Modeling Cyclic Change*

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Abstract. Database support of time-varying phenomena typically assumes that entities change in a linear fashion. Many phenomena, however, change cyclically over time. Examples include monsoons, tides, and travel to the workplace. In such cases, entities may appear and disappear on a regular basis or their attributes or location may change with periodic regularity. This paper introduces an approach for modeling cycles based on cyclic intervals. Intervals are an important abstraction of time, and the consideration of cyclic intervals reveals characteristics about these intervals that are unique from the linear case. This work examines binary cyclic relations, distinguishing sixteen cyclic interval relations. We identify their conceptual neighborhood graph, showing which relations are most similar and demonstrating that this set of sixteen relations is complete. The results of this investigation provide the basis for extended data models and query languages that address cyclically varying phenomena.

1 Introduction

The development of conceptual models that convey how objects change over space and time demands continued attention from software engineers and database system designers. Theoretical advances in the design of data models for geographic information systems (GISs), for example, have focused on increasing support for

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temporality [1-5] and spatial processes [6], including objects that experience identity changes [7] and objects that move [8]. At the same time, there has been an increased awareness of the necessity for a stronger cognitive element in software design [9]. Particular aspects of change, however, still remain beyond the scope of current data models. How do these models convey, for instance, spatio-temporal change associated with cycles of beach erosion and accretion due to tidal fluctuations and storms, or the planting cycles and crop rotations that are followed by farmers on a regular basis, or the cycles of monsoon rains in India? Queries based on any of these example scenarios must incorporate the cyclic nature of the phenomenon being studied. This paper develops a formal model for cycles based on cyclic intervals. Although we use examples of cycles drawn from geographic contexts of particular interest for spatiotemporal data modeling, this approach is also more generally useful for other applications that involve cyclic phenomena.

1.1 Linear vs. Cyclic Phenomena

Discussions within the database community on modeling time-varying phenomena have resulted in many models reflecting different views of the semantics associated with time [10]. Numerous approaches exist for modeling time, although time is most often discussed with respect to two key structural models: linear and branching models of time. The most general model of time in a temporal logic represents time as an arbitrary, partially-ordered set [11, 12]. The addition of axioms result in more refined models of time [11]. In the linear model, an axiom imposes total order on time, resulting in the linear advancement of time from the past, through the present, and to the future. The branching model, also known as the "possible futures" model, describes time as being linear from the past to the present, where it then divides into several time-lines, each representing a potential sequence of events.

Few of these models, however, explicitly treat cycles. Although current information systems are useful for producing a snapshot of a phenomenon at any one time, cyclically-varying phenomena require new solutions. The measurement scales-nominal, ordinal, interval, and ratio-frequently applied to geographic phenomena have been shown to provide less than complete coverage leaving out those measurements that are bounded within some range and repeat in a cyclic manner [13]. There are also cases of non-temporal cyclic change. Angles may at first seem to fit a ratio scale of measurement as there is a zero and the units are arbitrary (degrees, radians); however, an important characteristic of angles is that they repeat in a cyclic fashion [14]. Other examples of non-temporal cycles are color wheels and certain mathematical functions, such as the graphs of sine and cosine functions. The special nature of cycles has also been noted by cartographers exploring the role of cartographic animation as a technique for visualizing spatio-temporal change. Research on temporal legends that orient the user to a particular temporal framework [15] utilizes, for example, a time wheel designed to support querying of phenomena that exhibit cyclic variations. These efforts, however, are less common than the usual linear treatment of change.

1.2 Temporal Intervals

Temporal data models are commonly based on the primitive elements of either time points or time intervals. Time points typically describe a precise time when an event occurred. A linear model based on time points assumes a set of time points that are totally ordered [12]. When precise information on time is unavailable, time intervals become useful constructs. Reasoning about temporal intervals addresses the problem that much of our temporal knowledge is relative and methods are needed that allow for significant imprecision in reasoning [16]. This view does not require that all events occur in a known fixed order and it allows for disjunctive knowledge (e.g., event A occurred either before or after event B). Discussions about temporal points and intervals relate to conceptualizations of time that are discrete. Time, however, can be viewed as either discrete or continuous. The cycle of temperature change over the years, for instance, is a continuous phenomenon. The partitioning into seasons-winter, spring, summer, fall-forms discrete temporal concepts. Each season, modeled as an interval, forms a discrete temporal entity that becomes subject to cyclic reasoning. These discussions relate to those on spatial object and field models in the GIS domain [17, 18]. As people shift from a conceptualization based on continuous phenomena to discrete or vice versa as the task demands, they similarly switch from a view based on continuous time to time that is discrete.

In spite of this duality existing for many common geographic phenomena, a discrete model of time typically underlies most temporal database models. The reasons for such common usage are [11]: measures of time are inherently imprecise where even instantaneous events can only at best be measured as having occurred during a chronon, the highest resolution time unit; the discrete model is compatible with most natural language references to time; and any implementation of a data model with a temporal dimension will of necessity have to have some discrete encoding for time. Temporal database models also impose axioms that treat the boundedness of time. A finite encoding implies bounding from the left (i.e., the existence of a time origin) and from the right. Cycles, however, require a different treatment.

1.3 Structure of Paper

This paper focuses on modeling cyclic change. Frank [19] gives examples of nine cyclic interval relations, however, we show through a formalization of cyclic intervals and the relations between these intervals that there are more than nine relations. These relations are fundamental to reasoning about scenarios involving cyclic change. The remainder of the paper is organized as follows: Section 2 reviews and discusses the nature of cycles. An approach to modeling cycles based on cyclic intervals is introduced in Section 3. The model formalizes binary relations between cyclic intervals, distinguishing sixteen cyclic interval relations and their conceptual neighbors. An example scenario based on reasoning with cyclic intervals is presented in Section 4. Conclusions and future work are discussed in Section 5.

2 Cyclic Change

The linear or branching models of time do not treat the fact that certain events or phenomena may be recurring. The term *cycle* is used to capture the notion of recurring events. Conceptually, we talk about life cycles, work cycles, cycles of poems or songs, and the seasonal cycle, which is perhaps the most common example of a cycle (Figure 1).

Cycles may affect the existence of an object [7], the properties of an object, and the location of an object. In certain cases, a phenomenon, such as high tide, is existent for a period of time, becomes non-existent, and then it reappears again This cycle is repeated over time. Similarly, at regular (or irregular) intervals, a water body, such as a pond or stream, may dry up and become non-existent before rains or high water levels bring it back into existence.



Fig. 1. Seasonal activity cycle of 17th century Native peoples in Rupert's Land, Canada (after [20])

Cycles can also be described from the perspective of cyclic changes to *properties* of an object. Examples of properties that vary cyclically include the size, shape, and value of an object. The population of a small college town in the US, for instance, can increase or decrease according to whether the University is in session (students are resident in town) or not. During the summer, when the University is not in full session, the student population is often much smaller and the town's population is reduced. An understanding of the cyclic variation in population size is important in town planning, traffic planning, availability of accommodation, business decisions, etc.

An object's location can also vary in a cyclic pattern over time. People travel to their jobs each working day, for example, and then return to their homes in the evening, some people visit a grocery store on a regular basis, planning their excursion at approximately the same time every week, and trains, planes, and buses move in space according to schedules that are cyclic.

A formal approach to modeling cycles based on cyclic intervals is introduced in the next section.

3 Modeling Cyclic Intervals

The embedding space for a cycle *C* is a connected subset of the real numbers, IR^1 . The period $n \in Z$ describes the length of the full cycle C_n , such that $IR^1 \mod n$ captures all points that are part of the full cycle. A cyclic interval *I* is then a nonempty, connected, true¹ subset of C_n (i.e., $I \neq \emptyset$ and $I \subset C_n$). If $I \subset C_n$, then $C_n \setminus I$ is *I*'s complement, denoted by I^- (Figure 2).

Given $I \subset C_n$, the *interior* of I, denoted by I° , is defined to be the union of all open sets that are contained in C_n . In this paper, we assume that the interior of a cyclic interval is non-empty ($I^\circ \neq \emptyset$). The *closure* of I, denoted by \overline{I} , is the intersection of all closed sets that contain C_n . The *boundary* of I, denoted by ∂I , is the intersection of the closure of I and the closure of the complement of I (i.e., $\overline{I} \cap \overline{I^\circ}$).



Fig. 2. Cyclic interval A with start $(\partial_s A)$, interior (A°) , end $(\partial_e A)$, and complement (A^-)

The boundary of $I \subset C_n$ is disconnected, i.e., there are two distinct subsets of ∂I , called start $(\partial_{\alpha}I)$ and end $(\partial_{\alpha}I)$, satisfying the following three conditions:

- $\partial_s I \neq \emptyset$ and $\partial_e I \neq \emptyset$;
- $\partial_s I \cup \partial_e I = \partial I$; and
- $\partial_{\mathfrak{s}}I \cap \partial_{\mathfrak{s}}I = \emptyset$.

Based on these conditions, a cyclic interval is closed. It includes neither separations, nor a single point, nor an entire cycle.

The order of the underlying IR^1 implies an orientation for the sequence of the two parts of the boundary and the interior such that $\partial_s I < I^\circ < \partial_e I$. The ordering of C_n , however, does not establish an order relation, because when applied to a cyclic space such as C_n , the order relation \leq ("before or equal") is not necessarily transitive (i.e., for elements $a, b, c \in C_n$, $a \leq b$ and $b \leq c$ does not necessarily imply that $a \leq c$).

¹ We exclude here intervals that would extend through the entire cycle, since we are interested in modeling the prototypical cyclic relations. Our approach, however, is extendible to intervals that span the entire cycle.

Therefore, no information about the relative order of $\partial_s I$ and $\partial_e I$ can be derived from $\partial_s I < I^\circ < \partial_e I$.

Subsequently, we consider only cycles that have consistently the same orientation. We select a clockwise orientation, although the same results would apply for a consistent choice of a counterclockwise orientation.

3.1 Binary Relations Between Cyclic Intervals

Let A and B be a pair of cyclic intervals of the same cycle C_n (Figure 3a).



Fig. 3. (a) Cyclic intervals A and B and (b) the intersection matrix based on whether the intersections of start, interior, and end are empty or non-empty

The relation between A and B is described by the corresponding values of the set intersections of the intervals' boundaries and interiors. Since each cyclic interval has three distinct, mutually-exclusive parts ($\partial_s A$, A° , $\partial_e A$, and $\partial_s B$, B° , $\partial_e B$), there are a total of nine set intersections. They are concisely represented by a 3x3 matrix (Equation 1).

$$M = \begin{bmatrix} \partial_s A \cap \partial_s B & \partial_s A \cap B^o & \partial_s A \cap \partial_e B \\ A^o \cap \partial_s B & A^o \cap B^o & A^o \cap \partial_e B \\ \partial A_e \cap \partial_s B & \partial A_e \cap B^o & \partial A_e \cap \partial_e B \end{bmatrix}$$
(1)

Figure 3b shows an example of a cyclic interval relation and the corresponding matrix of empty (0) and non-empty (1) set intersections.

From among the 2^9 =512 possible combinations of empty and non-empty, a set of sixteen cyclic interval relations are realized (Figure 4). These relations are qualitative in nature as they do not capture any information, for example, about the cycle's periods, the lengths of the intervals, or the amount of overlap. We discuss the completeness of this set in section 3.3.

The matrices capture valuable information about the comparison of the relations. First, matrices that are mirror images along the main diagonal identify symmetric relations. This holds true for relations *disjoint*, *meets_twice*, *equals*, and *overlaps_twice*. Second, pairs of matrices that are identical if one matrix is transposed along the main diagonal identify converse relations. Among the sixteen relations, there are six pairs of converse relations: *meets* and *met_by*; *overlaps* and overlapped_by; passes and passed_by; starts and started_by; finishes and finished_by; and contains and contained_by.



Fig. 4. The sixteen cyclic interval relations with their corresponding intersection matrices. Orientation is clockwise

3.2 Conceptual Neighborhoods

The sixteen cyclic interval relations can be grouped according to their conceptual neighborhoods. Conceptual neighborhoods capture the similarities among the sixteen relations by linking those relations that are connected by an atomic change [21, 22]. Such a change is a single movement of one interval's start or end point from the other interval's boundary into its interior or exterior, or vice versa, moving the start or end point from the interior or exterior onto the boundary. Based on a computational model similar to that for topological line-region relations [23], the full set of all possible movements has been determined. It leads to a graph that corresponds to a lattice (Figure 5).

This regular figure is an indication that no relations located in the interior were missed. Further examination of the borders along the top and the bottom of the neighborhood graph (Section 3.3) demonstrate that the set of sixteen cyclic interval relations is actually complete, provided A and B are a true subset of IR^1 and none of their interiors are empty.

The conceptual neighborhood graph exposes some interesting properties. Beginning with the case where two cyclic intervals are separate (*disjoint*), all diagonal rows of relations that run from the top left to the bottom right of a diagonal (e.g., from *disjoint* to *contains*) are formed by moving the end of the outer cyclic interval in a clockwise direction. Diagonal rows of relations that run the opposite way—from the top right to bottom left of a diagonal (e.g., from *disjoint* to *overlapped_by*)—are formed by moving the start of the outer cyclic interval counterclockwise. Taken together, these relations form a double-diamond shape. *Overlapped_by* is drawn twice to demonstrate the regularity of the structure. Based on this grouping, each relation is

at least the conceptual neighbor of two relations (cases *disjoint*, *contained_by*, *overlaps_twice*, and *contains*) and at most four other relations (cases *overlapped_by*, *meets_twice*, *overlaps*, and *equals*).



Fig 5. Planar projection of the conceptual neighborhood graph, with relation *overlapped_by* drawn twice to demonstrate the regularity

3.3 Completeness of Set of Sixteen Cyclic Interval Relations

The analysis of the conceptual neighborhood graph already illustrated the underlying regularities along the diagonals. We use this pattern to demonstrate that any relations located along the fringes of the graph require cases of the intervals, which would violate at least one of the properties of a cyclic interval.

If one extends the diagonals beyond the border of the conceptual neighborhood graph, one provides 20 opportunities for additional relations (labeled A through T in Figure 6). If there was another cyclic interval relation, then it would have to be connected to the graph and would have to be located within the 20 slots. Four of these links point to existing cyclic relations (H to *met_by*, I to *passed_by*, R to *started_by*, and S to *finishes*); therefore, they can be discarded. From the regularity along the *disjoint-contains* diagonal—moving the end of the outer cyclic interval in a clockwise direction—it follows that K would be the relations can be realized for the cases O, J, and N (Figures 7b-d). Along the same diagonals the cases A, E, D, and T can be evaluated with the reverse information—from bottom right to top left, moving the end of the outer cyclic interval in a counterclockwise direction. This sequence implies for the four slots that the outer interval must collapse to a single point either in the outside of the inner interval (slot A, Figure 7e), in the inside (slot E, Figure 7f), or on the two boundaries (slot D, Figure 7g; and slot T, Figure 7h).

The corresponding analysis can be performed along the diagonals from top right to bottom left. It reveals that cases L, M, P, and Q would be occupied by relations with complete outer cycles, while cases B, C, F, and G would require the outer cycle to collapse to a single point. Since none of the twenty cases are occupied by a new interval relation, the set of sixteen (Figure 4) covers all possible cyclic interval relations.



Fig. 6. Set of cyclic interval relations plus cases where interval has been collapsed to a point or extended to a full cycle. Orientation is clockwise



Fig. 7. Additional cases of relations where the outer cyclic interval is extended to a full cycle with a start or end coinciding with (a) the outside of the inner interval, (b) the inside of the inner interval, (c) the start, and (d) the end, or the outer interval is collapsed to a single point (e) in the outside of the inner interval, (f) in the inside, (g) on the start boundary, and (h) on the end boundary

4 An Example

Cyclic relations are useful for reasoning about scenarios of change, for example, land use changes (Figure 8). Four different uses of land (timbering, fishing, hunting, and fruit gathering) vary cyclically over time with each cycle being one year in length. The orientation of the cycle of land use change is clockwise. The intervals representing the different land uses can be compared to one another (Figure 9). The interval for hunting, for instance, meets the interval for fishing at one end while overlapping at the other end. Both the ends of the interval for fishing overlap with the ends of the timbering interval.



Fig 8. Changes in land use: Land use includes timbering (October through mid-April), fishing (March through November), hunting (October through March), and fruit gathering (July through August).



Fig. 9. Comparison matrix for cyclic land use change.

5 Conclusions and Future Work

Many phenomena, such as tides, beach erosion, and monsoons, change in a cyclic fashion. The semantics associated with cycles, however, have yet to be incorporated in conceptual data models. Current spatio-temporal data models are based on a linear model of time assuming total ordering and do not offer explicit support for cycles. If users know *a priori* that their data vary cyclically, this information needs to be captured in a database and needs support for queries on cyclic-based intervals, such that any cyclic variation is explicitly returned.

A formalism of cycles based on cyclic intervals and the relations between these intervals distinguishes a set of sixteen cyclic interval relations, not including cases of full or empty cycles. This systematic derivation shows that there are more than the nine relations identified in Frank [19]. Analysis of the conceptual neighborhood graph demonstrates that these sixteen relations are complete, such that no relation exists between the nodes of the graph.

Study of the complete set of cyclic relations including the cases of empty and full cycles is underway. This work will include analysis of the conceptual neighborhoods associated with the complete set of cyclic relations. To enable more comprehensive cyclic reasoning it is necessary to establish the composition of the sixteen cyclic relations (e.g., *A meets_twice B* and *B contained_by C* implies *A overlaps_twice C*). Based on a method used for determining the composition of topological relations in IR^2 [24], we will derive all 256 compositions for the cyclic relations. Of particular interest will be the crispness of these compositions as compared to the crispness of the compositions for linear intervals [18]. Further extensions to the model are also possible, for example, future work will include extending the model to accommodate cycles with different period lengths.

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