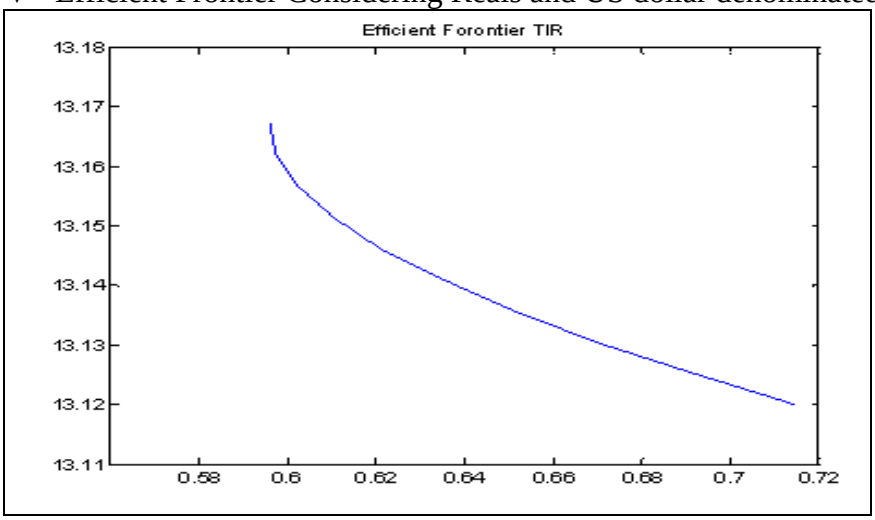
The total effect on the portfolio of a local currency or a foreign currency denominated debt can be seen in the table and graph below, which show different compositions on the efficient frontier.

Table IX - Cost and Risk for each portfolio

	Pre 10 US$	Pre 10 Real$	Risk	Cost
Portfolio 1	29%	71%	0,60%	13,17%
Portfolio 2	26%	74%	0,60%	13,16%
Portfolio 3	23%	77%	0,60%	13,16%
Portfolio 4	19%	81%	0,61%	13,15%
Portfolio 5	16%	84%	0,62%	13,15%
Portfolio 6	13%	87%	0,63%	13,14%
Portfolio 7	10%	90%	0,65%	13,14%
Portfolio 8	6%	94%	0,67%	13,13%
Portfolio 9	3%	97%	0,69%	13,13%
Portfolio 10	0%	100%	0,71%	13,12%

[18] Which mean, the government knows the debt service only in the payment day. The term "post-fixed" was used instead of "floating rate", since the last one usually reminds interest rate debt.

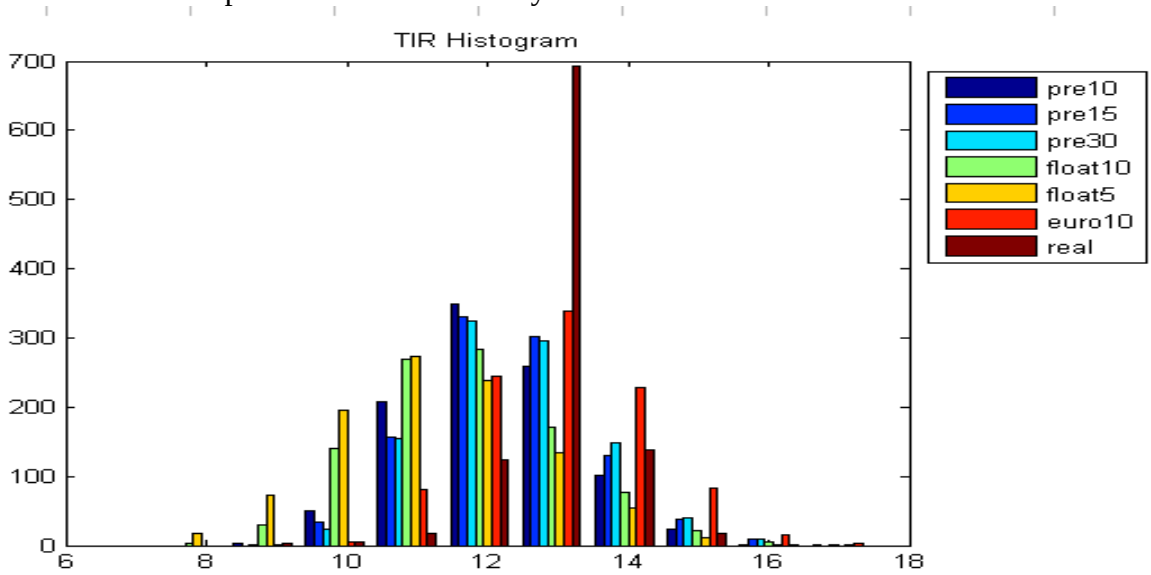
Graph V - Efficient Frontier Considering Reais and US dollar denominated bonds



5.4 Calculating the Efficient Frontier for the Entire Portfolio

The last exercise considers some stylized bonds and evaluates the optimal debt composition. The set of instruments includes fixed rate bonds denominated in Brazilian "reais", US dollars and Euros. Besides that, there are two US dollar floating rate bonds. It is important to emphasize that the exercise conclusions cannot be interpreted as a suggestion for the Brazilian debt composition since we used stylized bonds instead of the real portfolio and the spread among the different bonds possibly will not be maintained for strong changes in the composition[19].

Graph VI - Yield Probability Distribution for Each Bond



[19] As commented in the first unit, the government cannot make strong or fast movements in its portfolio without changing the relative price of the bonds, and, consequently, the shape of the efficient frontier and the optimal portfolio. The lower the demand for a specific bond the stronger the effect in the relative prices.

As expected, floating rate bonds in US dollars are, on average, less expensive and more volatile, comparatively with the fixed rate bonds in US dollars. Another interesting point refers to the long-term US dollar fixed-rate bond (30 years). This bond is more expensive than other shorter bonds, but much less volatile. In fact, it is refinanced only once during the whole simulation period. As a last point, it's important to comment that because the fixed-rate bond in "reais" is refinanced four times in 40 years its volatility is higher than someone could expect.

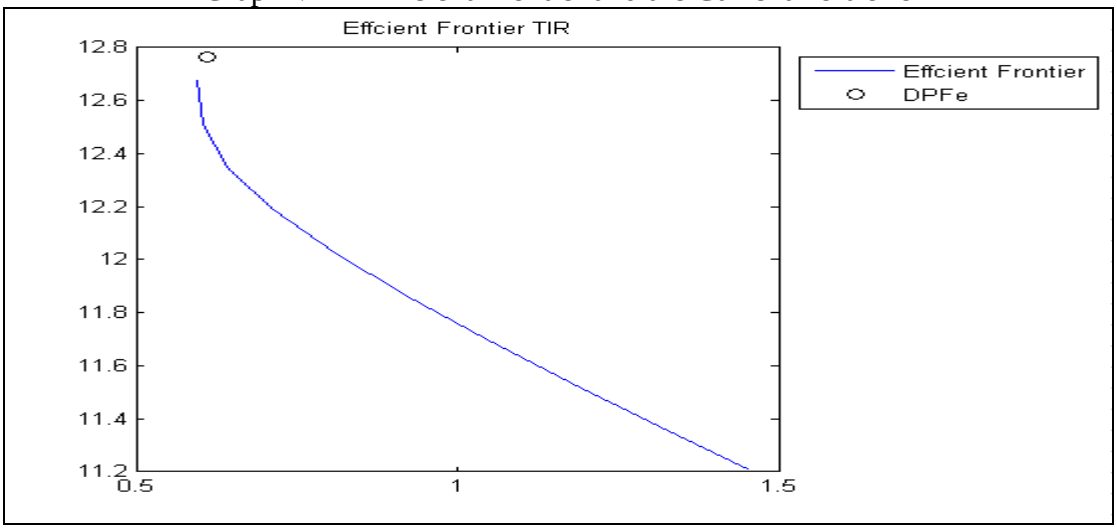
Again, we can observe that US dollar floating rate bonds have higher volatility if compared to other bonds. First, these assets are refinanced four and eight times during the exercise. Second, interest rate coupons are redefined every six months, according to the Libor level on that moment. These characteristics created the higher volatility observed. A last point to highlight refers to the volatility of the "reais" interest rate coupons, lower than other instruments.

Finally, the following table and graph show the efficient frontier for those stylized bonds, as well as the current Brazilian portfolio. As it can be seen from the graph, in this exercise the current Brazilian External Debt composition is not on the efficient frontier, which means there is room to improve it.

Table X - Cost and Risk for Different External Debt Portfolios and the Current One

	Pre 10 US$	Pre 15 US$	Pre 30 US$	Float 10	Float 5	Euro$ 10	Real$ 10	Risk	Cost
Portfolio 1	0%	0%	16%	0%	5%	4%	75%	0,61%	12,79%
Portfolio 2	13%	6%	0%	0%	10%	0%	71%	0,62%	12,43%
Portfolio 3	21%	0%	0%	0%	18%	0%	61%	0,66%	12,07%
Portfolio 4	24%	0%	0%	0%	26%	0%	50%	0,74%	11,72%
Portfolio 5	27%	0%	0%	0%	34%	0%	39%	0,84%	11,36%
Portfolio 6	30%	0%	0%	0%	42%	0%	28%	0,96%	11,00%
Portfolio 7	33%	0%	0%	0%	50%	0%	17%	1,09%	10,64%
Portfolio 8	36%	0%	0%	0%	58%	0%	6%	1,23%	10,28%
Portfolio 9	25%	0%	0%	0%	75%	0%	0%	1,39%	9,92%
Portfolio 10	0%	0%	0%	0%	100%	0%	0%	1,60%	9,57%
DPFe	52%	10%	8%	2%	8%	17%	2%		

Graph VII - Efficient Frontier and the Current Portfolio



6. Final Remarks

Different approaches have been used by DMO's to develop some analytical framework that could guide debt managers in the process of defining the public debt strategic planning and the optimal benchmark composition.

The aim of this paper was to, in the light of the more recent theories and technical tools related to public debt management, highlight the main features of the Brazilian Treasury framework and to discuss some relevant trade-offs, frequent faced by debt managers, which should be considered in any analytical model.

An important lesson is that standard portfolio theory must be adjusted to the reality of governments. This portfolio approach has not been widely explored in the literature on debt management, being mostly used by DMO's. However, the fact that different DMO's face, in general, very different challenges in the debt management and in the economy as a whole makes this a more difficult task, although also an exciting one.

Even similar analytical models could give very different answers for different economies depending on the type of shocks that are more probable to happen in that country. Other particularities include demand and legal constraints, level of development of the secondary market, financial market infrastructure, among others.

We presented a simplified model to illustrate some trade-offs often faced by public debt managers when assessing costs and risks of the public debt. We specifically pointed out issues such as the refinancing risk, the currency (local vs. external) and the importance of looking the debt portfolio as a whole.

Although presenting a simple model for the purposes said above, we are so far convinced that a public debt management model should include all relevant assets and liabilities of the government, reflecting more correctly the trade-off between cost and risk for the government, and, as last resort, for the taxpayers.

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Appendix

Table A I - Parameters for 10 *vs.* 20 years bond simulation

parametros	yield curve (10anos)	yield curve (20 anos)	Dolar	ipca_br	ipc_EUA
a (reversão à média)	0,02157	0,02157	0,09081	0,00000	0,00000
b (média)	0,05692	0,05692	3,04498	0,00383	0,00222
σ (vol)	0,01564	0,01564	0,21567	0,00530	0,00312
λ (lambda)	(0,01775)	(0,01775)	0,00000	0,00000	0,00000
r (taxa inicial)	0,03719	0,03719	2,40380	1,00000	1,00000
Indexador	1,00000	1,00000	3,00000	3,00000	3,00000
prazop	120,00000	240,00000	0,00000	0,00000	0,00000
cor	1,00000	1,00000	2,00000	3,00000	4,00000
modelo	0,00000	0,00000	1,00000	2,00000	2,00000

Inputs	PREFIXED 2	PREFIXED 10
	US$	US$
Inputs		
Títulos	1	2
Taxa de Emissão	0,085562698	0,0918475
Maturação	120	240
Index	1	1
Moeda	1	1

Mont	1.000.000.000,00

corr	Yield US 10	dolar	ipca_br	ipc_EUA
Yield US	100,00%	70,60%	-47,56%	-41,92%
dolar	70,60%	100,00%	16,47%	19,77%
ipca_br	-47,56%	16,47%	100,00%	97,63%
ipc_EUA	-41,92%	19,77%	97,63%	100,00%

Taxa de desconto
10%

PesosB	54,24%	10,00%	8,46%	1,61%	8,47%

prelibor	0,02909375

moedas	1

Table A II - Parameters for Real *vs.* US Dollar simulation

parametros	yield curve (10anos)	Real 10	Dolar	ipca_br	ipc_EUA
a (reversão à média)	0,0215699	0	0,090812	0	0
b (média)	0,0569170	0	3,044978	0,003833	0,002215
σ (vol)	0,0156380	0,029747946	0,215672	0,005296	0,003115
λ (lambda)	-0,0177506	0	0	0	0
r (taxa inicial)	0,0372	0,1275	2,4038	1	1
Indexador	1	4	3	3	3
prazop	120	120	0	0	0
cor	1	2	3	4	5
modelo	0	0	1	2	2

Inputs	PREFIXED 2	PREFIXADO 10
	US$	Real
Inputs		
Títulos	1	2
Taxa de Emissão	0,085562698	0,1275
Maturação	120	120
Index	1	1
Moeda	1	3

Mont	1.000.000.000,00

corr	Yield US 10	dolar	ipca_br	ipc_EUA	Real-10
Yield US	100,00%	70,60%	-47,56%	-41,92%	78,53%
dolar	70,60%	100,00%	16,47%	19,77%	50,31%
ipca_br	-47,56%	16,47%	100,00%	97,63%	-49,80%
ipc_EUA	-41,92%	19,77%	97,63%	100,00%	-40,27%
Real-10	78,53%	50,31%	-49,80%	-40,27%	100,00%

tx	Taxa de desconto
	10%

PesosB	54,24%	10,00%	8,46%	1,61%	8,47%

prelibor	0,02909375

moedas	2

Table A III - Parameters for the Whole Portfolio Simulation

par	parametros	yield curve (5 anos)	yield curve (10 anos)	yield curve (15 anos)	yield curve (30 anos)	swap curve (10 anos)	swap curve (5 anos)	libor	Euro soberana	Dolar	Euro	ipca_br	ipc_EUA	IPC_Euro	Real 10
1	a (reversão à média)	0,0215699	0,0215699	0,0215699	0,0215699	0,00451	0,00451	0,003703	0,0301272	0,090812	0,065241	0	0	0	0
2	b (média)	0,0569170	0,0569170	0,0569170	0,0569170	0,03383	0,03383	0,0616	0,038061	3,044978	3,368985	0,003833	0,002215	0,001835	0
3	σ (vol)	0,0156380	0,0156380	0,0156380	0,0156380	0,003368193	0,003368193	0,009967102	0,0193041	0,215672	0,233823	0,005296	0,003115	0,002691	0,029747946
4	λ (lambda)	-0,0177506	-0,0177506	-0,017750578	-0,017750578	-0,00085	-0,00085	0	-0,044399599	0	0	0	0	0	0
5	r (taxa inicial)	0,0372	0,0372	0,0372	0,0372	0,03492	0,03492	0,0291	0,029543203	2,4038	2,963995068	1	1	1	0,1275
6	Indexador	1	1	1	1	1	1	2	1	3	3	3	3	3	4
7	prazop	60	120	180	360	120	60	120	120	0	0	0	0	0	0
8	cor	1	1	1	1	2	2	3	4	5	6	7	8	9	10
9	modelo	0	0	0	0	0	0	0	0	1	1	2	2	2	0
		1	2	3	4	5	6	7	8	9	10	11	12	13	14

Inputs	PREFIXED 10	PREFIXED 15	PREFIXED 30	FLOAT 10	FLOAT 5	PREFIXED 10	PREFIXADO 10
	US$	US$	US$	US$	US$	Euro$	Real
Inputs							
Títulos	1	2	3	4	5	6	7
Taxa de Emissão	0,0789	0,082306	0,0848	0,0305	0,0221	0,0768	0,1275
Maturação	120	180	360	120	60	120	120
Index	1	1	1	2	2	1	1
Moeda	1	1	1	1	1	2	3

Mont	1.000.000.000,00

corr	Yield US 10	Yield Euro 10	SwapLibor 10	Libor	dolar	euro	ipca_br	ipc_EUA	IPC_Euro	Real_10
Yield US	100,00%	96,22%	1,04%	-61,11%	83,58%	39,16%	-74,72%	-63,36%	-68,51%	78,53%
Euro	96,22%	100,00%	-11,02%	-75,69%	92,14%	54,55%	-85,11%	-75,42%	-78,29%	71,64%
Swap	1,04%	-11,02%	100,00%	29,75%	-4,95%	8,73%	36,02%	41,75%	42,70%	-24,35%
Libor	-61,11%	-75,69%	29,75%	100,00%	-85,09%	-62,48%	94,02%	93,22%	90,15%	-22,73%
dolar	83,58%	92,14%	-4,95%	-85,09%	100,00%	77,06%	-85,69%	-79,70%	-79,17%	50,31%
euro	39,16%	54,55%	8,73%	-62,48%	77,06%	100,00%	-51,35%	-51,70%	-44,56%	6,73%
ipca_br	-74,72%	-85,11%	36,02%	94,02%	-85,69%	-51,35%	100,00%	97,57%	97,70%	-49,80%
ipc_EUA	-63,36%	-75,42%	41,75%	93,22%	-79,70%	-51,70%	97,57%	100,00%	98,29%	-40,27%
IPC_Euro	-68,51%	-78,29%	42,70%	90,15%	-79,17%	-44,56%	97,70%	98,29%	100,00%	-48,98%
Real-10	78,53%	71,64%	-24,35%	-22,73%	50,31%	6,73%	-49,80%	-40,27%	-48,98%	100,00%

tx	Taxa de desconto
	10%

PesosB	52,32%	10,00%	8,46%	1,61%	8,47%	17,21%	1,92%

prelibor	0,02909375

moedas	3