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FINANCIAL MARKET IMPERFECTIONS AND BUSINESS CYCLES

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ABSTRACT

This paper develops a simple model of macroeconomic behavior which incorporates the impact of financial market "imperfections," such as those generated by asymmetric information in financial markets. These information asymmetries may lead to breakdowns in markets, like that for equity, in which risks are shared. In particular, we analyze firm behavior in the presence of equity rationing and imperfect futures markets, in which there are lags in production. As a consequence, firms act in a risk-averse manner. We trace out the macroeconomic consequences, and show that they are able to account for many of the widely observed aspects of actual business cycles.

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This paper describes a simple model of macro-economic fluctuations based upon the kinds of informational imperfections -- chiefly related to adverse selection and moral hazard -- that have received substantial attention in the recent micro-economic literature.[1] A major consequence of these informational imperfections already studied extensively in connection with insurance, labor and financial markets, is that they interfere with the proper distribution of risk among economic agents.[2] In extreme cases, markets for sharing risk may break down completely.[3] As a result, individual agents must manage risks to some extent without depending on external markets and they are likely to do this in two ways that have significant macro-economic consequences. First, they can usually reduce risks by reducing their level of economic activity; reacting to unexpected "shocks" by adjusting this level of activity and, in the process, transmitting those "shocks" to other agents. Second, accumulated "net" asset balances, particularly liquid asset balances, can act as "buffer stocks" to absorb risks. Accordingly, increases in net asset balances result in increases in economic activity. Thus, fluctuations in "net" asset balances, which are likely to be "persistent", can lead to "persistent" fluctuations in the level of economic activity. In the paper, both

[1]This literature is now almost too vast to be summarised completely in a manageable number of citations. For a relatively recent survey covering the areas most closely related to this paper see Stiglitz [1985].

[2]See, for example, Arrow [1970], Wilson [1977], Rothschild-Stiglitz [1976] on insurance markets; Greenwald [1979], Grossman-Stiglitz [1976] on labor markets; Jaffee and Russell [1976], Stiglitz-Weiss [1981] and Ross [1977] on financial markets and Akerlof [1970] for a general discussion of the adverse selection issue.

[3]See, for example, Akerlof [1970].

these mechanisms are at work and they produce persistent macro-economic cycles whose characteristics bear a striking resemblance to those of observed business cycles.[4]

The model focuses on the behavior of firms[5] and the derivation of an aggregate supply function.[6] The central role of informational imperfections is to restrict a firm's ability to raise "equity" funds in external capital markets.[7] Specifically, we will assume that there is a fixed exogenous (and, considering signalling related dividend requirements, perhaps negative) flow of external equity to firms. At the same time, in order to separate the "risk" distribution issues that are at the heart of the model from traditional credit restriction questions, we will assume that there is a "perfect" loan market. The output decisions of firms are assumed to be made by "managers" who are averse to the possibility of "bankruptcy".[8] Finally, we will assume that futures markets do not exist and inputs

[4]In emphasizing problems of risk distribution this paper is in some sense a return to the traditional approach of Kalecki [1939].

[5]This is done partly for the sake of expositional convenience. Extension to the "supply" sectors of households is complicated by the intimate connection between production and consumption within households.

[6]This must, however, be sharply distinguished from a Lucas-type supply function (see Lucas [1979]) since supply fluctuations are related directly to the varying impact of "real" microeconomic market failures and are not simply dependent on fluctuations in labor supplies.

[7]For formal models of this phenomenon see Greenwald, Stiglitz and Weiss [1984], Majluf and Myers [1984] and, in a slightly different spirit, Leland and Pyle [1977]. The basic argument is that if the managers of firms, who are better informed about a firm's future prospects than equity investors at large, are willing to issue stock at the current market price, then "outside" investors ought to be unwilling to buy at that price. Hence, an equity issue announcement ought to be associated with a decline in a firm's current stock price which should in turn inhibit equity issues. This has, in fact, been observed (see Asqinth and Mullins [1983]) and is presumably related to the observation that internally generated funds are by far the predominant source of equity for most firms (see Taggart [1983]). Agency considerations reinforce these arguments (see Jensen and Meckling [1976]).

[8]The justification for this kind of assumption is again informational. When a firm becomes "financially distressed", it is usually impossible to tell whether this is due to bad luck with projects which were apriori properly undertaken or to bad management. Thus, managers will inevitably

must be paid for significantly before outputs are sold.[9] Thus, any decision to produce is inherently a risky investment decision.

The full macro-economic model into which the aggregate supply behavior is embedded concentrates primarily on the existence and nature of business cycles. Narrowly interpreted, the model does not involve either unemployment or, in any detail, the macro-economic role of monetary institutions and financial markets. However, simple and natural extensions of the model are capable of covering both these areas as well as a number of related macro-economic phenomena (e.g. price rigidities). The paper will, therefore, consist, beyond this introduction, of three parts. Section I describes the behavior of firms and aggregate supply. Section II incorporates the aggregate supply function into a complete macro-economic model and describes the dynamic behavior of that model. Finally, Section III discusses extensions of the model to cover a wide range of macro-economic phenomena including aggregate investment behavior, unemployment and price rigidities.

I. Firm Behavior and Aggregate Supply

Firms, identified by an index $i = 1,\ldots I$, will be assumed to make decisions at discrete intervals $t = 1,\ldots T$. At the beginning of each period, a firm inherits both a nominal level of debt, B_{t-1}^i, and a "real" level of output, q_{t-1}^i, from the previous period. We will assume that there is a one-period lag between the use (and

suffer a stigma associated with "financial distress".

[9]Again this assumption rests ultimately on informational failures, typically associated with product quality and terms of delivery, which inhibit the development and use of futures markets. In practice, futures markets are far from complete.

payment) of inputs and the availability of output. Thus, q^i_{t-1} results from production decisions made at the beginning of period $t-1$, but becomes available for sale only at the beginning of period t. For simplicity, we will assume also that output is perishable and q^i_{t-1} must all be sold at the beginning of period t. We will assume that the nominal debt, B^i_{t-1}, was incurred at the beginning of period $t-1$ in order to pay for the inputs that were required for producing q^i_{t-1}. Associated with this debt is a nominal contractual rate of interest R^i_{t-1} determined at that time. Thus, nominal contractual repayments owed to debtholders by firm i on entering period t are $(1 + R^i_{t-1})\, B^i_{t-1}$.

At the beginning of period t competitive goods markets for the sale of q^i_{t-1} open and clear. This determines the price P^i_t at which firm i sells its inherited output, q^i_{t-1}. The price P^i_t also determines the nominal "equity" position[10] of firm i at the beginning of period t since

$$A^i_t \equiv \text{Nominal Equity Position of firm } i \text{ at the beginning of period } t$$

$$\equiv P^i_t\, q^i_{t-1} - (1+R^i_{t-1})\, B^i_{t-1} \tag{1}$$

The level of A^i_t then determines the solvency of firm i. For some level of A^i_t sufficiently low (or negative) firm i would presumably be declared bankrupt and reorganized with appropriately negative consequences for the managers (or owners, if owner-managed) of the firm. For simplicity we will assume that $A^i_t < 0$ implies bankruptcy, although a non-zero (either positive or negative) threshold could have been used without fundamentally altering the implications of the

[10] For the moment we will ignore both equity sales and dividends.

model.[11]

Simultaneously with the clearance of the several goods markets at the beginning of period t, loan and labor markets open and clear. These markets determine w_t, the real wage[12] that firms must offer workers, and r_t, the expected real return required by lenders. The expected real return, r_t, then determines the terms on which loans will be made available to individual firms, typically a schedule[13] relating R_t^i to q_t^i and A_t^i for a given expected real return and expected rate of inflation. Combined with expectations concerning future output prices and A_t^i, these factor prices lead managers to select a level of output, q_t^i, which, once workers have been paid, leads to a level of debt, B_t^i, and a contractual nominal return, R_t^i, on that debt. Thus, this burst of simultaneous activity at the beginning of period t produces levels of q_t^i, B_t^i and R_t^i that firm i inherits at the beginning of period $t+1$, when the entire process is repeated.

Within this temporal context we will assume that

[A1] firms produce output using only labor as an input with $\ell_t^i = \phi(q_t^i)$ where ϕ is a labor requirements function[14] with $\phi' > 0$ and $\phi'' \geq 0$,

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[11]It should, however, be noted that the comparative static properties of a bankruptcy threshold below zero are both more complicated and less clearly determinate than those of a zero or positive threshold.

[12]Given the average price level determined by the individual P_t^i prices, real wage levels determine also an equilibrium nominal wage.

[13]See below for detailed discussion on this point.

[14]ϕ could, of course, easily be made to vary across firms. However doing this would merely complicate the notation with significantly altering the implications of the model. Note that ϕ^{-1} is a production function of the usual sort.

[A2] the price level, P_t^i, faced by an individual firm is determined by a sectoral random variable, $\tilde{u}_t^i$, and the overall price level, P_t, where

$$P_t^i = \tilde{u}_t^i P_t, \; E(\tilde{u}_t^i) = 1 \tag{2}$$

and $\tilde{u}_t^i$, the relative price of the output of firm i, is i.i.d. with a distribution function $F(\cdot)$, and density $f(\cdot)$,

[A3] if $A_t^i < 0$, firms go "bankrupt" and the entire proceeds from the sale of q_{t-1}^i are distributed without loss to debt-holders (i.e. there are no reorganization or liquidation costs to debt-holders).[15]

Given [A2] and [A3] lenders to firm i at the beginning of period t earn returns which are a random variable whose value is resolved only when prices are revealed at the beginning of period $t+1$. If P_{t+1}^i is high enough so that $A_{t+1}^i \geq 0$, then lenders receive a nominal return R_t^i. If P_{t+1}^i falls below the level at which $A_{t+1}^i = 0$, then lenders receive a nominal return $((P_{t+1}^i q_t^i / B_t^i) - 1)$ where, given [A1],[15a]

$$B_t^i = P_t w_t \phi(q_t^i) - A_t^i \; . \tag{3}$$

Firms go bankrupt if what they promise to pay exceeds their income; that is when

$$(1+R_t^i)B_t^i \geq P_{t+1}^i q_t^i$$

or, using (2) and (3),

[15]Introducing reorganisation costs has an impact on the results similar, but not quite identical, to the effect of a negative bankruptcy threshold. Also with reorganization costs firms will have an additional incentive (beyond the managerial penalty) to avoid bankruptcy.

[15a]If $P_t w_t \phi(q_t^i) > A_t^i$, then the firm is a net lender and the probability of bankruptcy is zero. For the remainder of the paper we will focus on the case where the reverse inequality holds.

$$\tilde{u}^i_{t+1} \leq (1+R^i_t) \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{w_t \phi(q^i_t) - a^i_t}{q^i_t}\right) \equiv \overline{u}^i_{t+1} \tag{4}$$

where

$$a^i_t \equiv \frac{A^i_t}{P_t} \equiv \text{ real equity level of firm } i \text{ at the beginning of period } t \ ,$$

and, thus,

$\overline{u}^i_{t+1} \equiv$ level of relative price in period $t+1$, $\tilde{u}^i_{t+1}$, at which firm i is just solvent.

Thus, real returns to lenders are,

$$(1+\tilde{R}^i_t) \left(\frac{P_t}{P_{t+1}}\right) = \left\{ \begin{array}{ll} (1+R^i_t) \left(\frac{P_t}{P_{t+1}}\right) & \text{if} \quad \tilde{u}^i_{t+1} \geq \overline{u}^i_{t+1} \\ \frac{\tilde{u}^i_{t+1} \cdot q^i_t}{w_t \phi(q^i_t) - a^i_t} & \text{if} \quad \tilde{u}^i_{t+1} < \overline{u}^i_{t+1} \end{array} \right\}, \tag{5}$$

Strictly speaking P_{t+1}, looking forward from the beginning of period t, is a random variable. However, in order to simplify the exposition, we will assume for the moment that there is relatively little uncertainty about future price levels (as opposed to the relative sectoral prices P^i_{t+1}) and, thus, that

$$P_{t+1} \cong P^e_{t+1} \equiv \text{Expected price level at the beginning of period } t+1 \text{ looking forward from the beginning of period } t \ . \tag{6}$$[16]

Given equation (5), the expected real return to lenders to firm i in period t is

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[16] This assumption may appear extreme and indeed will be violated in the next section of this paper. However, it can be relaxed without affecting the conclusions of the model in any fundamental way. Unfortunately, the price of such relaxation is considerable notational complexity since it requires definition of a bivariate price distribution covering both aggregate and sectoral prices; hence the use of the present assumption.

$$E\left[(1+\tilde{R}_t^i)\right]\left(\frac{P_t}{P_{t+1}^e}\right) = (1+R_t^i)\left(\frac{P_t}{P_{t+1}^e}\right)\left(1-F(\overline{u}_{t+1}^i)\right) + \frac{q_t^i}{w_t\phi(q_t^i)-a_t^i}\int_0^{\overline{u}_{t-1}^i} x\,dF(x)\,. \quad (7)$$

where P_{t+1}^e can now be substituted for P_{t+1} in the expression for $\overline{u}_{t+1}^i$. The first expression on the right-hand side of equation (7) represents the expected real return to lenders from those situations in which firm i is solvent in period $t+1$. The second expression then represents the expected real return to lenders from situations in which firm i is insolvent in period $t+1$. For determining the appropriate contractual rate of return, R_t^i, we next assume that

[A4] Lenders are perfectly informed[17] and risk neutral which implies that

$$E\left[1+\tilde{R}_t^i\right]\left(\frac{P_t}{P_{t+1}^e}\right) = 1 + r_t\ . \quad (8)$$

Equations (4) and (8) can be solved for the equilibrium level of the contractual nominal interest rate, R_t^i, and the solvency relative price, $\overline{u}_{t+1}^i$, as functions of q_t^i, a_t^i, w_t, r_t and P_t/P_{t+1}^e:

$$R_t^i = R_t^i(q_t^i,\ a_t^i,\ w_t,\ P_t/P_{t+1}^e,\ 1+r_t)\ , \quad (9a)$$

$$\overline{u}_t^i = \overline{u}_t^i(q_t^i,\ a_t^i,\ w_t,\ P_t/P_{t+1}^e,\ 1+r_t)\ . \quad (9b)$$

Then, substitution from (9b) into $F(u)$ yields

$$\text{Probability of Bankruptcy} \equiv F\left[\overline{u}_t^i(q_t^i,\ a_t^i,\ w_t,\ P/P_{t+1}^e,\ 1+r_t)\right]$$

giving the probability of bankruptcy as a function of the decision variable, q_t^i, the

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[17] Clearly for the informational imperfections that interfere with the issue of equity to exist, lenders must not be able to use their information to purchase equity. The best way to interpret [A4] is that lending is done through institutions that are legally enjoined from purchasing stock. In any event, imperfect information on the part of lenders would intensify rather than alleviate the problems embodied in the model.

state variable, a_t^i, and the parameters, w_t (wages), P_t/P_{t+1}^e (the expected change in the price level) and r_t (the real interest rate).

In deciding upon a level of output, we will assume that the objectives of a firm's managers are described by the assumption that,

[A5] firm's select q_t^i in order to maximize expected real profits (i.e. total sales minus repayment to lenders) minus an expected real cost of bankruptcy, i.e.

$$\max \left(\frac{1}{P_{t+1}^e}\right) E\left[P_{t+1}^i q_t^i - (1+\tilde{R}_t^i)\left(P_t w_t \phi(q_t^i) - A_t^i\right)\right] - c_t^i F(\bar{u}_{t+1}^i)\,. \qquad (10)$$

Equation (10) is a simple way of capturing the hypothesis that firms act to avoid bankruptcy. As we shall see, this bankruptcy avoidance behavior induces a kind of risk aversion;[17a] similar results obtain whether these bankruptcy costs are viewed as real (managerial) reorganization costs associated with bankruptcy or if we view firms as maximizing the expected utility of profits with the utility function characterized by a declining marginal utility of profits and decreasing absolute risk aversion.[17b]

We assume further that

[A6] Bankruptcy costs increase with the level of a firm's output:

$$c_t^i = c\, q_t^i\,. \qquad (11)$$

This assumption is made largely for analytic reasons; similar results hold for other bankruptcy cost functions as long as expected bankruptcy costs are convex in q_t^i.

[17a] Strictly speaking this is true only if $c_t^i F$ is appropriately convex in q_t^i. Later we will impose conditions which will ensure that this is true.

[17b] See Greenwald-Stiglitz [1987].

There are, however, three economic justifications which suggest that [A6] represents a plausible simplification. First, as firms become larger they presumably involve more managers whose loss of position, income and power in the event of insolvency is likely to increase. Bankruptcy should, therefore, be a more serious matter for General Motors than for a local grocery store. Since q_t^i is the only scale variable in the model, having bankruptcy costs increase with q_t^i is the only way to capture these scale effects. Second, a significant role of managers is choosing a level of output (in the model this is their only role). Bankruptcy with high levels of output should reflect unfavorably on their ability to do this. Since bankruptcy in the model is due to low prices, a high level of output in the face of these low prices may, retrospectively at least, imply unusually bad judgement by managers and may thus be unusually costly to their future prospects.[18] Third, having bankruptcy costs depend on q_t^i is necessary in order to ensure that the possibility of bankruptcy is never ignored. If there were a fixed cost of bankruptcy independent of the level of output, then profits, which are increasing in output, may grow so large relative to bankruptcy costs that bankruptcy becomes a negligible consideration.[19] Since the purpose of this paper is to investigate the macro-economic implications of conditions in which managers (or owners) are penalized for bad outcomes and are affected by the possibility of these penalties, assumption [A6] is a convenient way of ensuring that these conditions are met.

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[18] This seems especially likely to be true when managers suffer from degrees of financial distress short of bankruptcy.

[19] In any case, we must assume that there is an upper limit on output (or that c^i increases sufficiently rapidly) and the bankruptcy costs co-efficient c is sufficiently large that a maximum for the objective function in [A5] exists. These technical assumptions are discussed in Appendix I.

Moreover with the addition of fixed bankruptcy costs there are reasonable circumstances under which the fundamental implications of the model with [A5] continue to hold (see Appendix I).[20]

Given [A2] and [A4], the objective function of [A5] can be written as

$$\max_{q_t^i} \left[q_t^i - (1+r_t)\,(w_t \phi(q_t^i) - a_t^i) - c_t^i F(\overline{u}_{t+1}^i) \right] \tag{12}$$

Under these assumptions, a firm's real output is, therefore, determined by real wages, real interest rates, real equity holdings, and relative price uncertainty. The first order condition[20a] for an interior maximum can now be written as

$$1 - (1+r_t) w_t \phi' = \rho_t^i \tag{13}$$

where ρ_t^i is the *marginal bankruptcy risk* of firm i in period t, i.e.

$$\rho_t^i = \left(\frac{dc_t^i}{dq_t^i} \right) F + cf(\overline{u}_{t+1}^i)\, \frac{d\overline{u}_{t+1}^i}{dq_t^i} \,. \tag{14}$$

If ρ were zero, equation (13) would be the standard result that output should be increased to the point where the marginal product $(1/\phi')$ equals the wage, taking into account the fact that the wage is paid the period before the output is received (and hence in present value terms, viewed at the time of production, wage costs are $w_t(1+r_t)$). Since ρ is positive, the impact of "bankruptcy" risks are to restrict output; these risks drive a wedge between expected prices (i.e. 1) and marginal costs in the traditional sense (i.e. $(1+r_t)w_t\phi'$).

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[20] The implied restriction in [A5] to a single period horizon is a matter of expositional convenience. The multi-period maximization problem is examined in Appendix I.

[20a] There are several restrictions that have to be imposed to ensure that the second order conditions are satisfied. These are discussed in Appendix I.

More importantly, the variables on the left hand side of equation (13) — real interest rates, real wages and technology — have historically shown substantial stability over time, changing only slowly at relatively predictable rates. In contrast, the risk variables on the right hand side of equation (13) which include the financial position of firms, a_t^i, and the degree of uncertainty concerning future prices (i.e. the distribution function F) may change rapidly and unpredictably. It is these variables, many of which may be difficult to observe, which account for cyclical fluctuations in the model.

The Determinant of Marginal Bankruptcy Risk and Individual Firm Supply

The marginal bankruptcy risk, ρ_t^i, depends, of course, on the level of output. In addition, it is a function both of the level of "equity" of the firm as well as the subjective probability distribution of the random variable $\bar{u}_{t+1}^i$. We can thus represent the supply function of a firm by an equation of the form

$$q_t^i = g^i(w_t, r_t, a_t^i; v_t^i),$$

where v_t^i represents a measure of the riskiness of the distribution F. It is easy to verify that

$g_w^i < 0$: real wage increases depress supply;
$g_r^i < 0$: real interest rate increases depress supply

Our main concern, however, is with the effect of equity levels and uncertainty (risk) on production. It is possible to verify

Proposition 1. *The higher the level of equity, the lower the marginal bankruptcy* *cost (risk permium) ρ_t^i, and hence the higher the level of production.*

Proposition 2. *Increases in the degree of uncertainty result in an increase in the marginal bankruptcy costs (risk premium) and hence in a lower level of investment.*[20b]

Under the assumption that ϕ is linear, up to a capacity constraint, we can show that investment, as a function of the equity level a_t^i, appears as in Figure 1. For the range within which the constant returns assumption holds, the elasticity of supply with respect to firm equity is unity.[20c]

Accordingly,

Proposition 3. *At least near the capacity level, output is a concave function of equity levels.*

These three propositions are the heart of the analysis: they imply that, if for some reason, a firm's equity is reduced (e.g. because the prices at which the firm is able to sell its goods are lower than anticipated) then, in subsequent periods, the firm's output will be reduced.

Moreover, our analysis suggests that for highly levered economies the output multipliers associated with equity injections may be substantial. For example if in equilibrium, equity represents one third of total capital (which in this circulating capital world is slightly less than output), then with constant returns to scale a $1 increase in equity will yield $3 of increased output. Note that there are a

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[20b] The precise meaning of increases in uncertainty and the circumstances under which Proposition 2 is valid are discussed in Appendix I.

[20c] More generally, with diminishing returns, the elasticity of supply is less than or equal to unity.

variety of ways that such equity injections may occur; unanticipated increases in the rate of inflation (monetary policy) as well as certain pump priming activities can result in substantial increases in the equity base of firms.

Later, we shall show the not surprising result that losses in equity will not instantaneously be restored, and thus the model has the immediate implication of *persistence;* a loss of equity at time t results in lower output, not only at time t, but in subsequent periods as well.

The fact that the investment function is concave means that redistributions of wealth within the production sector may have deleterious consequences for production. Thus unanticipated increases in prices (say of oil) may have negative effects, and, at the same time, unanticipated decreases in prices of the same commodity may have negative effects. Propositions 2 and 3 together imply that increased uncertainty -- both ex ante (anticipated) and ex post -- depress production. This will be true whether the uncertainty is due to concerns about real shocks or to concerns with the instabilities of macro-economic policy.

Aggregate Supply. An aggregate supply function can be derived straightforwardly by summing the supply functions of individual firms. For simplicity, we shall assume that all firms have the same production functions (ϕ) and face the same uncertainty (F). We can then write aggregate output as

$$q_t = \hat{g}(w_t, r_t, a_t^1, \ldots\ldots\ldots; v)$$

We can approximate the expression by taking a Taylor series expansion around the average level of firm equity holdings (under our symmetry assumptions), giving us an aggregate supply function of the form

$$q_t = g(w_t, r_t, a_t; v, \sigma)$$

where σ^2 is the variance of firm equity levels. The comparative static properties of this aggregate supply function will, in general, mirror those of a representative firm's output (with the additional effect noted that an increase in the dispersion of equity ownership will generally lower output).

Since, in this model, output is restricted by failure in the market for sharing the risk of bankruptcy, it is plausible to think of higher output as unambiguously implying an improvement in social welfare.

II. Macro-economic Model and Dynamic Behavior

In order to embed the supply function of the previous section in a model which is as simple as possible, we will assume that consumer behavior can be described by the behavior of a single, infinitely-lived representative consumer. Furthermore, we will assume that this representative consumer may borrow and lend freely at the competitive real rate of interest, r_t, and consequently faces a single lifetime budget constraint of the form

$$\sum_{j=0}^{\infty} (z_{t+j} - w_{t+j}\ell_{t+j})\,\pi_{t,j} = n_t \tag{15}$$

where

$z_{t+j} \equiv$ real consumption in period $t+j$,

$\ell_{t+j} \equiv$ hours worked in period $t+j$,

$\pi_{t,j} \equiv \prod_{i=0}^{j} \left(\frac{1}{1+r_i}\right)$ (and 1 for $j = 0$)

and

$n_t \equiv$ real wealth in period t .

Finally, we will assume that the representative consumer has a utility function of the form

$$\sum_{j=0}^{\infty} \left(\frac{1}{1+\delta}\right)^j \left(z_{t+j} - \nu(\ell_{t+j})\right) \tag{16}$$

where $\nu' > 0$ and $\nu'' > 0$.

Under these circumstances, equilibrium in the aggregate market for goods and services is characterized by the conditions[21]

$$r_t = \delta \tag{17}$$

and consumption equals output,

$$z_t = q_t \ . \tag{18}$$

In addition, the supply of labor is an increasing function only of the wage in the current period, w_t. The demand for labor is strictly speaking the sum of the demands of the individual firms. However, in order to simplify the notation we will write this as $\phi(q_t)$ where q_t is aggregate output and ϕ now represents an aggregate labor requirements function. The real wage is then determined by an equilibrium in the labor market of the form

$$\phi(q_t) = s(w_t),\ s' > 0 \ . \tag{19}$$

This can, in turn, be solved to yield real wages as a function of aggregate output

[21] That is, the utility function (16) ensures that since the individual is willing to trade off a dollar of consumption at time $t+1$ for $1+\delta$ at t, regardless of the levels of consumption of goods or leisure, the market rate of interest must be δ.

of the form

$$w_t = \Psi(q_t) \tag{20}$$

where $\Psi' = (\phi'/s') > 0$. Finally, substitution from the labor and capital market equilibria into the aggregate supply function yields a relationship of the form

$$q_t = g(\Psi(q_t), \delta, a_t) \tag{21}$$

which can be solved to yield

$$q_t = H(a_t) \tag{22}$$

where $H' = (g_a/(1 - g_w \Psi')) = (g_a s'/(s' - g_w \phi')) > 0$. Thus, *in each period output is determined by the level of equity and movements in output over time will be driven by movements in the level of equity.*

Equity in period $t+1$ consists of equity in period t plus earnings on that equity plus new equity sales less dividends paid. In nominal terms,

$$\tilde{A}^i_{t+1} = \tilde{P}^i_{t+1}\, q^i_t - (1 + \tilde{R}^i_t)\left(P_t w_t \phi(q^i_t) - A^i_t\right) - \tilde{M}^i_{t+1}$$

where $\tilde{M}^i_{t+1}$ is a nominal random variable representing the nominal value of dividends paid less new equity issued. Summation over firms and the taking of expected values yields

$$E[A_{t+1}] = P_{t+1}\, q_t - \left(P^e_{t+1}\right) E\left[\frac{(1+\tilde{R}^i_t)P_t}{P^e_{t+1}}\right](w_t \phi(q_t) - a_t) - M_{t+1}$$

$$= P_{t+1} q_t - \left(P^e_{t+1}\right)(1 + \delta)\,(w_t \phi(q_t) - a_t) - M_{t+1}$$

where unsuperscripted variables now denote aggregate quantities. Division by P_{t+1} to convert to real terms yields an equation for real equity levels in period $t+1$ of the form

$$a_{t+1} = q_t - \left(\frac{P^e_{t+1}}{P_{t+1}}\right)(1+\delta)\ (w_t\phi(q_t) - a_t) - m_{t+1} \tag{23}$$

where m_{t+1} now denotes the real value of dividends less equity sales which we will assume is determined exogenously.[22] Equations (22) and (23) together with whatever determines "price shocks" (i.e. the variable P^e_{t+1}/P_{t+1}) now completely determines the dynamic behavior of output in the model.

In order that this dynamic behavior be at least minimally interesting the level of net equity outflows (i.e. m_{t+1}) must be sufficiently large -- especially at high levels of a_t -- so that the real equity level in the economy does not simply increase without bound. When a_{t+1} is plotted as a function of a_t (see figure 2), the curve must at some point fall below and stay below the forty-five degree line (it must also at some point obviously lie above that line). There are several ways of assuring that this occurs. One possibility is to assume that growth (in either the labor force or due to labor augmenting technical progress) occurs at a constant proportional rate. Then, when a_{t+1} and a_t are redefined in per capita terms the need to equip new workers with the existing level of equity per capita will tend to drive per capita equity levels toward a steady-state level. The difficulty

[22] Similar adverse selection and signalling arguments that have been used to argue that firms will not, in general, have recourse to equity markets have also been used to explain why dividend levels change so infrequently. For the short run analysis on which this paper, we can accordingly take these as fixed. More generally, dividend levels can be related to the state variables of the system, in particular, to a_t.

with this approach is that it requires an appropriate redefinition of the production relationship to accommodate growth which is not straightforward. An alternative, therefore, is to assume that m_{t+1} is an increasing function of a_t and, without going into detail, that this function increases sufficiently rapidly so that a steady-state equilibrium exists. Since equity scarcity is central to the kinds of equity (dividend) signalling models that occur in the micro-economic literature and underlie the model of this paper, the latter assumption is the one we will make.[23] Formally, therefore, we will assume

$$a_{t+1} = q_t - \left(\frac{P^e_{t+1}}{P_{t+1}}\right)(1+\delta)\,(w_t\phi(q_t) - a_t) - m(a_t) \tag{24}$$

cuts the forty-five degree line from above as shown in figure 2.[23a]

Cycles

Cycles (in the sense of persistent fluctuations) may occur in this model for two reasons. First, even with $P_{t+1} = P^e_{t+1}$ in every period, deterministic cycles of multiple periodicity may occur if the slope of the curve

$$a_{t+1} = q_t - (1+\delta)\,(w_t\phi(q_t) - a_t) - m(a_t) \equiv G(a_t) \tag{25}$$

is sufficiently highly negative when it crosses the forty-five degree line (see figure

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[23]The argument here is that when firms set dividend policy in order to distinguish themselves from "weaker" firms they will do so in a way that tends to generate equity scarcity among the population of firms as a whole. Otherwise dividend policy would ultimately not be an effective signalling device.

[23a] If we treat P_{t+1} as a random variable, equation (24), together with equation (22), defines a stochastic difference equation. The limiting properties of the stochastic process are studied elsewhere. Here, we simply note that along each sample path, the property of persistence, to be described in the next subsection, will be observed.

2).[23b] Since

$$G' = (1+\delta) - m' - \Big((1+\delta)\,(\Psi'\phi + \phi'w) - 1\Big)H'$$

and $(1+\delta)\phi'w < 1$ (from the first order condition of firms), this requires that m' be large and that the impact of increased output on wages (i.e. $\Psi'\phi$) also be large. However, if these conditions are met the resulting "real" cycles bear at least a casual resemblance to the "wage-shock" models which have been discussed, at least informally, in the empirical literature. Prosperity in the form of rising output and firm equity levels leads to both rising wages, which reduces profits and internal funds flows, and rising dividend requirements. These in turn ultimately reduce equity levels and output, which both restores profitability (as wages fall) and reduces dividend requirements, causing the cycle to begin again.

If G' is always greater than zero, then no such cycles are possible and convergence to the steady-state is monotone. For future reference we will denote this steady-state level of equity by a^* where

$$a^* = G(a^*)\,. \tag{26}$$

Persistence

Second, random price shocks, which lead to unexpected fluctuations in the real value of debt obligations and hence in real equity level, will lead to output fluctuations which persist over several periods. Consider, for example, an unexpectedly low level of P_{t+1} (i.e. $P_{t+1} < P^e_{t+1}$). From equation (24), this will lead to an immediate and substantial drop in equity levels away from the steady-state

[23b] See Grandmont [1985] for a discussion of these cycles in a slightly different context.

level, a^*, (assuming that the economy started at a^*) with an associated drop in output. The economy will return to a^* (and the associated "full-employment" level output) only slowly as a result of successive positive increments to a_t. Moreover, these increments (and the associated increments in wages) will be fully anticipated. They may not, however, be arbitraged away nor the process short-cut because doing so would involve levels of economic activity that require firms to bear unacceptably high levels of risk. Firms will do this only as they acquire equity funds which, for informational (risk) reasons, they are assumed to do only slowly over time.

There are innumerable possible ways to model the sources of these price shocks. The simplest is to assume that output is sold on a large international market and international prices vary in response to forces which are external to the economy in question. A more traditional source of such "shocks" would be a monetary sector which determines the aggregate price level. From this perspective, an unexpectedly low level of P_{t+1} might be associated with either an unexpectedly low level of money supply or, for some money demand specifications, with an unexpectedly low level of consumption demand. Explorations of these phenomena are contained in Greenwald-Stiglitz [1986], but they add relatively little (at the cost of some complexity) to understanding of the basic characteristics of the model in question.

In all cases, as in many monetary models with real effects, the source of the real effect is a redistribution of assets as a result of the "price shock". In most of these models (see, for example, Grossman [1985]), the redistribution involved is a redistribution of wealth among households and it is difficult to believe that such a redistribution would have significant macro-economic consequences. In contrast, the redistribution that occurs in the model of this paper is between the equity and debt of firms. It is for practical purposes a redistribution between managers and/or owners who make production decisions and passive investors and/or lenders. Given the important information asymmetries, this should more readily be expected to have a significant impact on output. In the model, the effects of any associated redistributions of wealth among consumers are, given the underlying assumptions on consumer behavior, nonexistent.

A second variety of shock that might be expected to have persistent consequences is what might be referred to as an "uncertainty shock". An increase in the perceived uncertainty of future relative prices will, in general, lead to a reduction in the level of output (i.e. a downward shift in the function $H(a_t)$). If the increase in uncertainty is permanent, then the drop in output will be permanent. However, if

$$1 - (1+\delta)w_t\phi' - (1+\delta)\,\Psi'\phi < 0\,,$$

then the drop in output is associated with a simultaneous upward shift in the function $G(a_t)$; assuming no change in the net dividend function $m(a_t)$, lower output raises profitability which increases the flow of new equity. As $G(a_t)$ shifts upward, a^* increases and there is a gradual adjustment to the new higher steady-state level of equity. This is accompanied by a slow, steady, but perhaps incomplete recovery from the initial drop in output. The pattern is again one of slow, persistent, fully-anticipated recovery.

III. Applications and Extension

It should be stressed that the formal model presented above is a model of persistent cycles only in the sense that recovery from the trough of a recession follows an extended path involving a predictable sequence of positive increments to output. The model does not describe the kind of slow descent into that trough which also appears to be characteristic of actual business cycles. Nor, although it does involve fluctuations in real output, does the model explicitly involve unemployment. However, as noted in the introduction to this paper, simple extensions of the model appear able to account for these and other widely noted cyclical phenomena. These extensions are described in this section of the paper and include the following.

Fixed Investment Behavior

One widely observed characteristic of business cycles is that fluctuations are disproportionately severe in the investment goods sectors -- business fixed investment and construction, residential construction, consumer durables -- of a typical industrial economy. Peak-to-trough variations in activity in these sectors are greater than variations in the economy as a whole. This presents a puzzle for traditional models since, in theory at least, investment projects should be less intensely subject to the pressures of cyclical variations than shorter term undertakings. If firms are risk neutral and on average forecast future output accurately (including a future recovery from the current recession), then even firms facing constraints on current demand should undertake some countercyclical investment (for plausible values of the relevant cost curves). The costs of installation and bringing plants on line should be lower in slack than tight periods; long lead times for many investments argue against basing projections of demand at the time of plant completion on current capacity levels and relatively small reductions in investment goods prices should be sufficient to shift timing decisions. Thus, the fact that investment is so strongly pro-cyclical suggests that more is involved than simply a passive response to fluctuations in aggregate demand. The paradoxical nature of these investment fluctuations is compounded by the observation that the fluctuations often appear largest in those sectors where traditional arguments for fixed prices (and wages) appear to be weakest, e.g. home construction.

There are at least two ways in which a simple extension of the model of this paper can account for these phenomena. First, investment goods sectors may for informational reasons (i.e. they produce highly complex goods with high private information content) have particularly restricted access to equity markets. At the same time technology in these industries may be highly capital intensive and firms in these industries operate with high degrees of leverage. Demand disturbances which reduce the equity bases of firms in these sectors will, thus, produce particularly large output responses. Residential construction appears to be a prominent case in point.

Second, if fixed investment is thought of as current expenditures which yield output several periods in the future, the relative price uncertainty surrounding these expenditures is likely to be far greater than that associated with expenditures on next period's output. As depletion of a firm's equity base requires a reduction in the risk that management is willing to bear, this reduction should fall disproportionately on the firm's riskiest activities.[24] Hence, the model should predict disproportionate cyclical variations in the demand for investment goods regardless of supply conditions in the sectors in question.

Inventory Fluctuations and Interfirm Interactions

Traditional theory has found it particularly difficult to account for the cyclical pattern of inventory fluctuations. (See Blinder [1986].) While firms typically accumulate excess inventories in the early part of a recession, they typically not only reduce inventories in later parts of recessions, but even reduce inventory-sales ratios. With traditional production models, inventories should serve as buffers, allowing firms to engage in production smoothing. Given the low levels of real interest rates, and their relative invariance over time, even if real wages (and other factor costs) did not vary much over the business cycle, one would expect inventories to move countercyclically; a fortiori, if shadow wages (reflecting labor hoarding) are lower in recessions, then inventory accumulation should be greater.

Although our model, as it stands, does not directly incorporate inventories, a simple extension of the model offers some insight into the problem. Inventory accumulation early in the business cycle may simply reflect the desire of firms to take advantage of the lower marginal costs accompanying a reduction in demand. They do this until their asset position (achieved by converting financial into real assets in the process of inventory accumulation) so increases their potential

[24] Formally, the magnitude of the response to changes in equity levels is an increasing function of the ratio α/β where α and β are defined in the Appendix. If firms are operating with bankruptcy points in the lower tail of the relative price distribution, then an increase in uncertainty (i.e. the spread of the F distribution) should increase α/β. Thus, firms which are subject to high relative price uncertainty -- e.g. firms with high levels of investment and hence large investments in far future outputs -- should react most strongly to reductions in their equity bases. However, the foregoing arguments apply to the case of decreasing returns-to-scale. With constant returns-to-scale all firms reduce all activities proportionately.

exposure to financial distress that the process stops and, in the face of continuing short falls in demand, is reversed. This may be augmented by the effect of transfers of risk as output shifts (or does not shift) to other firms in a vertical chain. For example, the initial inventory accumulations in recession environments may be due to the slow speed of information transmission and/or interfirm interactions along a vertical supply chain.

Unemployment

Unemployment can be incorporated in the model in several possible ways. Depletion of the equity bases of firms effectively reduces the "net" marginal product of labor. This "net" product is the marginal physical product of labor less the risk that firms must assume in the process of hiring and production. As this risk rises, the "net" product of labor falls. In the model this appears as a decline in wages and output (which increases the marginal physical product of labor along the marginal product curve -- see figure 3). However, in a number of settings this could easily involve a degree of unemployment. A sharp rise in the "risk" cost of incremental output could well reduce the "net" product of workers below the disutility of the labor involved. Thus, lay-offs would be an appropriate response to the loss of firm equity. Ideally, of course, workers would continue to produce and firms would accumulate inventory for future sale. However, with a depleted equity base the risk for firms of doing this is prohibitive. In theory, workers could circumvent this difficulty by accepting deferred and/or contingent wage payments (contingent on the realized price of the output when sold). But there appear to be two arguments against this. First, the commitments of the firm to pay higher wage in the future may not be credible. Second, the individual in this position is, in effect, a supplier of a kind of equity capital: there is no fixed commitment to repay, even if the firm has the best of intentions. Thus, the usual moral hazard and adverse selection difficulties arise: it is precisely those firms who are in the worst financial position (i.e., Eastern Airlines) who will be most anxious to borrow from their employees, and offer seemingly the most attractive terms.

Moreover, where workers are already unemployed and searching for jobs, the existence of significant hiring and training costs may prevent the striking of mutually acceptable wage bargains with potential employers. In making a wage offer (presumably involving an extended period of employment), a firm will count these costs highly since the associated outlays will carry a high "risk" premium at a time when the firm's equity base is severely depleted. In order to recoup this "risk" cost, the firm will require a substantial saving in future wages in order to justify countercyclical hiring. From the perspective of the worker, these wage sacrifices may not be justified. Unless his financial position is so impaired so that immediate employment is a necessity, it may well pay the worker to attempt to outwait the recession. He knows that as the financial positions of firms improve, the effective cost to a firm of hiring and training will decline and wage offers will improve accordingly. Thus, the model provides a "waiting" motive for unemployment; in a recession a worker may rationally wait for an improvement in economic conditions (until his financial resources are depleted and he must accept a job).[25] The worker could, of course, himself offer to bear hiring and training costs but the same difficulties arise in this case as in the case of wage deferral considered above. Indeed, since the unemployed worker is, if anything, less familiar with the firm in question the difficulties should be intensified.

Finally, in the context of efficiency wage models, the decline in "net" worker productivity may appear as increased unemployment rather than lower wages.

Price Rigidities

Price rigidities may arise endogenously in the model whenever firms are imperfect competitors. Assume that these firms face demand curves that are inelastic in the short run but are characterized by sufficiently high long-run elasticities to restrain prices.[26] Thus, raising prices increases current profits, but as customers are induced to search to find alternative suppliers, future profits are reduced. The restraint of this long run elasticity depends on the rate at which future profits are discounted. As a firm's current equity base is depleted, the value of these uncertain future profits will fall relative to the immediate returns available from raising current prices. As a result, prices may increase despite low levels of current demand. And as all firms act in this way, the upward pressure on prices may be reinforced. In the context of empirical observation, this may well look like downward price rigidity.

[25]The implied pattern of increased probability of job acceptance as a worker's financial resources are depleted by a long spell of unemployment appears to be characteristic of observed job search behavior.

[26]See Phelps and Winter [1970] for a model of this kind of firm.

It should finally be noted that all these phenomena may be intensified by expectations cycles[27] which may be added within the framework of the basic model. The interaction of the basic mechanisms of the model with these expectations cycles would then produce something that closely resembles Keynesian descriptions of business cycles (although not perhaps Keynesian models).

Concluding Comments

This paper has explored the consequences of three simple, but we believe plausible, characteristics of the economy: (a) firms act as if they are equity rationed; (b) firms act as if they are averse to risk, to increasing the likelihood of bankruptcy; and (c) there are imperfect futures markets, production takes time, and inputs have to be paid before goods are sold. Accordingly, every production decision is a risk decision.

Under these circumstances, the level and distribution of equity among firms has real macro-economic implications. The simple model we presented can generate cyclical behavior and can explain why a variety of shocks to the economy (such as price shocks) have persistent effects. Moreover, the theory provides a set of explanations for a variety of phenomena which, at best, are difficult to explain within the traditional neoclassical paradigm. Most notably, while in the traditional model, investment, particularly in inventories, should have a smoothing effect, dampening fluctuations in demand, the variability of investment seems to contribute to these fluctuations. Such fluctuations could only be consistent with

[27]See Woodford [1986].

plausible assumptions concerning technology (adjustment costs, etc.) if the cyclical movements were accompanied by greater variations in real interest rates and pro-cyclical movements in (shadow) real wages than is in fact observed.[28] While Keynes was willing to let Animal Spirits serve as the Deus ex Machina to retrieve an explanation of investment variability, our theory provides a more plausible explanation of the variability in investment, or at least one which is more consistent with currently fashionable strictures on hypotheses concerning expectation formations.

We have noted too that traditional theory both failed to provide a rationale for price (and wage) rigidity, and while ascribing considerable importance to these rigidities, failed to explain why output fluctuations were greater in the seemingly flexible price sectors. Our theory provides a rationale for price rigidities and an explanation for why sectors with flexible prices may suffer greater output variability.

Our model provides an explanation of another long standing conundrum in economics. Keynes argued that firms should be on their supply functions, and that accordingly, when output was reduced real wages should increase. Soon after the publication of the *General Theory* Dunlop and Tarshis argued that this did not in fact happen. While there is some controversy over the exact numbers, it is clear that real wages certainly do not increase to the extent that standard production function models would have predicted.[29] The resolution provided by

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[28] In fact, with the exception of isolated episodes, real interest rates have not varied much over the past hundred years. Keynes' focuses on the real interest rate as the critical variable determining investment seems, at best, injudicious, as the subsequent criticism of Andrews indicated.

much of the recent literature (Solow-Sitglitz, Barro-Grossman, Benassy, Malinvalud, etc.), that firms are demand constrained (just as in the labor market, workers are demand constrained), is not totally plausible, even to the original proponents of these models: why, in a competitive environment, should firms that are able to solve the complex production problems which these models postulate that they can, not be able to discover that by lowering their prices, they could steal customers away from their rivals and hence increase their profits?

The explanation provided by our model is that firms may, indeed, be on their supply function; but that the supply function has shifted to the left (see figure 3), not because of the disappearance of capital, but because of increases in uncertainty and the dispersion of equity holdings.[30]

Macro-economic phenomena -- unemployment, the variability in output and investment, the lack of variability in wages and prices -- are complex. No single model, or even simple set of explanations, is likely to inform us concerning all of its important aspects.

In this paper, we have argued that understanding capital markets -- the constraints that imperfect information imposes on the ability of individuals to divest themselves of risk -- is essential to understanding certain aspects of macroeconomic behavior. Other informational problems and the constraints to which they give rise -- including credit rationing -- are, we would argue, essential to

[29] Notice that implicit contract theory has been concerned with explaining why real wages do not fall in a recession; our concern here is why they do not rise.

[30] This is not to say that we would contend that there might not be short periods, during which the economy was off its supply function. Firms may lower their prices only gradually, and if wages fall, "real" prices may fall even more gradually. During such times, the demand constrained equilibrium is relevant.

understanding other aspects of these phenomena.

Appendix I

Firm Supply Behavior

Suppressing the time and firm subscripts for the sake of expositional convenience, a representative firm's optimization problem is to

$$\max_q \left[q - (1+r)(w\phi - a) - cqF(\bar{u}) \right] \tag{A-1}$$

Also we can rewrite equations (7) and (8) giving the nominal contractual rate of interest charged the representative firm as

$$h \equiv (1+r) \left(\frac{w\phi(q)-a}{q} \right) = \bar{u}(1-F(\bar{u})) + \int_0^{\bar{u}} x dF(x) = z(\bar{u}). \tag{A-2}$$

In examining this decision problem it will be useful to look first at the constant-returns-to-scale case, in which with a suitable choice of units

$$\phi(q) = q \; . \tag{A-3}$$

Given (A-3), the decision problem of the representative firm can be rewritten

$$\max_q \left(a(1+r) + q[1 - (1+r)w - cF(\bar{u})] \right) \tag{A-4}$$

subject to (A-2) where $h = (1+r)(w - (a/q))$. The first order condition can thus be written as

$$1 - (1+r)w = c \left[F + \frac{(1+r)(a/q)f}{1-F} \right] \equiv \rho(\bar{u}(q)) \tag{A-5}$$

where we have made use of the fact that, from (A-2),

$$z' = 1 - F \tag{A-6}$$

and

$$\frac{d\bar{u}}{dq} = (1/z')\,\frac{dh}{dq} = -(1/z')(h - (1+r)w\phi')/q \qquad \text{(A-7)}$$

or, in the case under consideration where $\phi = q$,

$$d\bar{u}/dq = \frac{(1+r)a}{q^2(1-F)} \; . \qquad \text{(A-7A)}$$

Equation (A-5) can be rewritten as

$$q = ac(1+r)f\,/[(1-F)\,(m - cF)] \qquad \text{(A-8)}$$

where $m \equiv 1 - (1+r)w$. The RHS of (A-8) is just a function of $\bar{u}$, which, from (A-7), is just an (increasing) function of q.

Solving for the Equilibrium Level of Output

(a) We first show that, under a fairly weak condition to be given below, there exists a bounded solution to q. To derive this condition, we first need to observe that as q increases toward infinity, h tends toward $(1+r)w$ and $\bar{u}$ approaches a unique finite limit $\bar{u}_o$ which solves the equation

$$(1+r)w = \bar{u}_o(1 - F(\bar{u}_o)) + \int_0^{\bar{u}_o} x\,dF(x)\,. \qquad \text{(A-9)}$$

The last step in this argument follows from the facts that in any equilibrium with positive output $(1+r)w < 1$ and that the right hand side of (A-9) increases continuously and monotonically to a limit $E(x) = 1$ as $\bar{u}_o$ goes toward infinity. Thus, as q goes toward infinity, the probability of bankruptcy $F(\bar{u})$ approaches a finite limit $F(\bar{u}_o) \equiv F_o$. In order that a maximum to the firm's decision problem exist, it must then be the case that, at the equilibrium level of real wages and interest rates,

$$1-(1+r)w - cF_{o} < 0\ . \tag{A-10}$$

Otherwise the firm's objective function, (A-4), can be increased without bound. Thus, we will assume that c is sufficiently large that (A-10) holds and consequently that there is a finite optimal level of output.

(b) For $a > 0$, optimal output is positive (since $a > 0$ and $1 > (1+r)w$ imply positive profits with no risk of bankruptcy for small positive q); while for $a = 0$, optimal $q = 0$ since, under those circumstances, $h = (1+r)w$ and $F(\overline{u}) = F_o$ for all q. Thus, if F is sufficiently smooth, the firm's objective function is locally continuous and twice differentiable at the optimal level of output.[31]

Second Order Condition

With a constant-returns-to-scale technology, the second order condition takes the form

$$-\frac{c(1+r)^2a^2}{(1-F)^2q^3}\left[f' + \frac{f^2}{1-F}\right] < 0 \tag{A-11}$$

where f' is the first derivative of the density function f evaluated at the optimal bankruptcy point. At the optimal level of output, therefore,

$$f' + \frac{f^2}{1-F} > 0\ . \tag{A-12}$$

Note that since, in practice, bankruptcies appear to be relatively rare for

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[31] We will ignore the possibility that two locally separate values of q produce the same optimal value of the firm's objective function.

moderate and large-sized firms (i.e. they occur with probability less than one half in any decision period), firms operate with bankruptcy levels in the lower tail of the price distribution; if that distribution is single peaked, f' will be positive at relevant levels of output. This, in turn, means that (A-12) is satisfied.

Note, too, that if the distribution F is characterized by an increasing hazard function (i.e. $f/1-F$ is monotonically increasing), then (A-11) is satisfied globally.

Graphical Solution and Comparative Statics

With constant returns to scale, the marginal return to production, ignoring bankruptcy costs, is fixed (at what we have called m), while the marginal bankruptcy cost, ρ, increases with q. At any maximum, the $\rho(q)$ curve cuts m from below; the discussion in the preceding paragraph argued that normally there will be only one relevant intersection, and provided a global condition for a unique intersection (see figure 4).

The simplicity of the structure of the first order conditions makes comparative statics analysis relatively easy.

First, note that since, from (A-2), $\bar{u}$, the bankruptcy relative price, is a function only of a/q, ρ is a function only of a/q. Hence, an increase in a accompanied by an equiproportionate increase in q leaves ρ unchanged: With constant returns, $d\ln q/d\ln a = 1$.

Secondly, an increase in w reduces m (the marginal return from production, ignoring bankruptcy costs), while from (A-2), at any q, $\bar{u}$ increases; so long as the second order condition is satisfied, this implies that ρ, the marginal bankruptcy cost, is increased.[32] Thus, as figure 4 illustrates, output is unambiguously reduced.

Thirdly, an increase in r reduces m, while, from (A-2), at any q, $\bar{u}$ increases; again, so long as the second order condition is satisfied, this implies that ρ, the marginal bankruptcy costs, is increased, and output is reduced.

Finally, we examine the effect of an increase in uncertainty. This shifts the values of f and F corresponding to any set of values of the other parameters. Straightforward differentiation shows that, at the optimum,

$$d\rho/dF = c + (m - cF)/(1-F) > 0 .$$

At the same time

$$d\rho/df = (m - cF)/f > 0 .$$

We focus our attention on the situation where bankruptcy probabilities (in any period) are low. If an increase in uncertainty increases the likelihood of bad events (F and f both increase), then ρ will increase, and output will be reduced. Even if F increases, but f decreases uncertainty will increase ρ (reduce output) so long as the hazard rate is increased.[33]

The General Case

The first and second order conditions for the general case ($\phi'' < 0$) as well as the comparative static properties can easily be derived. We simply present the relevant formulae.

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32 $d\rho/dw = cf\ d\bar{u}/dw + (d\rho/dq)q^2/a$ where $d\bar{u}/dw = (1+r)/1-F$ and $(d\rho/dq) > 0$ by the second order condition.

33 Denote the change in the distribution by dy. Then we can write the total effect on ρ as $d\rho/dy = cdF/dy + (m-cF)d\ln\{f/1-F\}/dy$. Hence, so long as the cumulative of the bad states is increased and the hazard function is increased, ρ is increased.

(a) First order condition

$$1-(1+r)w\ \phi' = cF + \frac{cf(1+r)}{q(1-F)}\left[a + w(\phi' q - \phi)\right].$$

Note that $\phi' q - \phi > 0$ since $\phi'' > 0$ and $\phi(0) = 0$.

(b) The second order condition for an interior maximum is,

$$-(1+r)\ w\phi''\beta - \alpha \leq 0 \qquad \text{(A-13)}$$

where

$$\beta \equiv 1 + \frac{cf}{1-F} > 0,$$

$$\alpha \equiv \frac{cq}{(1-F)^2}\left(\frac{dh}{dq}\right)^2\left(f' + \frac{f^2}{1-F}\right),$$

$$\frac{dh}{dq} = \frac{(1+r)w}{q^2}\left[\phi' q - \phi + a/w\right]$$

and the various distribution and density functions (i.e. F, f and f', the derivative of the density function) are evaluated at $\bar{u}$. A sufficient condition for this is that (A-12) holds.

The comparative statics follow along similar lines to those presented before. If we define $m(q)$ as the marginal return to production, ignoring bankruptcy costs, now $m(q)$ is a decreasing function of q. Moreover, now, ρ is a function of w not only indirectly, through the effect of w on $\bar{u}$, but directly. Nonetheless, the basic qualitative properties remain.

First, for a change in real equity,

$$\frac{dq}{da} = \left(\frac{q}{a}\right)\left(\frac{\alpha\zeta}{\alpha + (1+r)w\phi''\beta}\right) \geq 0 \qquad \text{(A-14)}$$

where

$$\varsigma \equiv \left(\frac{1}{1+(w/a)(\sigma' q-\sigma)}\right).$$

With decreasing returns $(a/q)(dq/da) < 1$ and proportional increases in equity lead to less than proportional increases in output. In particular, note that if marginal labor requirements increase greatly beyond some point, $\sigma' q-\sigma$ becomes large and hence ς becomes small. Moreover q/a is becoming smaller and smaller. Hence, dq/da eventually becomes quite small.[34]

For changes in wages,

$$\frac{dq}{dw} = -\left(\frac{q}{w}\right)\left[\frac{\alpha\gamma + (1+r)w(\sigma'/q)\beta}{\alpha + (1+r)w\sigma''\beta}\right] \leq 0 \tag{A-15}$$

where

$$\gamma \equiv \left(\frac{w\sigma}{a}\right)\varsigma \equiv w\,\frac{\sigma}{a+w(\sigma' q-\sigma)}.$$

If $\sigma'' = 0$, then $(w/q)(dq/dw) < -1$, since $w\sigma > a$. On the other hand, if marginal costs increase very rapidly as a firm approaches its "capacity" limit (i.e. $\sigma'' >> \sigma'/q$, $\sigma' q >> \sigma$), then near "capacity" $(w/q)(dq/dw) > -1$.

For changes in real interest rates,

$$\frac{dq}{dr} = -\left(\frac{q}{1+r}\right)\left[\frac{\alpha\eta + (1+r)w(\sigma'/q)\beta}{\alpha + (1+r)w\sigma''\beta}\right] \leq 0 \tag{A-16}$$

where

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34 On the other hand, one cannot ensure that q is everywhere a concave function of a. The second derivative of q with respect to a involves, among other things, terms in σ''', about which we have so far made no assumptions.

$$\eta \equiv \left(\frac{w \phi}{a} - 1 \right) \varsigma = \gamma - \varsigma \,.$$

Thus $(w/q)(dq/dw) \leq ((1+r)/q)(dq/dr)$ and wages have a greater proportional negative impact on real output than interest rates.

The analysis of the effects of changes in uncertainty parallels that of our earlier discussion. Inspection of the first order condition shows that so long as the uncertainty increases the cumulative probability of bad states (F) and the hazard rate $f/1-F$, the marginal bankruptcy cost is increased and output is reduced.

More General Bankruptcy Cost Functions

An obvious extension of the form of bankruptcy costs is to add a fixed component to the bankruptcy cost function so that it becomes

$$C(q) = c_o + c_1 q \tag{A-17}$$

The comparative static results become, in general, more ambiguous. However, there are reasonable circumstances in which those results continue to hold.

For the case of constant-returns-to-scale in production, as above, a necessary condition for the existence of an optimal level of output is that

$$1 - (1+r)w - c_1 F_o < 0 \tag{A-18}$$

We will assume that (A-18) holds (and in particular that $c_1 > 0$). With the now modified bankruptcy cost function, the second order condition takes the form

$$-\alpha + \frac{2c_o f}{q(1-F)} \left(\frac{dh}{dq} \right) < 0 \tag{A-19}$$

where α and dh/dq are defined above with $C(q)$ in its present form substituted for cq. Since the second term in (A-19) is positive, condition (A-12), while necessary for (A-15) to hold, is no longer sufficient (N.B. with CRTS, the second order condition for the decision problem of the body of the paper is $-\alpha < 0$).

For an interior maximum at which (A-19) holds the impact of a change in the firm's equity level is described by the equation

$$\frac{dq}{da} = \left(\frac{q}{a}\right)\left(\frac{\alpha - \mu}{\alpha - 2\mu}\right) > 0 \tag{A-20}$$

where $\mu \equiv (c_0 f/q)(d\bar{u}/dh)(dh/dq)$ and, since the second order condition requires that $\alpha - 2\mu > 0$, $\alpha - \mu > 0$. Thus, with constant-returns-to-scale, adding a fixed bankruptcy penalty does not alter the direction of a firm's response to an increase in equity (it remains positive), but it does intensify the magnitude of the firm's response (since $(\alpha - \mu)/(\alpha - 2\mu) > 1$). Similar results hold for output responses to changes in real wages, real interest rates and the price distribution F. Therefore, to the extent that constant returns to scale characterize a firm's production technology -- either because we are studying scale decisions or because we are concerned with levels of output below capacity over which marginal costs are roughly constant -- inclusion of a fixed bankruptcy cost does not fundamentally alter the implications of the model.

More General Utility Functions

A final obvious extension of the basic model is to settings in which firms' managers maximize over a horizon which is longer than a single decision period. For analytical purposes, this involves considering an objective function of the form

$$\max_{q_t} \left\{ q_t - (1+r_t)(w_t \phi_t - a_t) - c q_t F(\bar{u}_t) + E[V(a_{t+1})] \right\} \qquad \text{(A-21)}$$

subject to (A-2), where E is a mathematical expectation, a_{t+1} is the firm's equity level entering period $t+1$ and V is a valuation function of the usual sort. Formally,

$$a_{t+1} = \tilde{p}_{t+1} \cdot q_t - (1+\tilde{r}_{t+1})(w_t \phi_t - a_t)$$

where $1 + \tilde{r}_{t+1}$ is the random real return to lenders.[35] Thus, (A-21) can be rewritten as

$$\max_{q_t} E\left[a_{t+1} + V(a_{t+1}) \right] - c q_t \; F(\bar{u}_t) \; . \qquad \text{(A-22)}$$

subject to (A-2) with appropriate t subscripts on the variables.

In examining this decision problem, the case of constant returns to scale is again the easiest starting point. With CRTS,

$$q^*_t \equiv \text{optimal } q_t \equiv k_t a_t \; .$$

Therefore

$$h^*_t \equiv \text{optimal level of } h_t = (1+r_t)(w_t - a_t / q^*_t) = (1+r_t)(w_t - 1/k_t)$$

which means that h^*_t, $\bar{u}^*_t$ and $F(\bar{u}^*_t)$ are independent of a_t. Thus, $cq^*_t F(\bar{u}^*_t)$ is linear in a_t and the valuation function for the multiperiod decision problem is linear in a_t. With constant returns to scale, therefore, the multiperiod decision problem is qualitatively identical to a single period problem and the extension to

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[35] This ignores dividends. However, including them would complicate the analysis without altering its implications fundamentally.

multiperiod decision-making is straightforward, involving nothing more than a rescaling of the bankruptcy cost factor c.

Unfortunately the same simplicity does not apply to the general case and here only the most general principals can be articulated. The flavor of these is captured best by abandoning the specific formulation of "bankruptcy" constraints since these no longer yield unambiguous results and simply assuming that managers choose output to

$$\max_{q_t} E[V(a_{t+1})]$$

where V is a general utility function, a_{t+1} is the end of period value of a firm's equity

$$a_{t+1} = \tilde{p}_{t+1} \cdot q_t - (1+r_t)(w_t c(q_t) - a_t) ,$$

and a_{t+1} is now allowed to become negative in order to repay lenders. For this problem it is straightforward to show that (1) risk aversion leads to a reduction in output below what a risk neutral firm would produce and (2), if V exhibits decreasing absolute risk aversion, then greater firm equity levels lead to greater output. Thus, if a multiperiod decision problem generates a valuation function characterized by decreasing absolute risk aversion, then in general we should expect the results of the model (with respect to equity levels and output) to apply without change.

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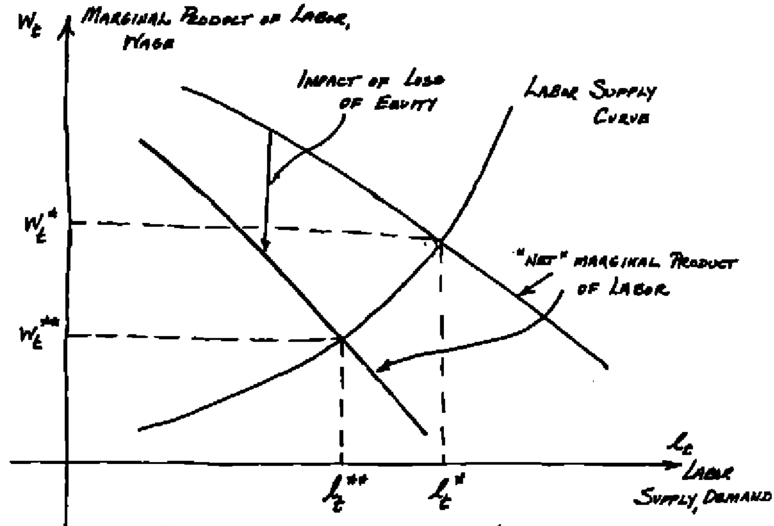
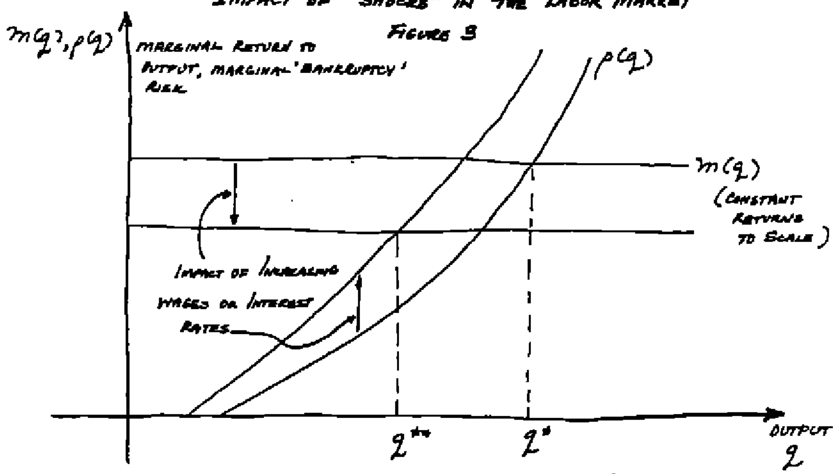


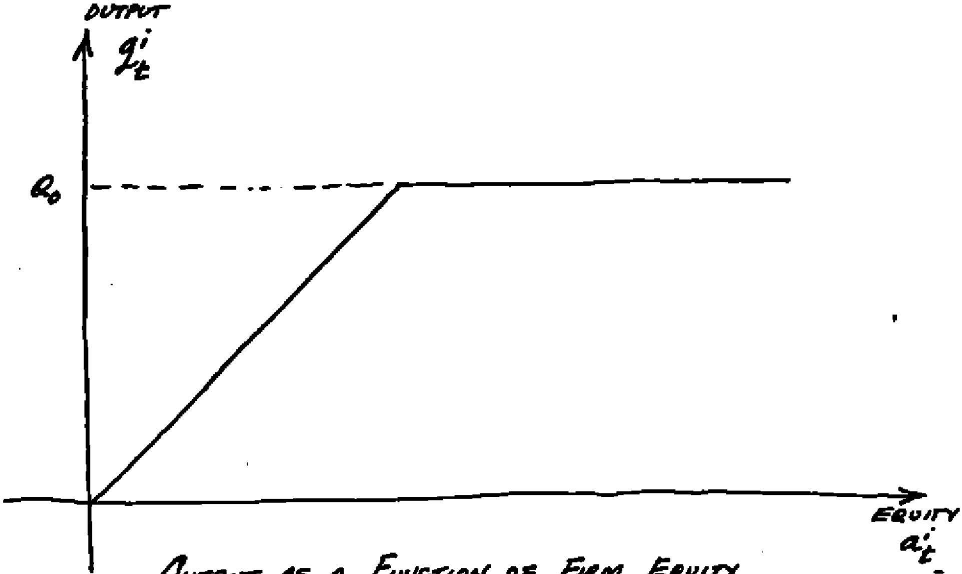
IMPACT OF "SHOCKS" IN THE LABOR MARKET

FIGURE 3



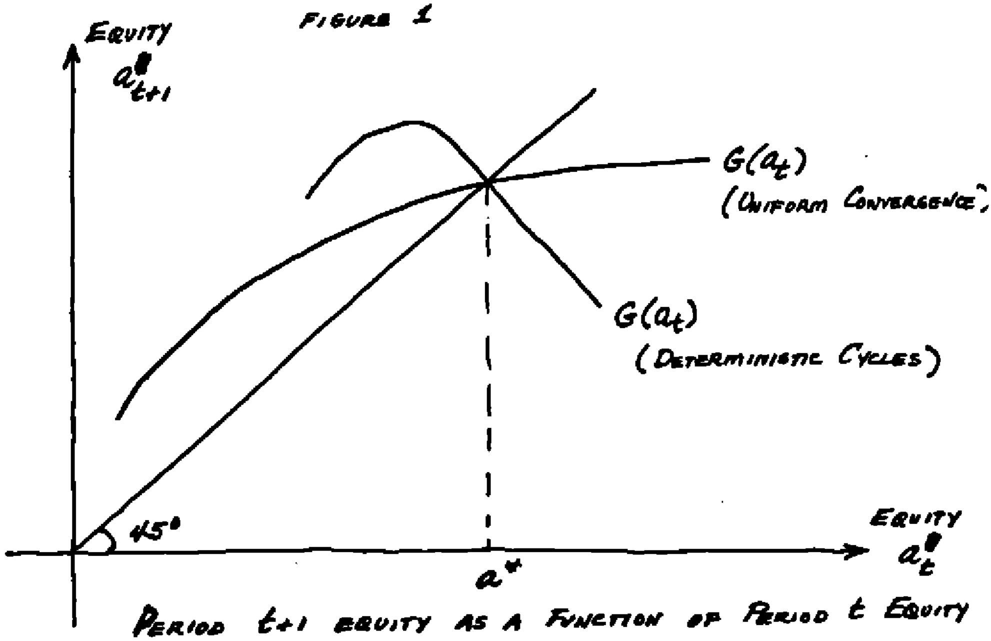
DETERMINATION OF FIRM OUTPUT LEVELS

FIGURE 4



OUTPUT AS A FUNCTION OF FIRM EQUITY
(CONSTANT RETURNS UP TO CAPACITY Q_0)

FIGURE 1



PERIOD $t+1$ EQUITY AS A FUNCTION OF PERIOD t EQUITY

FIGURE 2