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FUZZY ECONOMIC PRODUCTION QUANTITY MODEL FOR ITEMS WITH IMPERFECT QUALITY

Shan Huo Chen
Department of Information Management
Ching Yun University
229, Chien-Hsin Rd., Jung-Li 320, Taiwan
shchen_im@cyu.edu.tw

Chien-Chung Wang
Department of Finance, Management College
National Defense University
70, Sec.2, Ehongyang North Rd., Peitou District, Taipei 11258, Taiwan
jameswangcc@yahoo.com

Shu Man Chang
Department of Shipping Business Management
China College of Marine Technology and Commerce
212, Sec.9, Yan-Pin N. Rd., Taipei 111, Taiwan
smchang@mail.ccmtc.edu.tw

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Abstract. *In the real world, vague phenomenon is quite common in the production/inventory models. In order to process the vagueness, a production/inventory model that can be more closely related to the real vagueness and can take account of the vague factors that contribute to production costs, is required. The model must be extended or altered to fit in with the fuzzy situation. Since items with imperfect quality, during production or inventory procedure, are unavoidable, we also consider this situation. In order to treat the case in the vague environment, we propose a Fuzzy Economic Production Quantity (FEPQ) model with imperfect products that can be sold at a discount price. In this model, costs and quantities are expressed as trapezoidal fuzzy numbers. Moreover, we use Function Principle to manipulate arithmetical operations, Graded Mean Integration Representation method to defuzzify, and Kuhn-Tucker conditions to find the optimal economic production quantity of the fuzzy production inventory model. Finally, an application of an electronics industry example gives a satisfactory result.*
Keywords: Fuzzy inventory model, Economic production quantity, Function principle, Graded mean integration representation method, Optimization, Imperfect production, Defective products

1. **Introduction.** Fuzzy set concept has been widely used to treat the classical inventory model. Park [31] considered fuzzy inventory cost in the economic order quantity model. Chang [3] discussed how to get the economic order quantity, when the quantity of demand is uncertain. Hsieh [17], Lee et al. [23], Lin et al. [25] also wrote some articles about fuzzy production model, but all of them have not developed an inventory model with imperfect products.

In the real world, imperfect products are inevitable during most production processes. It would be interesting to discuss an inventory model with imperfect products. Recently, Salameh et al. [35], Mohamed [28], Lin et al. [24], Chung et al. [14], and Lee [22] have written papers about imperfect production processes, but they have not discussed fuzzy costs under vague situations. Therefore, we propose a Fuzzy Economic Production Quantity model with imperfect products that can be sold at a discount price to treat both vague and imperfect production problems.

Function Principle [5], instead of Extension Principle, is used to calculate the fuzzy total revenue (FTR), Graded Mean Integration Representation Method [7,10] is used to defuzzify the FTR, and Kuhn-Tucker condition [36] is applied to find the optimal economic production quantity. Then an application of an electronics industry example is proposed and the solution is quite satisfied.

This paper is organized as follows. In Section 2, the three required methodologies of the proposed model are introduced. In Section 3, we describe the production problems, and then propose the necessary notations and assumptions. We also describe the solving procedure of the fuzzy economic production quantity model with imperfect products in this section. An application of this model is shown in Section 4, and finally Section 5 concludes this paper.

2. **Methodology.**

In this paper, function principle and graded mean integration representation method are applied to calculate the optimal economic production quantity in the fuzzy inventory model. When the quantities are fuzzy numbers, we can use the kuhn-tucker conditions to solve the model. Therefore now we introduce these three methodologies as follows.

2.1. **Fuzzy arithmetical operations under function principle.**

Function principle is introduced by Chen [5] to treat the fuzzy arithmetical operations of trapezoidal fuzzy numbers. We apply this principle as the operation of addition, multiplication, subtract, division of trapezoidal fuzzy numbers, because (1) function principle is easier to calculate than extension principle, (2) function principle will not change the shape of a trapezoidal fuzzy number after the multiplication of two trapezoidal fuzzy numbers, but the multiplication of two trapezoidal fuzzy numbers will become drum like shape fuzzy number by using extension principle, (3) if we have to multiply more than four trapezoidal fuzzy numbers, extension principle can not solve the operation, but function principle can easily find the result by pointwise computation. Here we describe some fuzzy arithmetical operations under function principle as following:

Suppose $\tilde{A}= (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then,

(1) The addition of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4),$$

where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(2) The multiplication of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \otimes \tilde{B} = (c_1, c_2, c_3, c_4),$$

where $T = \{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $T_1=\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, $c_1 = \min T$, $c_2 = \min T_1$, $c_3 = \max T_1$, $c_4 = \max T$.

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all nonzero positive real numbers, then

$$\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4).$$

(3) $-\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$, then the subtraction of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \Theta \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1),$$

where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(4) $1/\tilde{B} = \tilde{B}^{-1} = (1/b_4, 1/b_3, 1/b_2, 1/b_1)$,

where b_1, b_2, b_3 and b_4 are all positive real numbers.

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all nonzero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \emptyset \tilde{B} = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1).$$

(5) let $\alpha \in R$, *then*

$$\begin{cases} (i) & \alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_1, \alpha_2, \alpha a_3, \alpha a_4) \\ (ii) & \alpha < 0, \alpha \otimes \tilde{A} = (\alpha a_4, \alpha_3, \alpha a_2, \alpha a_1). \end{cases}$$

Note: We do not introduce a new addition symbol, as the sum under Extension Principle is the same as Function Principle in Figure 1. For a mathematically minded reader, we observe that the Extension Principle is a form of convolution (Chen, [6]) while the Function Principle is akin to a pointwise multiplication as Figure 2.

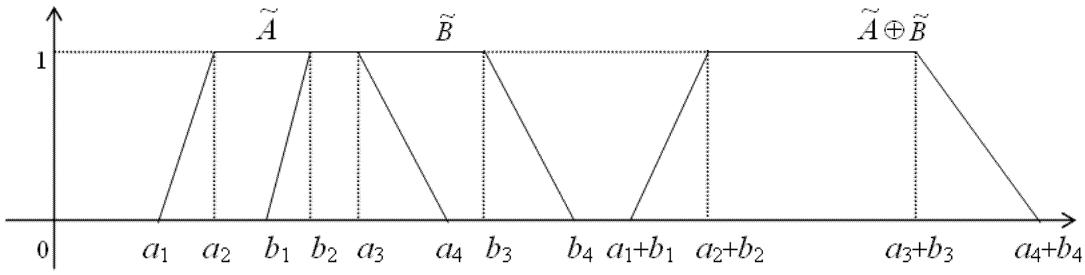

FIGURE 1. Fuzzy addition operation using function principle and extension principle

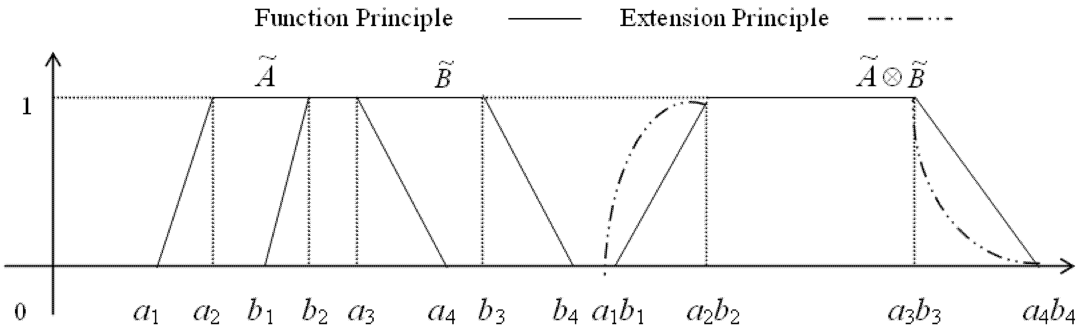

FIGURE 2. Comparison of fuzzy multiplication operation under Function Principle and Extension Principle

Remark 2.1. *In Figure 2, we can see that the result of multiplication of two trapezoidal fuzzy numbers using extension principle is a drum-like shape fuzzy number, but the result of using function principle is still a trapezoidal fuzzy number.*

Remark 2.2. *In figure 2, we can see that the result of the multiplication of two trapezoidal fuzzy numbers with extension principle or function principle has same four vertical points.*

Remark 2.3. *If more than four positive trapezoidal fuzzy numbers are multiplied, extension principle can not solve this operation, but function principle can easily calculate the result by pointwise computation.*

2.2. **Graded mean integration representation method.** Chen et al. [7,10,12] introduced graded mean integration representation method based on the integral value of graded mean h-level of a generalized fuzzy numbers for defuzzifing generalized fuzzy number. They also found this method is better than the methods of Adamo [1], Campos et al. [2], Yager [37], Kaufmann et al. [19], Liou et al. [26], and Heilpern [15]. Now, we describe a generalized fuzzy number as following.

Suppose $\tilde{A}$ is a generalized fuzzy number as shown in Figure 1. It is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions:

(1) μ_A is a continuous mapping from R to the closed interval $[0, 1]$,
(2) $\mu_A = 0, -\infty < x \leq a_1$,
(3) $\mu_A = L(\text{x})$ is strictly increasing on $[a_1, a_2]$,
(4) $\mu_A = w_A, \; a_2 \leq x \leq a_3$,
(5) $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$,
(6) $\mu_A = 0, \; a_4 \leq x < \infty$,

where $0 < w_A \leq 1$, and a_1, a_2, a_3, and a_4 are real numbers.

Also this type of generalized fuzzy numbers be denoted as $\tilde{A}= (a_1, a_2, a_3, a_4; w_A)_{LR}$, when w_A=1, it can be simplified as $\tilde{A}= (a_1, a_2, a_3, a_4)_{LR}$.

In graded mean integration representation method, L^{-1} and R^{-1} are the inverse functions of L and R respectively, and the graded mean h-level value of generalized fuzzy number $\tilde{A}= (a_1, a_2, a_3, a_4; w_A)_{LR}$ is $h(L^{-1}(h)+R^{-1}(h))/2$ as shown in Figure 3. Then the graded mean integration representation of $\tilde{A}$ is $P(\tilde{A})$ with grade w_A where

$$P(\tilde{A}) = \frac{\int_0^{w_A} h(\frac{L^{-1}(h)+R^{-1}(h)}{2})dh}{\int_0^{w_A} hdh} \quad (1)$$

with $0 < h \leq w_A$ and $0 < w_A \leq 1$.

Throughout this paper, only normal trapezoidal fuzzy numbers as the type of all fuzzy parameters are used in our proposed fuzzy production inventory models. Let $\tilde{B}$ be a trapezoidal fuzzy number, and be denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then we can get the graded mean integration representation of $\tilde{B}$ by formula (1) as

$$P(\tilde{B}) = \frac{\int_0^{1} h(\frac{b_1+b_4+(b_2-b_1-b_4+b_3)h}{2})dh}{\int_0^{1} hdh} = \frac{b_1+2b_2+2b_3+b_4}{6} \quad (2)$$

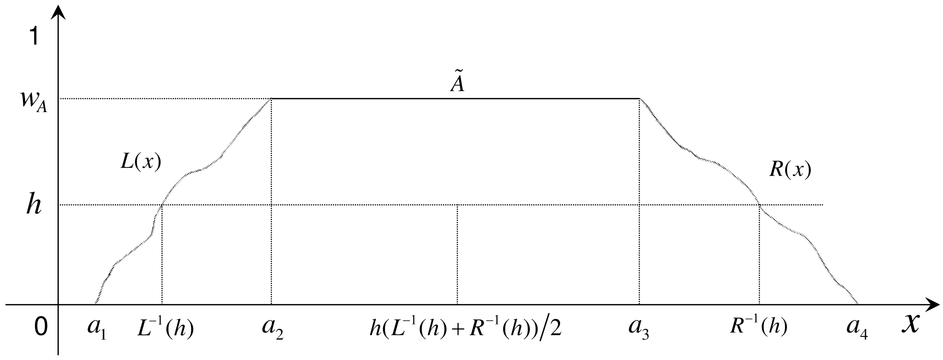

FIGURE 3. The graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$

2.3. **The Kuhn-Tucker conditions.** Taha [36] discussed how to solve the optimum solution of nonlinear programming problem subject to inequality constraints by using the Kuhn-Tucker conditions. The development of the Kuhn-Tucker conditions is based on the Lagrangean method.

Suppose that the problem is given by

Maximize $y = f(x)$
Subject to $g_i(x) \leq 0$, $i = 1, 2, \ldots, m$.

The nonnegative constraints $x \geq 0$, if any, are included in the m constraints.

The inequality constraints may be converted into equations by using nonnegative slack variables. Let's S_i^2 be the slack quantity added to the ith constraint $g_i(x) \leq 0$. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$, $g(x)=(g_1(x), g_2(x), \ldots, g_m(x))^T$and $S^2=(S_1^2, S_2^2, \ldots, S_m^2)^T$. The Lagrangean functions is thus given by

$$L(x, s, \lambda) = f(x) - \lambda[g(x) + S^2]$$

Given the constraints $g_i(x) \leq 0$.

Taking the partial derivatives of L with respect to x, S, and λ, we obtain

$$\frac{\partial L}{\partial X} = \nabla f(x) - \lambda \nabla g(x) = 0;$$

$$\frac{\partial L}{\partial S_i} = -2\lambda_i S_i = 0, \quad i = 1, 2, \ldots, m;$$

$$\frac{\partial L}{\partial \lambda} = -g(x) - S^2 = 0, \quad i = 1, 2, \ldots, m.$$

From the second and third sets of equations, it shows that

$$\lambda_i g_i(x) = 0, \quad i = 1, 2, \ldots, m.$$

The Kuhn-Tucker conditions need x and λ to be a stationary point of the minimization problem which can be summarized as follows:

$$\begin{cases} \lambda \geq 0; \\ \nabla f(x) - \lambda \nabla g(x) = 0; \\ \lambda_i g_i(x) = 0, \quad i = 1, 2, \dots, m; \\ g_i(x) \leq 0, \quad i = 1, 2, \dots, m. \end{cases} \tag{3}$$

3. **Fuzzy Inventory Model.** We thereby discuss the case of imperfect products that can be sold at a discount price with fuzzy holding cost, fuzzy setup cost, fuzzy yearly demand, and fuzzy order quantity. Throughout this paper, we use the following variables in order to simplify the treatment of the fuzzy production inventory model:

$\tilde{H}$: fuzzy daily holding cost per unit,
$\tilde{K}$: fuzzy setup cost,
c : the unit production cost,
s : unit selling price of items of good quality,
v : unit selling price of defective items ($v < c$),
x : screening rate,
a: unit screening cost,
p : the percentage of defective items in a production lot,
$\tilde{D}$: fuzzy daily demand over the planning time period [0, 365],
$\tilde{Q}$: fuzzy production quantity,
T: cycle time.
Inventory quantity

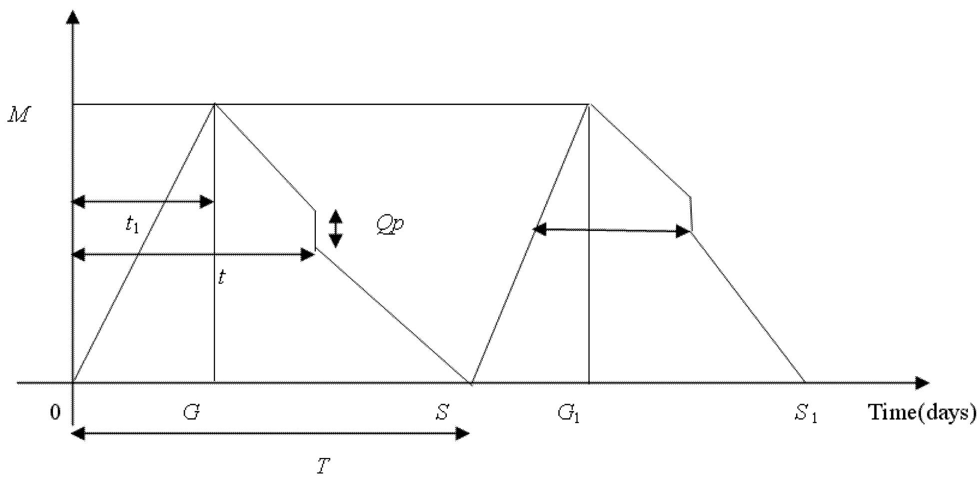

FIGURE 4. Inventory control and the production process

Remark 3.1. *Figure 4 indicate that the manufacturer produces and sells products in the time $OG, SG_1, \dots$, etc., and he only sells products in the time $GS, G_1S_1, \dots$, etc.*

Remark 3.2. *Qp denotes the selling amount with discount price for imperfect items at some point within time GS.*

Remark 3.3. *t_1 denotes the producing and selling time length.*

Remark 3.4. *t denotes the selling time of imperfect items.*

Remark 3.5. *M denotes the largest amount of inventory.*

Let $FN(\tilde{Q},p)$ be the fuzzy number of good items, and it approximates to $FN(\tilde{Q},p) = \tilde{Q}\Theta p\otimes\tilde{Q} = (1-p)\otimes\tilde{Q}$. To avoid shortage, it is assumed that the number of good items is at least equal to the demand during screening time t, that is $FN(\tilde{Q},p)\geq\tilde{D}\otimes t$. Replacing t by $\tilde{Q}\emptyset x$, the value of p is restricted to $p\leq 1\Theta\tilde{D}\emptyset x$. Now the fuzzy total revenue $FTR(\tilde{Q})$ is the approximation to $s\otimes\tilde{Q}\otimes(1-p)\oplus v\otimes\tilde{Q}\otimes p$. The fuzzy total cost per cycle FTC($\tilde{Q}$) approximates to the sum of procurement cost per cycle, screening cost per cycle and holding cost per cycle that is $\tilde{K}\oplus c\otimes\tilde{Q}\oplus a\otimes\tilde{Q}\oplus\tilde{H}\otimes\{(\tilde{Q}\otimes(1-p)\otimes T)\emptyset 2\oplus p\otimes\tilde{Q}^2\emptyset x\}$. The total profit $FTP(\tilde{Q})$ is the approximation to $s\otimes\tilde{Q}\otimes(1-p)\oplus v\otimes\tilde{Q}\otimes p\Theta$ $\{\tilde{K}\oplus c\otimes\tilde{Q}\oplus a\otimes\tilde{Q}\oplus\tilde{H}\otimes\{(\tilde{Q}\otimes(1-p)\otimes T)\emptyset 2\oplus p\otimes\tilde{Q}^2\emptyset x\}\}$.

Firstly, the fuzzy total profit per unit time,

$$\begin{aligned}FTPU(\tilde{Q}) &= TP(\tilde{Q})\emptyset T\\ &\doteq (approximatesto)\tilde{D}\otimes(s\Theta v\oplus\tilde{H}\otimes\tilde{Q}\emptyset x)\oplus\tilde{D}\\ &\quad\otimes(v\Theta\tilde{H}\otimes\tilde{Q}\emptyset x\Theta c\Theta a\Theta\tilde{K}\emptyset\tilde{Q})(1/(1-p))\Theta[\tilde{H}\otimes\tilde{Q}\otimes(1-p)]\emptyset 2, \qquad (4)\end{aligned}$$

where $\oplus$,Θ, $\otimes$ and $\emptyset$ are the fuzzy arithmetical operations under Function Principle.

Here, we suppose $\tilde{H}=(h_1,h_2,h_3,h_4)$, $\tilde{K}=(k_1,k_2,k_3,k_4)$, $\tilde{D}=(d_1,d_2,d_3,d_4)$, $\tilde{Q}=(q_1,q_2,q_3,q_4)$ are non-negative trapezoidal fuzzy numbers. Then we solve the optimal fuzzy total profit per unit time formula (4) as the following steps.

Firstly,

$$\begin{aligned}FTPU(\tilde{Q}) = (&d_1s+pv/(1-p)d_1-pd_4h_4q_4/(1-p)-cd_4/(1-p)-ad_4/(1-p)\\ &-\ d_4k_4/(q_1(1-p))-(1-p)h_4q_4/2,\\ &d_2s+pv/(1-p)d_2-pd_3h_3q_3/(1-p)-cd_3/(1-p)-ad_3/(1-p)-d_3k_3/(q_2(1-p))\\ &-(1-p)h_3q_3/2,\\ &d_3s+pv/(1-p)d_3-pd_2h_2q_2/(1-p)-cd_2/(1-p)-ad_2/(1-p)-d_2k_2/(q_3(1-p))\\ &-(1-p)h_2q_2/2,\\ &d_4s+pv/(1-p)d_4-pd_1h_1q_1/(1-p)-cd_1/(1-p)-ad_1/(1-p)-d_1k_1/(q_4(1-p))\\ &-(1-p)h_1q_1/2)\end{aligned}$$

Secondly, we defuzzify the fuzzy total profit per unit time by using graded mean integration representation method. The result is

$$\begin{aligned}P(FTPU(\tilde{Q})) = \{&s(d_1+d_4)+pv/(1-p)(d_1+d_4)-p/(1-p)(d_4h_4q_4+d_1h_1q_1)\\ &-c/(1-p)(d_1+d_4)-a/(1-p)(d_1+d_4)-1/(1-p)(d_1k_1/q_4+d_4k_4/q_1)\\ &-(1-p)/2(h_4q_4+h_1q_1)+2[s(d_2+d_3)+pv/(1-p)(d_2+d_3)\\ &-p/(1-p)(d_3h_3q_3+d_2h_2q_2)-c/(1-p)(d_2+d_3)-a/(1-p)(d_2+d_3)\\ &-1/(1-p)(d_2k_2/q_3+d_3k_3/q_2)\\ &-(1-p)/2(h_2q_2+h_3q_3)]\}/6 \qquad (5)\end{aligned}$$

where $0<q_1\leq q_2\leq q_3\leq q_4$.

It will not change the meaning of formula (5), if we replace the inequality conditions $0<q_1\leq q_2\leq q_3\leq q_4$ with the following inequality constrains:

$$q_1 - q_2 \leq 0,\ q_2 - q_3 \leq 0,\ q_3 - q_4 \leq 0 \text{ and } -q_1 < 0$$

Thirdly, let

$L(\tilde{Q}) = P\ (FTPU(\tilde{Q}))$-$\lambda_1(q_1 - q_2 + S_1^2)$-$\lambda_2(q_2 - q_3 + S_2^2)$ -$\lambda_3(q_3 - q_4 + S_3^2)$-$\lambda_4(-q_1 + S_4^2)$, then the Kuhn-Tucker condition is used to find the solution of q_1, q_2, q_3 and q_4 to maximize $P(FTPU(\tilde{Q}))$ in formula (5), subject to $q_1 - q_2 \leq 0$, $q_2 - q_3 \leq 0$, $q_3 - q_4 \leq 0$, and $-q_1 < 0$. The Kuhn-Tucker conditions are thus given as formula (3):

$$\lambda \geq 0,$$
$$\nabla f(P(FTPU(\tilde{Q}))) - \lambda \nabla g(Q) = 0,$$
$$\lambda_i g_i(Q) = 0,$$
$$G_i(Q) \leq 0,$$

These conditions simplify to the following

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0 \quad (6\text{-}1)$$
$$1/6[-p/(1-p)d_1h_1 + d_4k_4/(q_1^2(1-p)) - (1-p)/2h_1)] - \lambda_1 + \lambda_4 = 0 \quad (6\text{-}2)$$
$$1/6\{2[-p/(1-p)d_2h_2 + d_3k_3/(q_2^2(1-p)) - (1-p)/2h_2)]\} + \lambda_1 - \lambda_2 = 0 \quad (6\text{-}3)$$
$$1/6\{2[-p/(1-p)d_3h_3 + d_2k_2/(q_3^2(1-p)) - (1-p)/2h_3)]\} + \lambda_2 - \lambda_3 = 0 \quad (6\text{-}4)$$
$$1/6[-p/(1-p)d_4h_4 + d_1k_1/(q_4^2(1-p)) - (1-p)/2h_4)] + \lambda_3 = 0 \quad (6\text{-}5)$$
$$\lambda_1(q_1 - q_2) = 0 \quad (6\text{-}6)$$
$$\lambda_2(q_2 - q_3) = 0 \quad (6\text{-}7)$$
$$\lambda_3(q_3 - q_4) = 0 \quad (6\text{-}8)$$
$$\lambda_4 q_1 = 0 \quad (6\text{-}9)$$
$$q_1 - q_2 \leq 0 \quad (6\text{-}10)$$
$$q_2 - q_3 \leq 0 \quad (6\text{-}11)$$
$$q_3 - q_4 \leq 0 \quad (6\text{-}12)$$
$$-q_1 < 0 \quad (6\text{-}13)$$

Because $q_1 > 0$, and $\lambda_4 q_1 = 0$, then $\lambda_4 = 0$. If $\lambda_1 = \lambda_2 = \lambda_3 = 0$, then $q_4 < q_3 < q_2 < q_1$, it does not satisfy the constrains $0 < q_1 \leq q_2 \leq q_3 \leq q_4$. Therefore $q_2 = q_1, q_3 = q_2$ and $q_4 = q_3$, that is $q_1 = q_2 = q_3 = q_4 = Q$. Hence, from formula (6-2), (6-3), (6-4), and (6-5), we find the optimal production quantity Q^* by the above equation as

$$Q^* = \sqrt{\frac{2(d_1k_1 + 2d_2k_2 + 2d_3k_3 + d_4k_4)}{2p(d_1h_1 + 2d_2h_2 + 2d_3h_3 + d_4h_4) + (1-p)^2(h_1 + 2h_2 + 2h_3 + h_4)}} \quad (7)$$

When demand, production and costs are real numbers, that is $h_1 = h_2 = h_3 = h_4 = H, k_1 = k_2 = k_3 = k_4 = K$, and $d_1 = d_2 = d_3 = d_4 = D$, then formula (7) can be revised as

$$Q^* = \sqrt{\frac{2KD}{(2pD + (1-p)^2)H}}.$$

when $p = 0$, then $Q^* = \sqrt{\frac{2KD}{H}}$.

Remark 3.6. *When holding cost, setup cost and demand are all real values, then the model becomes a certain environment production model.*

Remark 3.7. *Moreover, when the percentage (p) of imperfect items in a production is zero, then the model become a simple production model.*

Remark 3.8. *In 2000, M.K. Salameh, and M. Y. Jaber [35] have written a paper titled "Economic production quantity model for items with imperfect quality" to discuss the* *imperfect quality under certain environment. But we extend the case into vaguer environments with some fuzzy values to meet the real world situation.*

4. **Example.** ABC manufacturing company produces commercial television units in batch. The firm estimated that the fuzzy daily storage cost ($\tilde{H}$) per unit is about NT$1, the fuzzy setup cost ($\tilde{K}$) is about NT$100,000, the unit production cost (c) is NT$5,000, the percentage of defective items in a lot (p) is about 1%, the screening rate (x) is 1unit/10mins, the screening cost (a) is NT$20/unit, the unit selling price of items of good quality (s) is NT$10,000, the unit selling price of defective items (v) is NT$3,000, the fuzzy total demand over the planning time period [0,365] ($\tilde{D}$) is greater or less than 20,000 units. How many television units should ABC manufacturing company produce in each batch?

Solving: Here we use a general rule to transfer the linguistic data, "greater or less than X'' and "about X'', into trapezoidal fuzzy numbers as

"greater or less than $X'' = (0.9X,\ 0.95X\ ,\ 1.05X\ ,\ 1.1X)$, and

"about $X'' = (0.95X,\ X, X,\ 1.05X)$.

By using the above rule, the fuzzy parameters in this example can be transferred as follows:

$$\tilde{H} = (0.95,\ 1,\ 1,\ 1.05),\ \tilde{K} = (95000,\ 100000,\ 100000,\ 105000),$$
$$\text{c} = 5000, \qquad \tilde{D} = (18000,\ 19000,\ 21000,\ 22000).$$

Replace the above fuzzy parameters values into formula (1), we find the optimal fuzzy production quantity

$$\tilde{Q} = (2234.25, 2234.25, 2234.25, 2234.25) \approx (2234, 2234, 2234, 2234).$$

Then, the minimization fuzzy total production inventory cost is

$$FTPU(\tilde{Q}) = (67423065, 82667569, 114246867, 128233884).$$

The result shows that per batch production units for ABC manufacturing company is 2234.

5. **Conclusion.** In the real world, vague phenomenon is quite common and defective products are inevitable in most production processes. It would be very interesting and reasonable to develop a fuzzy production model considering imperfect products and fuzzy costs. The proposed model is applicable when inventory continuously flows or builds up over a period of time after an order has been placed and units are produced and sold simultaneously. The result of the example applying this model is quite feasible and satisfactory. This model could be applied in more practical and sophisticated cases. For a special case that all variables are set as real numbers, the result will be the same as the traditional non-fuzzy model.

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