

# Controlling the Bullwhip Effect in a Supply Chain Network with an Inventory Replenishment Policy Using a Robust Control Method

mahdi Ghaffari<sup>a,\*</sup>, Nikbakhsh Javadian<sup>b</sup>, Reza Tavakkoli-Moghaddam<sup>c</sup>

<sup>a</sup> Instructor, Department of Industrial Engineering, Mazandaran University of Science and Technology, Babol, Iran

<sup>b</sup> Assistant Professor, Department of Industrial Engineering, Mazandaran University of Science and Technology, Babol, Iran

<sup>c</sup> Professor, Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

Received 23 April, 2014; Revised 28 May, 2014; Accepted 03 August, 2014

## Abstract

This paper develops a mathematical model using differential equations and considers a bullwhip effect in a supply chain network with multiple retailers and distributors. To ensure the stability of the entire system and reduce the bullwhip effect, a robust control method and an inventory replenishment policy are proposed. This shows that the choice of the output matrix may reduce the bullwhip effect. It was also observed that the inventory replenishment mechanism may be a negative impact on the robustness of the bullwhip effect. However, the inventory replenishment behavior may lead to the bullwhip effect on the presented model. This means that the complex supply relationships may have a significant role in controlling or reducing the bullwhip effect of fluctuations.

*Keywords:* Robust Control, Bullwhip Effect, Inventory Replenishment, Supply Network.

## 1. Introduction

The bullwhip effect is a term used to describe progressive fluctuations in customer demand along a supply chain from downstream to upstream. Essentially, larger phenomena produce a bullwhip effect because the process of information dissemination is constantly being distorted. This has a step-by-step effect on the upstream supply chain in that manufacturers and suppliers of raw materials are supplied with distorted information and then decisions can easily lead to over-production and inventory errors. Various factors contribute to the bullwhip effect such as lead-time, type of inventory policy, and information on demand forecasting. This article focuses on retailers with an inventory replenishment policy strategy and incorporates robustness of the bullwhip effect in a supply network using parameters of uncertain demand behavior.

Chuang and Huang (2004) declared that inventory-replenishment decision is vital to the supply chain performance. The selection of an appropriate inventory policy would not only reduce the total inventory costs but also would satisfy the downstream customers and the final customer in a supply chain environment. In this aspect, most of the existing researches have been devoted to the supply chain inventory decision to a single known demand distribution, such as normal or uniform.

However, in a supply chain environment, there are always multiple downstream customers with different probability distributions of a demand quantity.

Daganzo (2003), Nagatani and Helbing (2004), Surana et al. (2005), Helbing et al. (2006) aimed to develop a realistic model to describe nonlinear interactions and to represent dynamics of the flow of the materials through networks. Helbing et al. (2004) proposed a supply network governed by balance equations and equations for the adaptation of production speeds and studied the stability and dynamics of supply networks. Daganzo (2004) examined the stability of multistage supply chains under arbitrary demand conditions and presented commitment-based policies that can maintain any desired inventory level for any demand rate. A supply network based on a stochastic discrete-time controlled dynamical system was proposed by Laumanns and Lefebvre (2006), in which an explicit state-feedback control policy was derived to control the material flow of the supply network. Ouyang and Li (2010) analyzed the propagation and amplification of order fluctuations in supply chain networks and based on inventory management policies. They proposed robust analytical conditions to predict the presence of the bullwhip effect for any network structure. Yang et al. (2009) developed a model of a general closed-loop supply chain network and optimized the equilibrium state of the network by using the variation inequalities method. Dong et al. (2011) proposed a supply chain

\* Corresponding author Email address: ghaffari@ustmb.ac.ir

network model with reentrant nodes based on partial differential equation, which can accurately reflect the impacts of the reentrant degree of the product on the system performance. Zhang and Zhou (2012) established nonlinear complementarity formulation for supply chain network equilibrium models and formulated the equilibrium state of the network by using the variation inequalities method. Dejonckheere et al. (2003) proved that the severity of the bullwhip effect depends more on the internal structure of the supply chain system. Ouyang and Li (2010) provided a theoretical basis for the transfer function of some scholars to basis, the amplitude-frequency characteristic curve by plotting the calculated noise bandwidth of the classical control theory of the bullwhip effect to any demand made under the measurement and interpretation of research inventory strategy, the impact of demand forecasting and information sharing and other factors on the bullwhip effect. Song et.al. (1999) studied an application of robust  $H_\infty$  control considering the bullwhip effect in a supply chain system.

In summary, the existing literature does not clearly indicate the bullwhip effect for the simple structure of the supply chain system and complex factors using a lateral transshipment policy between members of supply chain strategies (e.g. impact of bullwhip effect mechanisms). A robust control theory considered in this paper is based on research needs in an uncertain environment, multiple retailers and multiple distribution. Additionally, by considering supply network features for a hierarchical structure and properties of heterogeneous nodes, dynamic equations were established for retailers and distributors to build a unified state space model suitable for application in robust control theory. An application of a robust  $H_\infty$  control method is in ordering a policy designed a network of each node through numerical simulation to study the effects of different control methods on robustness of the bullwhip effect, and analyzing the impact of demand in modeling the bullwhip effect.

## 2. Model

Considering a cycle and inventory situation, the supply network consists of multiple levels of distributors and multi-retailer. It is assumed that there are  $n$  distributors and  $m$  retailers. All distributors have unlimited supply capacity of upstream suppliers and each retail manufacturer has its downstream customers with many interrelations between supply retailers and distributors in order to reduce stock risk and improve the level of customer service. Retailers lateral transshipment policy inventory cooperation is based on the differences in order to simplify modeling and analysis. It is assumed that retailers and distributors of goods orders arrived at the beginning of the next cycle, which are an order lead-time cycle affected by transmission of information and by

factors delays, lateral transshipment policy between the time delay  $\tau$  retailers cycles, where  $\tau$  is an integer.

Retailers and distributors operate according to the following sequence of events: 1) at the beginning of each cycle, retailers and distributors place an order to reach the receiver and storage; 2) the retailer's inventory is made according to differences in lateral transshipment policy cooperation, in which retailers and distributors manufacturers are in inventory orders on the basis of their inventory information; 3) during the remaining period of time, retailers and distributors both backordered manner downstream of customer needs and made horizontal transshipment policy orders.

Supply network node collection includes a set of  $N_R$  retailers nodes and  $N_D$  distributors nodes, distributors node indicated by  $i, 1 \leq i \leq m$  and retailers node indicated by  $j, m + 1 \leq j \leq m + n$ , where  $j \in N_R, i \in N_D$ .  $x_i(t)$  represents distributor's node  $i$  at the first  $t$  net inventory cycle and  $x_j(t)$  represents the net inventory  $j$  at the retailer node in the  $t$ -th cycle. Matrix  $A = (a_{ij})$  describes the supply relationship between retailers and distributors. If  $a_{ij} = 1$ , then the distributors  $i$  for retailers  $j$  relationships with upstream and downstream supply. If  $a_{ij} = 0$ , then the distributors  $i$  for retailers  $j$  supply relationship does not exist because the supply network has a hierarchical structure characteristics of the supply network nodes have heterogeneous characteristics were established retailers and distributors dynamic equation. Distributors inventory changes for each cycle node as computed by:

$$x_i(t) = x_i(t-1) + u_i(t-1) - \sum_{j \in N_R} a_{ij} u_{ij}(t-1) \quad (1)$$

where  $i \in N_D, u_i(t-1)$  Representatives distributors order to reach capacity,  $u_{ij}(t-1)$  Representatives distributors  $i$  for retailers  $j$  supply capacity.

Lateral transshipment policy retailer cooperation is based on differences in stock, the capacity of the lateral transfer of the retailer  $j_1$  for retailers  $j_2$ , which is  $h_{j_1 j_2}(t)$  computed by:

$$h_{j_1 j_2}(t) = r_{j_1 j_2}(x_{j_2}(t-\tau) - x_{j_1}(t-\tau)) \quad (2)$$

Deliveries lateral shift library determines the sign of the direction,  $r_{j_1 j_2}$  as co-factor, the specific value is determined by negotiation of bilateral cooperation,  $\tau$  lateral transshipment policy behalf of the time delay, so the retailer  $j$  inventory differential equation can be expressed by:

$$x_j(t) = x_j(t-1) + \sum_{i \in N_R} a_{ij} u_{ij}(t-1) - d_j(t-1) + \sum_{j' \in N_R} a_{j j'}(t) \quad (3)$$

where  $d_j(t)$  facing the  $j$  retailer customer needs.

When  $d_j(t)$  is constant  $d_j^\infty$ , for a stable supply network, its inventory and order quantity is a constant steady state shall,  $\lim_{t \rightarrow \infty} x_i(t) = x_i^\infty, \lim_{t \rightarrow \infty} x_j(t) = x_j^\infty, \lim_{t \rightarrow \infty} u_i(t) = u_i^\infty$ . On this basis, the definition of the state vector is shown below:

$$x(t) = [x_1(t) - x_1^\infty, x_2(t) - x_2^\infty, \dots, x_m(t) - x_m^\infty, x_{m+1}(t) - x_{m+1}^\infty, \dots, x_{m+n}(t) - x_{m+n}^\infty]^T \quad (4)$$

In this study, in terms of the supply network, all retailers and distributors orders are quantified as a control vector. Structure control vector order and supply network are assigned a direct relationship because each distributor has its unique external suppliers, and each retailer has at least one distributor for its supply of goods, so it may be assumed that the steady-state minus the constant use of all distributors and retailers re-order quantity number to get control vector  $u(t)$ . Then,  $u(t)$  is an order the number of vector  $m + \sum_{i=1}^m \sum_{j=m+1}^{m+n} a_{ij}$ . Input vector is defined by:

$$d(t) = [d_{m+1}(t) - d_{m+1}^\infty, d_{m+2}(t) - d_{m+2}^\infty, \dots, d_{m+n}(t) - d_{m+n}^\infty]^T \in R^n \quad (5)$$

In the definition of state vector, the basic control vector and the input vector on the supply network can be established by the following unified state model:

$$\begin{cases} x(t) = A_1 x(t-1) + A_2 x(t-\tau) + B_1 u(t-1) + B_2 d(t-1) \\ y(t) = Du(t) \end{cases} \quad (6)$$

where  $A_1, A_2$  are  $(m+n) \times (m+n)$  matrix,  $B_1$  is  $(m+n) \times (m + \sum_{i=1}^m \sum_{j=m+1}^{m+n} a_{ij})$  matrix,  $B_2$  is  $(m+n) \times n$  matrix,  $y(t)$  is the output vector,  $D$  is the output matrix of the system. Due to robustness of the output variable, indicators and systems are closely linked, therefore the value of the output matrix and managers expected performance of the supply network have a direct relation. For example, managers can choose a value for a distributors order set by the output matrix as the system output, focusing on suppression orders fluctuations distributor side.

### 3. Robustness Index and Inventory Strategy

#### Algorithm

The main task of robust control for a variety of supply networks is to manage uncertainty. The application of a robust inventory control strategy designed node enterprises to meet the performance requirements of the system administrator. A supply network uncertainty does

not consider demand as uncertainty. The uncertainty of supplying raw materials, in advance of uncertainty and structural uncertainty due to demand uncertainty is often a source of uncertainty generated by the other. This paper focuses on the robust demand environment in uncertain supply network control problems. According to Wei et al. (2013), consideration of robust indicators is determined as follows:

$$W_I = \sup_{\forall d(k) \neq 0} \left( \frac{\sum_{k=1}^{\infty} y(k)^T y(k)}{\sum_{k=1}^{\infty} d(k)^T d(k)} \right)^{1/2} \quad (7)$$

From the definition of  $W_I$  can be seen that the demand for any form, as long as the lower value of  $W_I$ , we can improve the output variables of the input variables interference ability to meet the requirements of a robust supply network if the output vector  $y(t)$  contains only the distributors order quantity, and its steady-state value for the order of the matrix,  $W_I$  can be expressed by:

$$W_I = \lim_{N \rightarrow \infty} \sup_{\forall d(k) \neq 0} \left( \frac{\sum_{i=1}^m \sum_{k=1}^N (u_i(k) - u_i^\infty)^2}{\sum_{j=m+1}^{m+n} \sum_{k=1}^N (d_j(k) - d_j^\infty)^2} \right)^{1/2} = \sup_{\forall d(k) \neq 0} \left( \frac{\sum_{k=1}^{\infty} \text{Var}(u_i)}{\sum_{k=1}^{\infty} \text{Var}(d_j)} \right)^{1/2} \quad (8)$$

As can be seen,  $\sum_{k=1}^{\infty} \text{Var}(u_i) / \sum_{k=1}^{\infty} \text{Var}(d_j)$  is the classic expression of the bullwhip effect, which means that demand for the unknown, as long as possible to minimize the value of  $W_I$ , distributors will be able to end the bullwhip effect, or at least to reduce it to the minimum. Specifically, in circumstances when  $W_I < 1$ , the bullwhip effect can completely control an entire network. It is appropriate to apply  $W_I$  bullwhip effect robustness index.

In the field of control engineering,  $W_I$  robustness index is also known as the  $H_\infty$  norm. For a control system, the goal of robust  $H_\infty$  control is through the controller design, to ensure stability of the system, based on the  $H_\infty$  norm minimization. The value of  $H_\infty$  norm, there are two methods; namely, the frequency-domain method and the time-domain method. In the frequency domain, a transfer function is based on a system. If the transfer function is  $G(z)$ , then its  $H_\infty$  norm is  $\max |G(e^{j\omega})|_{\omega \in (0, 2\pi)}$ , calculation of the time domain by defining a suitable Lyapunov function, and a robust control problem into the LMI problem solving. This idea in a robust control theory is widely used.

Since this construct is a state-space model for calculating time-domain method, robustness indicators used to optimize robustness of the indicator optimization problem can be transformed into a linear matrix inequality constraints due to Wei et al. (2013). This gives a general model of robust control algorithms, in which direct reference to ideas in the literature present the following conclusions.

**Theorem 1:** Estimation for the upper bounds of a system, Eq. (4) robustness index  $W_I$  can be transformed into the following optimization problem if there exists  $\gamma > 0$ ,  $(n + m) \times (n + m)$  is positive definite matrix  $P, Q, L$  (i.e.,  $P > 0, Q > 0, L > 0$  satisfies the following optimization problem):

$$\text{Min } \gamma \tag{9}$$

$$\begin{bmatrix} -P & 0 & 0 & PA_1^T + L^T B_1^T & \sqrt{\tau}P & L^T D^T \\ 0 & -Q & 0 & QA_2^T & 0 & 0 \\ 0 & 0 & -\gamma^2 I_q & B_2^T & 0 & 0 \\ A_1 P + B_1 L & A_2 Q & B_2 & -P & 0 & 0 \\ \sqrt{\tau}P & 0 & 0 & 0 & -Q & 0 \\ DL & 0 & 0 & 0 & 0 & -I_N \end{bmatrix} < 0 \tag{10}$$

where  $I_N$  is the identity matrix of order  $n + m$ , if there is an optimal solution of the above optimization problem, then the order policy  $u(t) = LP^{-1}x(t)$  is determined to ensure system stability, and to make the robust indicators meet  $W_I < \gamma$ .

The above optimization algorithm can be used in the LMI toolbox of MATLAB to quickly solve the optimization algorithm. It has the following outstanding advantages. As long as there is an optimal solution optimization algorithm can not only ensure stability of the

system, but also ensure its robustness, when an entire network remains stable as long as customer demand is bounded fluctuations supply network inventory and order all the nodes are bound, which avoids the cost of excessive or unnecessary losses due to inventory in order to bring a certain extent, and reduces risk. Furthermore, since guaranteed  $WI < \gamma$ , it means that the demand for the unknown, the bullwhip effect can be suppressed in different applications.  $D$  has different values of the matrix corresponding robustness index calculation expression will be different, thus the optimization algorithm can get different control strategies based on different values of  $D$  matrix to meet the needs of management practices.

#### 4. Numerical Analysis

Robustness of the following numerical simulation output matrix  $D$  is a major concern as well as a horizontal transshipment policy strategy of the bullwhip effect supply network, considering a supply network that consists of three levels of distributors and five retailers. It is a collection of nodes distributors  $N_D = \{1, 2, 3\}$ , in which retailers node set is  $N_R = \{4, 5, 6, 7, 8\}$ .

Construction of Eq. (6) is shown in the supply network state space model, in which  $A_1$  is a unit matrix of the order 8. The other value of the coefficient matrix is as follows:

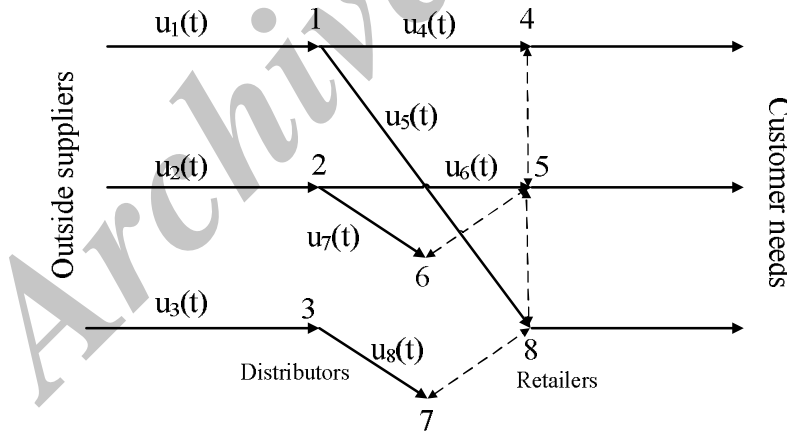


Fig. 1. Supply network structure

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r_{45} & r_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & r_{54} & -(r_{54} + r_{56} + r_{58}) & r_{56} & 0 & r_{58} \\ 0 & 0 & 0 & 0 & r_{65} & -r_{65} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -r_{78} & r_{78} \\ 0 & 0 & 0 & 0 & r_{85} & 0 & r_{87} & -r_{85} - r_{87} \end{bmatrix} \tag{11}$$

$$B_1 = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (13)$$

Considering the existence of inventory replenishment between behavior and output matrix according to the retailer, different values of  $D$  are controlled according to four cases.

**Case 1:** Select the retailer's order quantity from output vectors (i.e.,  $D = \text{diag}(I_{A1}, I_{B1})$ ) among  $I_{A1}$  value of all the elements 0 three square matrix and  $I_{B1}$  for the fifth-order unit matrix. Furthermore, it is assumed  $A_2 = 0$ , that behavior is not considered as inventory replenishment among retailers.

**Case 2:** Select output variables and shapes the same situation,  $D = \text{diag}(I_{A2}, I_{B2})$ , in which  $I_{A2}$  there are elements of the value taken 0 to 3 order of party array,  $I_{B2}$  as 5-order unit matrix; consider inventory replenishment strategies among retailers, inventory replenishment between the retailer for delay  $\tau = 2$ , set  $r_{45} = r_{54} = 0.0025$ ,  $r_{56} = r_{65} = 0.0015$ ,  $r_{56} = r_{65} = 0.005$ ,  $r_{78} = r_{87} = 0.002$ .

**Case 3:** Select distributors order quantity from the output vector  $D = \text{diag}(I_{A3}, I_{B3})$ , among  $I_{A3}$  as 3 order unit matrix,  $I_{B3}$  for all elements 0 to 5 square matrix. There is no consideration for inventory replenishment between retailers, namely  $A_2 = 0$ .

**Case 4:** Select output variables and circumstances 3 the same, that  $D = \text{diag}(I_{A4}, I_{B4})$ , among  $I_{A4}$  as 3 order unit matrix,  $I_{B4}$  for all elements 0 of 5 square matrix and circumstances 3 different strategies are considered for inventory replenishment among retailers, inventory replenishment between the retailer for delay  $\tau = 2$ , set  $r_{45} = r_{54} = 0.0025$ ,  $r_{56} = r_{65} = 0.0015$ ,  $r_{56} = r_{65} = 0.005$ ,  $r_{78} = r_{87} = 0.002$ .

According to Eqs. (7) and (8), an optimization algorithm using by the MATLAB of LMI toolbox is considered to determine value of the inventory control policy, in which  $\gamma$  for four different above-mentioned cases is minimized. Corresponding to these cases, the minimum value of  $\gamma$  is shown in Table 1.

Table 1  
Minimum value of  $\gamma$  per each case

Min of $\gamma$	Case 1	Case 2	Case 3	Case 4
	4.0	6.355	5.656	7.514

This table also shows an inventory replenishment policy strategy does not significantly improve robustness of the bullwhip effect in supply networks. In order to further reveal the advantages of robust control methods, using the specific needs of the bullwhip effect model to study the particular problem before simulation needs to apply the bullwhip effect, retailers and distributors are given the bullwhip effect metric expression.

$$BW_R = \frac{\sum_{j \in N_R} \text{Var}(u_j(t))}{\sum_{j \in N_R} \text{Var}(d_j(t))} \quad (14)$$

$$BW_D = \frac{\sum_{i \in N_D} \text{Var}(u_i(t))}{\sum_{j \in N_R} \text{Var}(d_j(t))} \quad (15)$$

$BW_R$  and  $BW_D$  are represented retailers and distributors bullwhip effect expression is worth noting that in numerical simulation, variance calculation and simulation length.

## 5. Results and Discussion

When demand obedience considers 20 retailers, variance 100 under normal distribution, Fig. 2 shows that in Case 4 there is a steady state under three distributors on the dynamic curve. This figure also shows that based on robust  $H_\infty$  control of the supply network design strategies to ensure stability of the order of the entire supply network helped reduce ordering volatility curve which can order to a steady state at a faster rate. It means that the system can respond more quickly to customer needs. In addition, Fig. 3 shows the convergence curve for the

bullwhip effect indicators (Eqs. 14 and 15). The demand model shows clearly that there are good results for robust control in terms of suppressing the bullwhip effect in a supply network.

The following equation is considered for time process of the retailer's customer needs.

$$d_j(t) = \mu + \rho d_j(t - 1) + \sigma \varepsilon(t), j = 4, 5, \dots, 8 \quad (16)$$

where  $\mu = 4, \sigma = 2, \varepsilon(t) \sim N(0, 1)$  is a normal random variable independent and identically distributed,  $\rho$  is the correlation coefficient in Cases 1 to 4, an application of a robust control method for a dynamic simulation model of the bullwhip effect can be calculated demand indicator (Eq. 16). The calculated results are shown in Table 2 and Figs. 3 and 4.

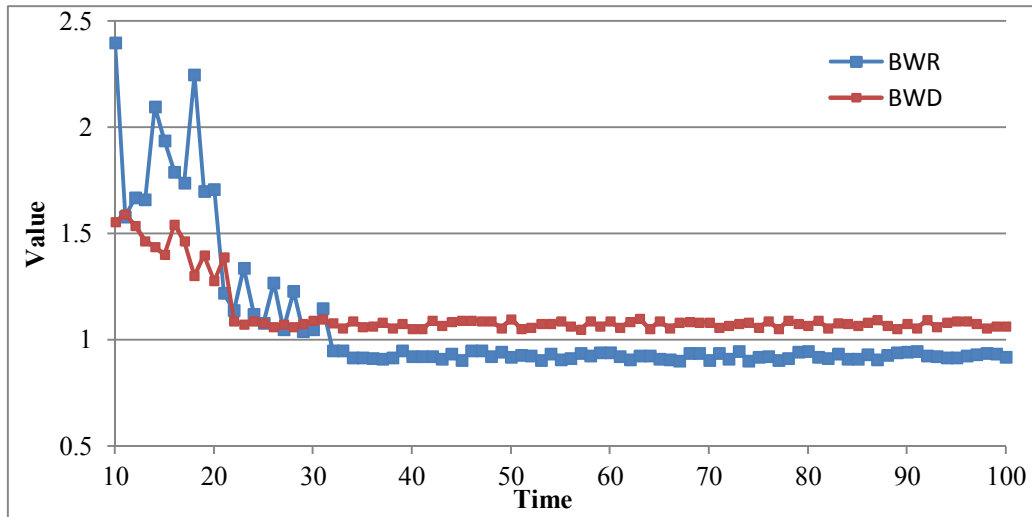


Fig. 2. Dynamic curves bullwhip effect under Case 4

Table 2  
Bullwhip effect and  $\gamma$  per each case

Case	$\gamma$	$\rho = -0.4$		$\rho = 0$		$\rho = 0.4$	
		$BW_D$	$BW_R$	$BW_D$	$BW_R$	$BW_D$	$BW_R$
1	4.0000	1.0141	1.0005	1.0103	1.0015	1.0086	1.0040
2	6.3350	0.9035	0.9174	0.9502	0.9418	0.9712	0.9665
3	5.6569	0.9694	1.0837	0.9773	1.0608	0.9882	1.0393
4	7.5146	0.8606	1.3337	0.8986	1.2445	0.9394	1.1505

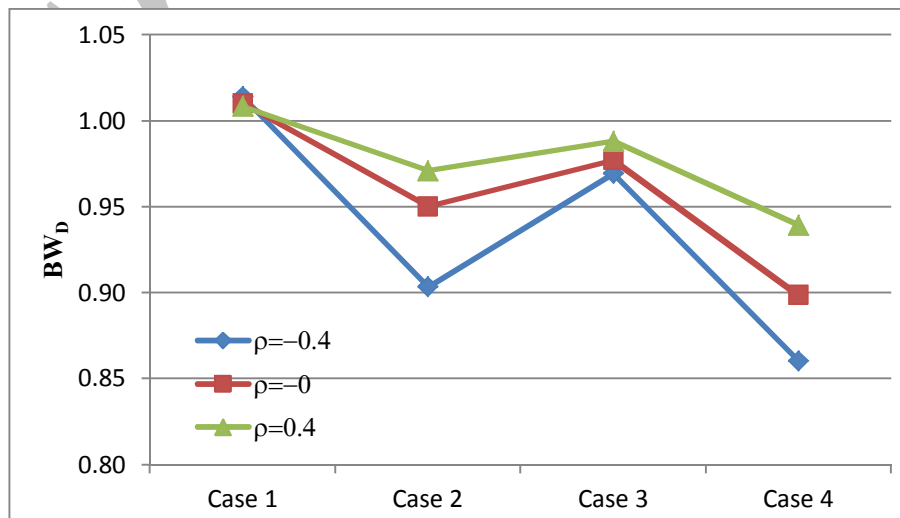


Fig. 3. Bullwhip effect  $BW_D$  per each case and three correlation coefficients

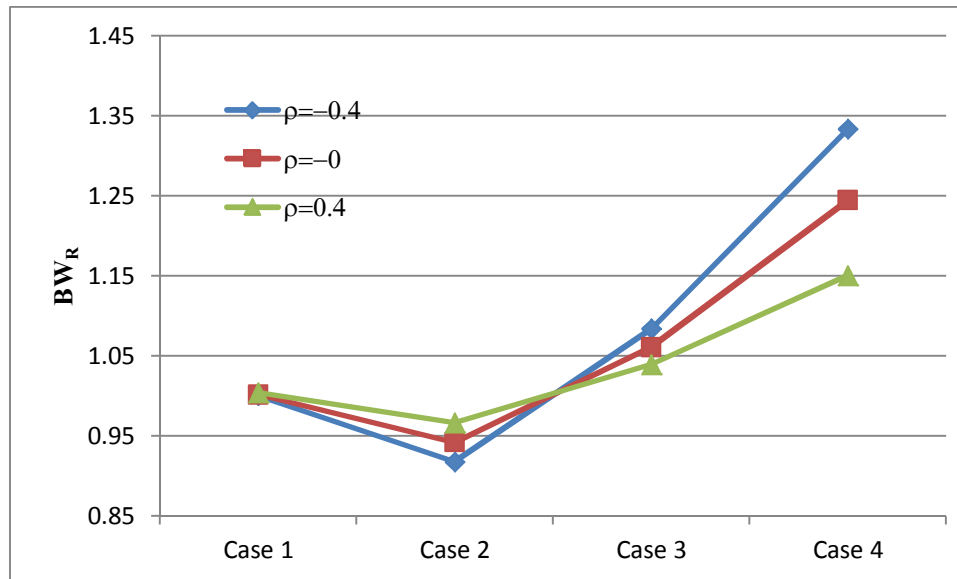


Fig. 4. Bullwhip effect  $BW_R$  per each case and three correlation coefficients

From Figs. 3 and 4 and Table 2, the following conclusions can be drawn: 1) Comparison of cases shows that in Cases 1 and 3 in terms of the bullwhip effect, the value of retailers and distributors bullwhip effects have a significant number. Cases 2 and 4 are compared to the situation bullwhip effect from end retailers to smaller. Its distributors bullwhip effect shows a significant number, which means the robust control design strategies for an inventory can be targeted to select some quantities from companies orders as an output variable to reduce the effect of these enterprises on the bullwhip effect. 2) Cases 1 and 2 compared with Cases 3 and 4, there is an amount of the order retailer output variables and inventory replenishment strategies help reduce the bullwhip effect in the supply networks. When selecting distributors, order quantity of output variables, the inventory replenishment strategy helps to alleviate the bullwhip effect in distribution but fluctuations are caused by retailers in the orders found in front of the inventory replenishment strategy, although the bullwhip effect does not necessarily enhance robustness of the supply network, it was able to suppress the bullwhip effect nicely under specific demand model. 3) Whether customer needs or demands are positively correlated or negatively related to the design of robust control method for inventory strategies helps to curb the overall bullwhip effect in a supply network and improve the speed of a response of the supply network.

## 6. Conclusion

A two-echelon supply network was modeled for retailers and distributors with an application of robust  $H_\infty$  control method designed according to ordering policies at each node. Studies showed that the application of robust control for inventories functions to control the bullwhip effect might well inhibit a supply network by setting the

output matrix. It could be targeted to inhibit the target node bullwhip effect and reduced production costs and inventory costs. The results of numerical simulation showed that the strategy of inventory replenishment, although not significantly, might reduce supply bullwhip effect of a network, but demand for a particular form, can be effective in terms of inhibiting the bullwhip effect. This study described the complex supply relationships that might be an important factor in inhibiting the bullwhip effect in a supply network research; however, some shortcomings remained. Moreover, considering only two levels of a supply network, the structure in reality a supply network had more complexity.

## References

- [1] Chuang, P.T. and Huang, S.C. (2004). A Supply Chain Inventory-Replenishment Policy for Multiple Demand Patterns. Proceedings of the Fifth Asia-Pacific Industrial Engineering and Management Systems Conference (APIEMS). 16.19.1-16.19.9.
- [2] Daganzo, C.F. (2003). A Theory of supply chains. Vol. 526. Springer, Berlin, Germany.
- [3] Daganzo, C.F. (2004). On the stability of supply chains. Operations Research. 52(6). 909–921.
- [4] Dejonckheere, J., Disney, S.M. and Lambrecht, M.R. (2003). Measuring and avoiding the bullwhip effect: A control theoretic approach. European Journal of Operational Research, 147(3). 567–590.
- [5] Dong, M., He, F.L. and Shao, X.F. (2011). Modeling and analysis of material flows in re-entrant supply chain networks using modified partial differential equations. J. of Applied Mathematics. 2011. 14 pages.

- [6] Helbing, D., Armbruster, D., Mikhailov, A.S. and Lefebvre E. (2006). Information and material flows in complex networks. *Physica A*. 363(1). 11–16.
- [7] Helbing, D., Lammer, S., Seidel, T., Seba, P. and Płatkowski, T. (2004). Physics, stability, and dynamics of supply networks. *Physical Review E*. 70(6). 6 pages.
- [8] Laumanns, M. and Lefebvre, E. (2006). Robust optimal control of material flows in demand-driven supply networks. *Physica A*. 363(1). 24–31.
- [9] Nagatani, T. and Helbing, D. (2004). Stability analysis and stabilization strategies for linear supply chains. *Physica A*. 335(3). 644–660.
- [10] Ouyang, Y.F. and Li, X.P. (2010). The bullwhip effect in supply chain networks. *Eur. J. of Operational Research*. 201(3). 799–810.
- [11] Ouyang, Y.F., Li, X. (2010). The bullwhip effect in supply chain networks. *European Journal of Operational Research*. 201(3). 799–810.
- [12] Song, S., Kim, J.K. and Yim, C.H. (1999).  $H_\infty$  control of discrete-time linear system with time-varying delay in state. *Automatica*. 35(1). 1587–1591.
- [13] Surana, A., Kumara, S., Greaves, M. and Raghavan, U.N. (2005). Supply-chain networks: A complex adaptive systems perspective. *Int. J. of Production Research*. 43(20). 4235–4265.
- [14] Wei, Y.C., Wang, H.W., Qi, C. (2013). On the stability and bullwhip effect of a production and inventory control system. *Int. J. of Production Research*. 51(1). 154–171.
- [15] Yang, G.F., Wang, Z.P. and Li, X.Q. (2009). The optimization of the closed-loop supply chain network. *Transportation Research, Part E*. 45(1). 16–28.
- [16] Zhang, L. and Zhou, Y. (2012). A new approach to supply chain network equilibrium models. *Computers & Industrial Engineering*. 63(1). 82–88.

Archive of SID