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Formulas and Identities of Trigonometric Functions

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The articles [2], [5], [1], [6], [3], and [4] provide the terminology and notation for this paper.

In this paper t_1, t_2, t_3, t_4 are real numbers.

One can prove the following propositions:

(1) If $\cos t_1 \neq 0$, then $\operatorname{cosec} t_1 = \frac{\sec t_1}{\tan t_1}$.

(2) If $\sin t_1 \neq 0$, then $\cos t_1 = \sin t_1 \cdot \cot t_1$.

(3) If $\sin t_2 \neq 0$ and $\sin t_3 \neq 0$ and $\sin t_4 \neq 0$, then $\sin(t_2 + t_3 + t_4) = \sin t_2 \cdot \sin t_3 \cdot \sin t_4 \cdot ((\cot t_3 \cdot \cot t_4 + \cot t_2 \cdot \cot t_4 + \cot t_2 \cdot \cot t_3) - 1)$.

(4) If $\sin t_2 \neq 0$ and $\sin t_3 \neq 0$ and $\sin t_4 \neq 0$, then $\cos(t_2 + t_3 + t_4) = -\sin t_2 \cdot \sin t_3 \cdot \sin t_4 \cdot ((\cot t_2 + \cot t_3 + \cot t_4) - \cot t_2 \cdot \cot t_3 \cdot \cot t_4)$.

(5) $\sin(2 \cdot t_1) = 2 \cdot \sin t_1 \cdot \cos t_1$.

(6) If $\cos t_1 \neq 0$, then $\sin(2 \cdot t_1) = \frac{2 \cdot \tan t_1}{1 + (\tan t_1)^{\mathbf{2}}}$.

(7) $\cos(2 \cdot t_1) = (\cos t_1)^{\mathbf{2}} - (\sin t_1)^{\mathbf{2}}$ and $\cos(2 \cdot t_1) = 2 \cdot (\cos t_1)^{\mathbf{2}} - 1$ and $\cos(2 \cdot t_1) = 1 - 2 \cdot (\sin t_1)^{\mathbf{2}}$.

(8) If $\cos t_1 \neq 0$, then $\cos(2 \cdot t_1) = \frac{1 - (\tan t_1)^{\mathbf{2}}}{1 + (\tan t_1)^{\mathbf{2}}}$.

(9) If $\cos t_1 \neq 0$, then $\tan(2 \cdot t_1) = \frac{2 \cdot \tan t_1}{1 - (\tan t_1)^{\mathbf{2}}}$.

(10) If $\sin t_1 \neq 0$, then $\cot(2 \cdot t_1) = \frac{(\cot t_1)^{\mathbf{2}} - 1}{2 \cdot \cot t_1}$.

(11) If $\cos t_1 \neq 0$, then $(\sec t_1)^{\mathbf{2}} = 1 + (\tan t_1)^{\mathbf{2}}$.

(12) $\cot t_1 = \frac{1}{\tan t_1}$.

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(13) If $\cos t_1 \neq 0$ and $\sin t_1 \neq 0$, then $\sec(2 \cdot t_1) = \frac{(\sec t_1)^{\mathbf{2}}}{1-(\tan t_1)^{\mathbf{2}}}$ and $\sec(2 \cdot t_1) = \frac{\cot t_1+\tan t_1}{\cot t_1-\tan t_1}$.

(14) If $\sin t_1 \neq 0$, then $(\operatorname{cosec} t_1)^{\mathbf{2}} = 1 + (\cot t_1)^{\mathbf{2}}$.

(15) If $\cos t_1 \neq 0$ and $\sin t_1 \neq 0$, then $\operatorname{cosec}(2 \cdot t_1) = \frac{\sec t_1 \cdot \operatorname{cosec} t_1}{2}$ and $\operatorname{cosec}(2 \cdot t_1) = \frac{\tan t_1+\cot t_1}{2}$.

(16) $\sin(3 \cdot t_1) = -4 \cdot (\sin t_1)^3 + 3 \cdot \sin t_1$.

(17) $\cos(3 \cdot t_1) = 4 \cdot (\cos t_1)^3 - 3 \cdot \cos t_1$.

(18) If $\cos t_1 \neq 0$, then $\tan(3 \cdot t_1) = \frac{3 \cdot \tan t_1-(\tan t_1)^3}{1-3 \cdot (\tan t_1)^{\mathbf{2}}}$.

(19) If $\sin t_1 \neq 0$, then $\cot(3 \cdot t_1) = \frac{(\cot t_1)^3-3 \cdot \cot t_1}{3 \cdot (\cot t_1)^{\mathbf{2}}-1}$.

(20) $(\sin t_1)^{\mathbf{2}} = \frac{1-\cos(2 \cdot t_1)}{2}$.

(21) $(\cos t_1)^{\mathbf{2}} = \frac{1+\cos(2 \cdot t_1)}{2}$.

(22) $(\sin t_1)^3 = \frac{3 \cdot \sin t_1-\sin(3 \cdot t_1)}{4}$.

(23) $(\cos t_1)^3 = \frac{3 \cdot \cos t_1+\cos(3 \cdot t_1)}{4}$.

(24) $(\sin t_1)^4 = \frac{(3-4 \cdot \cos(2 \cdot t_1))+\cos(4 \cdot t_1)}{8}$.

(25) $(\cos t_1)^4 = \frac{3+4 \cdot \cos(2 \cdot t_1)+\cos(4 \cdot t_1)}{8}$.

(26) $\sin(\frac{t_1}{2}) = \sqrt{\frac{1-\cos t_1}{2}}$ or $\sin(\frac{t_1}{2}) = -\sqrt{\frac{1-\cos t_1}{2}}$.

(27) $\cos(\frac{t_1}{2}) = \sqrt{\frac{1+\cos t_1}{2}}$ or $\cos(\frac{t_1}{2}) = -\sqrt{\frac{1+\cos t_1}{2}}$.

(28) If $\sin(\frac{t_1}{2}) \neq 0$, then $\tan(\frac{t_1}{2}) = \frac{1-\cos t_1}{\sin t_1}$.

(29) If $\cos(\frac{t_1}{2}) \neq 0$, then $\tan(\frac{t_1}{2}) = \frac{\sin t_1}{1+\cos t_1}$.

(30) $\tan(\frac{t_1}{2}) = \sqrt{\frac{1-\cos t_1}{1+\cos t_1}}$ or $\tan(\frac{t_1}{2}) = -\sqrt{\frac{1-\cos t_1}{1+\cos t_1}}$.

(31) If $\cos(\frac{t_1}{2}) \neq 0$, then $\cot(\frac{t_1}{2}) = \frac{1+\cos t_1}{\sin t_1}$.

(32) If $\sin(\frac{t_1}{2}) \neq 0$, then $\cot(\frac{t_1}{2}) = \frac{\sin t_1}{1-\cos t_1}$.

(33) $\cot(\frac{t_1}{2}) = \sqrt{\frac{1+\cos t_1}{1-\cos t_1}}$ or $\cot(\frac{t_1}{2}) = -\sqrt{\frac{1+\cos t_1}{1-\cos t_1}}$.

(34) If $\sin(\frac{t_1}{2}) \neq 0$ and $\cos(\frac{t_1}{2}) \neq 0$ and $1 - (\tan(\frac{t_1}{2}))^{\mathbf{2}} \neq 0$, then $\sec(\frac{t_1}{2}) = \sqrt{\frac{2 \cdot \sec t_1}{\sec t_1+1}}$ or $\sec(\frac{t_1}{2}) = -\sqrt{\frac{2 \cdot \sec t_1}{\sec t_1+1}}$.

(35) If $\sin(\frac{t_1}{2}) \neq 0$ and $\cos(\frac{t_1}{2}) \neq 0$ and $1 - (\tan(\frac{t_1}{2}))^{\mathbf{2}} \neq 0$, then $\operatorname{cosec}(\frac{t_1}{2}) = \sqrt{\frac{2 \cdot \sec t_1}{\sec t_1-1}}$ or $\operatorname{cosec}(\frac{t_1}{2}) = -\sqrt{\frac{2 \cdot \sec t_1}{\sec t_1-1}}$.

Let us consider t_1. The functor $\coth t_1$ yielding a real number is defined as follows:

(Def. 1) $\coth t_1 = \frac{\cosh t_1}{\sinh t_1}$.

Let us consider t_1. The functor $\operatorname{sech} t_1$ yielding a real number is defined by:

(Def. 2) $\operatorname{sech} t_1 = \frac{1}{\cosh t_1}$.

Let us consider t_1. The functor $\operatorname{cosech} t_1$ yields a real number and is defined as follows:

(Def. 3) $\operatorname{cosech} t_1 = \frac{1}{\sinh t_1}$.

We now state a number of propositions:

(36) $\coth t_1 = \frac{\exp t_1 + \exp(-t_1)}{\exp t_1 - \exp(-t_1)}$ and $\operatorname{sech} t_1 = \frac{2}{\exp t_1 + \exp(-t_1)}$ and $\operatorname{cosech} t_1 = \frac{2}{\exp t_1 - \exp(-t_1)}$.

(37) If $\exp t_1 - \exp(-t_1) \neq 0$, then $\tanh t_1 \cdot \coth t_1 = 1$.

(38) $(\operatorname{sech} t_1)^{\mathbf{2}} + (\tanh t_1)^{\mathbf{2}} = 1$.

(39) If $\sinh t_1 \neq 0$, then $(\coth t_1)^{\mathbf{2}} - (\operatorname{cosech} t_1)^{\mathbf{2}} = 1$.

(40) If $\sinh t_2 \neq 0$ and $\sinh t_3 \neq 0$, then $\coth(t_2 + t_3) = \frac{1 + \coth t_2 \cdot \coth t_3}{\coth t_2 + \coth t_3}$.

(41) If $\sinh t_2 \neq 0$ and $\sinh t_3 \neq 0$, then $\coth(t_2 - t_3) = \frac{1 - \coth t_2 \cdot \coth t_3}{\coth t_2 - \coth t_3}$.

(42) If $\sinh t_2 \neq 0$ and $\sinh t_3 \neq 0$, then $\coth t_2 + \coth t_3 = \frac{\sinh(t_2 + t_3)}{\sinh t_2 \cdot \sinh t_3}$ and $\coth t_2 - \coth t_3 = -\frac{\sinh(t_2 - t_3)}{\sinh t_2 \cdot \sinh t_3}$.

(43) $\sinh(3 \cdot t_1) = 3 \cdot \sinh t_1 + 4 \cdot (\sinh t_1)^3$.

(44) $\cosh(3 \cdot t_1) = 4 \cdot (\cosh t_1)^3 - 3 \cdot \cosh t_1$.

(45) If $\sinh t_1 \neq 0$, then $\coth(2 \cdot t_1) = \frac{1 + (\coth t_1)^{\mathbf{2}}}{2 \cdot \coth t_1}$.

(46) If $t_1 > 0$, then $\sinh t_1 \geqslant 0$.

(47) If $t_1 < 0$, then $\sinh t_1 \leqslant 0$.

(48) $\cosh(\frac{t_1}{2}) = \sqrt{\frac{\cosh t_1 + 1}{2}}$.

(49) If $\sinh(\frac{t_1}{2}) \neq 0$, then $\tanh(\frac{t_1}{2}) = \frac{\cosh t_1 - 1}{\sinh t_1}$.

(50) If $\cosh(\frac{t_1}{2}) \neq 0$, then $\tanh(\frac{t_1}{2}) = \frac{\sinh t_1}{\cosh t_1 + 1}$.

(51) If $\sinh(\frac{t_1}{2}) \neq 0$, then $\coth(\frac{t_1}{2}) = \frac{\sinh t_1}{\cosh t_1 - 1}$.

(52) If $\cosh(\frac{t_1}{2}) \neq 0$, then $\coth(\frac{t_1}{2}) = \frac{\cosh t_1 + 1}{\sinh t_1}$.

References

[1] Pacharapokin Chanapat, Kanchun, and Hiroshi Yamazaki. Formulas and identities of trigonometric functions. *Formalized Mathematics*, 12(**2**):139–141, 2004.

[2] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[3] Rafał Kwiatek. Factorial and Newton coefficients. *Formalized Mathematics*, 1(**5**):887–890, 1990.

[4] Takashi Mitsuishi and Yuguang Yang. Properties of the trigonometric function. *Formalized Mathematics*, 8(**1**):103–106, 1999.

[5] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.

[6] Yuguang Yang and Yasunari Shidama. Trigonometric functions and existence of circle ratio. *Formalized Mathematics*, 7(**2**):255–263, 1998.

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