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Intangible Capital in a Real Business Cycle Model

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1. INTRODUCTION

A number of existing studies have attempted to identify the drivers of economic growth in knowledge-based economies. Recent studies have started to closely examine the role of intangible capital. Most existing studies however focus on measuring the size of intangible capital in an economy. For instance, Corrada, Hulten and Sichel (2006) suggest that intangible capital is not fully included in the published US data. They argue that the observed pattern of growth in the US economy can be better explained once intangible capital is accurately measured and taken into account. McGrattan and Presscott (2010) argue that neo-classical growth theory fits well to the post-war US data prior to 1990. But in 1990s it predicts depressed economy while US economy was booming. They argue that this inconsistency disappears once investment in intangible capital is taken into account.

This paper utilizes a theoretical model that allows one to examine the role of intangible capital. Specifically, we introduce intangible capital into a standard real business cycle (RBC) model. Firms expend resources to create intangible capital, which is an additional input in the production process. Within the context of a standard RBC model, firms do not earn abnormal profits as the market structure is assumed to be competitive and production technology exhibits constant returns to scale. A distinctive feature of the model used in this paper is that firms earn abnormal profits despite operating in a perfectly competitive environment. As volatility in corporate profits earned by firms is an important feature of US business cycles, the model used in this paper is more realistic. A similar framework is used by Hou and Johri (2009).

The theoretical model utilized in this paper allows one to address some interesting issues. For example, is the investment in intangible capital pro-cyclical? Do firms prefer investment in intangible over investment in tangible capital? In response to a permanent or transitory productivity shock, do firms accumulate more intangible capital? How do firms behave in response to a preference shock?

The analysis presented in this paper suggests that investment in intangible capital is procyclical. We find that in response to a permanent or transitory productivity shock, firms increase their investment in intangible capital. Furthermore permanent technology shock result in higher factor share of labor and capital allocated to create intangible capital which decreases profits in the current period. However, higher investment in intangible capital raises future profits. A permanent technology shock results in a decrease in labor hours.[1]

The paper is structured as follows. The next section gives a brief overview of the literature. Section 3 presents the RBC model with intangible capital, steady-state and dynamic solution, and calibration. Section 4 discusses the results from effects of transitory and permanent technology shocks, and preference shocks. Section 5 presents the conclusion. The paper also includes three appendixes.[2] Appendix A discusses the effect of trend stationary technology shock; Appendix B shows impulse responses for normalized variables and Appendix C do the sensitivity analysis for three important parameters.

[1] Gali (1999), Basu, Fernald and Kimball (2006), Francis and Ramey (2005) and Galí (2005) suggest a permanent technology shock decreases labor hours.
[2] This online appendix is available at:
http://lums.edu.pk/shssl/economics/content/economics-working-papers.

2. LITERATURE REVIEW

Approximately, 78% of the world's wealth is attributed to intangible capital (World Bank, 2005). In developing nations, intangible capital accounts for 59% of the wealth, while in OECD countries this share is approximately 80%. A number of empirical studies have attempted to estimate the impact of intangible capital on economic growth. For example, Prescott and Parente (1994) argued that barriers to technology adaption can cause slower economic growth. They conclude that a large unmeasured investment exists in the business sector, which can be viewed as technology adoption investment or intangible capital investment. They estimated this unmeasured investment to be equal to approximately 40 percent of the measured output. Prescott (1998) suggests that investment in intangible capital is well above the assumed/implied 32% of the measured output.

Recent studies such as Corrado, Hulten and Sichel (2006) (from here on CHS) suggest that investment in intangible capital in the US for the period of 1998 to 2000 was approximately 12% of GDP. There seems to be no disagreement concerning the importance of investment in intangible capital. However, there is no agreement on the size of intangible capital within an economy. Both micro and macro-economist have found evidence to support the presence of investment in intangible capital.[3] It is perhaps worthwhile to look at the precise definition of intangible capital investment. CHS (2006) definition identifies three important categories– computerized investments, innovative property investments and investment in economic competencies. The computerized investment is embedded in computer programmes and computerized databases, innovative property reflects the scientific knowledge embedded in patents, licenses and general know-how (CHS 2006). CHS (2006) define economic competencies category of intangibles as "the value of brand names and other knowledge embedded in firm-specific human and structural resources". It comprises of expenditures on advertising, market research, firm-specific human capital and organizational change. The above definitions suggest that investment in intangible capital stimulates productivity growth through provision of knowledge, increase in sales of a product and development of processes and a productive environment for actual physical production of a good. In other words, due to investment in intangible capital, products and services become more knowledge-intensive (Corrado et al. 2009).

The concept of intangible capital is also explored in the organizational capital literature. Within the context of this literature, a firm's accumulation of unobserved input can be attributed to learning by doing or learning on the job (Lucas 1993). Cooper and Johri (2002) and Johri (2009) using dynamic general equilibrium models have further explored this idea. However, the two approaches differ in that organizational capital is a by-product of production whereas intangible capital is created by allocating resources to its production.

Human capital is a type of intangible capital. Earlier empirical studies, for instance Mankiw et al. (1992), suggest that an extended version of the neoclassical growth model (i.e., the model that includes human capital) can explain most variation in the cross-country output. Other studies, for example Klenow and Rodriguez-Clare (1997), Hall and Jones (1999), Hulten and Isaksson (2007) and Hashmi (2013), argue that one also needs to consider other factors to fully understand disparity across country output.

Danthine and Jin (2007) argue that if intangible capital translates into capital stochastically then plausible levels of macroeconomic volatility are compatible with highly variable corporate

[3] For microeconomics based studies, see Brynjolfsson et al. (2002) and the references therein. For macroeconomic perspective, see McGrattan and Prescott (2010).

valuations, price earnings ratios and returns. Jinnai (2009) suggests that intangible capital producing technology is important for medium-frequency business cycle. Perli and Sakellaris (1998) examined the implications of human capital for economic fluctuations. Similarly, Jones, Manuelli and Siu (2005) examined the properties of business cycle within the context of an endogenous growth model with human capital accumulation. Comin and Gertler (2006) examined the intellectual antecedents of medium-term business cycles. By including intangible capital in the final good production function, Hou and Johri (2009) showed that profits are not only more volatile than output but also positively correlated with output. Pyo, Chun and Rhee (2012) examined the rising proportion of intangible capital and economic growth in the US, Japan and Korea. Borgo, Goodridge, Haskel and Pesole (2012) measured the contribution of intangible capital to growth from the UK from 2000-2008. They found that intangible capital contributed to 23 percent of labor productivity growth.

In summary, a number of empirical studies have examined various aspects of investment in intangible capital. These studies highlight the importance of intangible capital in real economies. In the next section, we utilize a model where intangible capital appears as an additional input into the aggregate production function. Within the context of a RBC model, we examine the role of investment in intangible capital.

3. THE MODEL

In this section, we utilize a dynamic general equilibrium model where all agents operate within a competitive market environment.

3.1 Households

Consider a representative infinitely lived household who maximizes his/her lifetime utility as follows:

$$\max \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1-N_t, B_t) \qquad (1)$$

where β is the discount factor, C_t is the consumption in current period, N_t is the labor supply and B_t is a measure of preferences shock that follows a first-order autoregressive process with an *iid* error term

$$\ln B_t = \rho_b \ln B_{t-1} + \varepsilon_{bt} \qquad (2)$$

The utility flow function is as follows:

$$U(c_t, N_t) = \ln c_t + B\eta(1-N_t) \qquad (3)$$

Equation (3) takes into account labor-leisure trade-off, where leisure in period t is $L_t = 1 - N_t$. In each period, the representative household supplies labor and physical capital to the firm, given the wage rate w_t and the rental rate on capital, r_t. In addition, as the owner of the firm, the household receives real profits earned by the firm, . The budget constraint for the household utility maximization problem is given by

$$C_t + I_t = w_t N_t + r_t K_t + \pi_t \qquad (4)$$

where I_t is investment which augments capital stock over as follows:

$$K_{t+1} = I_t + (1-\delta)K_t \qquad (5)$$

where δ (0,1) is the rate of depreciation rate of physical capital.

Equations (4) and (5) can be combined as follows:

$$C_t + K_{t+1} = w_t N_t + r_t K_t + (1-\delta)K_t + \pi_t \qquad (6)$$

Given the initial values, household chooses (C_t, N_t, K_{t+1}) to maximize the utility function subject to budget constraint given by equation (6). The dynamic programming problem can be written as follows:

$$V(k) = \max_{c,n,k'} E\left[\ln c + B\eta(1-N) + \beta E V(k')\right]$$

$s.t.$

$$C + K' = w\, N + r\, K + (1-\delta)K + \pi$$

The first order optimality conditions are as follows:

$$U_c = \lambda \qquad (7)$$

$$\frac{B\eta}{1-N} = w\lambda \tag{8}$$

$$\beta EV(\kappa') = \lambda \tag{9}$$

$$V_k = (r + ((1-\delta))\lambda \tag{10}$$

Equations (7) to (10) can be reduced to the following equations:

$$w = \frac{cB\eta}{1-N} \tag{11}$$

$$1 = \beta E\left\{\frac{c}{c'}\left(r' + (1-\delta)\right)\right\} \tag{12}$$

3.2 Firms

The representative firm operates under conditions of perfect competition. The output is produced using labor, physical capital and intangible capital under constant returns to scale technology:

$$Y_t = e^{\ln\theta_t}(A_t s_{tN} N_t)^{\phi}(s_{tK} K_t)^{1-\phi-\tau} Z_t^{\tau} \tag{13}$$

where N_t , K_t and Z_t are labor, physical capital and intangible capital, respectively.

The presence of the intangible capital distinguishes this model from a typical business cycle model. Intangible capital is produced by means of labor and capital. s_N and s_K denote respectively the fraction of labor and physical capital, which firm allocates to produce the final good. Hence, the remainder of the labor and physical capital, i.e., $(1-s_N)$ and $(1-s_K)$ are used to produce intangible capital. The permanent technology shock, A_t , is assumed to follow a random walk with persistence parameter ρ_a and drift process v_a:

$$\ln A_t = v_a + \rho_a \ln A_{t-1} + \varepsilon_{at} \tag{14}$$

where ε_{at} are *iid* shocks.

The transitory productivity shock is given by a first-order autoregressive process with an *iid* error term:

$$\ln\theta_{t+1} = \rho_\theta \ln\theta_t + \varepsilon_{\theta,t+1} \tag{15}$$

The stock of intangible capital in period *t*+1 depends on labor, physical capital and intangible capital in the current period as follows:

$$Z_{t+1} = \left[\left(A_t(1-s_{tN})N_t\right)^{\mu}\left((1-s_{tK})K_t\right)^{1-\mu}\right]^{1-\omega} Z_t^{\omega} \tag{16}$$

The parameter ω has an interesting interpretation. This captures the idea that knowledge depreciates with time as economic environment passes through various transformations.[4] $\omega \in (0,1)$, which implies that contribution of the past intangible capital decays the further back in time it was created. $\omega = 1$ implies that intangible capital is constant over time. While $\omega=0$ implies that intangible capital present in the current period makes no contribution to the intangible capital in the next time period. The productivity shock A_t appears in the intangible evolution equation to ensure a balanced growth path where increase in labor productivity over time occurs in both the final good and intangible capital good sectors. $\mu(1-\omega)$ is the elasticity of labor hours that are used in the current period to create intangible capital with respect to intangible capital in the next period. If $\mu = 1$, means physical capital is not used in the creation of intangible capital. When $\mu = 0$, firm allocates labor only to final goods production and no labor is used in the creation of intangible capital.

3.3 Normalization

In order to solve for the steady state and to compute the decision rules for the representative agent in this economy, the model needs to be normalized. The normalized system of equations is as follows, where tilde is placed over each of the normalized variables.

$$C + a'K' = wN + rK + (1-\delta)K + \pi$$

$$\tilde{I} = a'K' - (1-\delta)K$$

[4] This idea is consistent with the notion of organizational forgetting explored in Benkard (2000) and the depreciation of organizational capital discussed in the learning literature

$$a' = \beta E\left\{\frac{C}{C'}\left(r' + (1-\delta)\right)\right\}$$

$$Y = e^{\ln\theta}\,(s_N N)^{\phi}\,(s_K K)^{1-\phi-\tau} Z^{\tau}$$

$$Z'a' = \left[\left((1-s_N)N\right)^{\mu}\left((1-s_K)K\right)^{1-\mu}\right]^{\omega} Z^{\omega}$$

3.4 Firm optimization

In each period, the firm maximizes the present value of its real profits subject to the production function for output and intangible capital; i.e., equations (13) and (16). The firm optimization problem can be written as follows:

$$V(Z) = \underset{N,K,s_N,s_K,z'}{Max}$$

$$E\left[U_c\left[(s_N N)^{\phi}(s_K K)^{1-\phi-\tau} Z^{\tau} - wN - rK\right] + \beta V(Z')\right]$$

$$+\lambda\left[\left[\left((1-s_N)N\right)^{\mu}\left((1-s_K)K\right)^{1-\mu}\right]^{\omega} Z^{\omega} - Z'a'\right]$$

The first order conditions are as follows

$$\beta E(V_z') = \lambda a' \qquad (17)$$

$$wU_c = U_c\left[\phi\frac{Y}{N}\right] + \lambda\left[\frac{\mu(1-\omega)\,a'Z'}{N}\right] \qquad (18)$$

$$rU_c = (1-\phi-\tau)\left[\frac{Y}{K}\right] + \lambda\left[\frac{(1-\mu)(1-\omega)\,a'Z'}{N}\right] \qquad (19)$$

$$U_c\left[\frac{\phi Y}{s_N}\right] = \left[\frac{\lambda\mu(1-\omega)a'Z'}{1-s_N}\right] \qquad (20)$$

$$U_c\left[\frac{(1-\phi-\tau)Y}{s_K}\right] = \left[\frac{\lambda(1-\mu)(1-\omega)a'Z'}{1-s_K}\right] \qquad (21)$$

The envelope condition is as follows:

$$V_z = U_c\left[\tau\frac{Y}{Z}\right] + \lambda\left[\omega a'\frac{Z'}{Z}\right] \qquad (22)$$

Substituting equation (22) in equation (17) results in

$$\beta\left[U_c\left[\tau\frac{Y'}{Z'}\right] + \lambda'\left[\omega a''\frac{Z''}{Z'}\right]\right] = \lambda a' \qquad (23)$$

Since the model includes intangible capital and not all labor and capital is used in the production of the final good, the marginal productivities of labor and capital do not equal the respective factor prices. In fact within the context of our model, factor prices are higher than the marginal productivities. In the absence of intangible capital all inputs will be used solely in the production of the final good. Equations (18) and (19) suggest that firm allocates capital and labor to the production of intangible capital in a way that marginal decrease in the output of the final good offsets the marginal increase in intangible capital available to the firm. Using equation (20) in (18) and equation (21) in (19) results the following equations.

$$w = \phi\frac{Y}{s_N N} \qquad (24)$$

$$r = (1-\phi-\tau)\frac{Y}{s_K K} \qquad (25)$$

It is clear from the above equations that factor prices exceed their marginal product in the final good production.[5] Equation (23) represents the marginal value of an extra unit of intangible capital. Investment in intangible can be described by equation (26) as follows:

$$I_z = wN\left(1-s_N\right) + rK\left(1-s_K\right) \qquad (26)$$

Using equations (24) and (25), equation (26) can be written as follows:

$$\frac{I_z}{Y} = \left(\frac{1}{s_N} - 1\right)\phi + \left(\frac{1}{s_K} - 1\right)(1-\phi-\tau) \qquad (27)$$

It is clear from the above relationship that investment in intangible capital is increasing in output and decreasing in shares of factors of labor and capital. The presence of intangible capital leads to profit as follows:

$$\pi = Y - wN - rK \qquad (28)$$

[5] Labor and capital share act as a time varying wedge between factor prices and marginal products.

Using equations (24), (25) and (27) in equation (29) leads to the following relationship between profit and intangible investment

$$\pi = \tau Y - I_z \tag{29}$$

The above shows a trade-off between current period profit and investment in intangible capital.

3.5 Steady State

In the steady state all variables are constant in different time periods, i.e., $C'=C$, $Y'=Y$, $K''=K'=K$, and $Z''=Z'=Z$, etc. Accordingly, household Euler equation (12) can be solved for optimal value of r as follows:

$$\bar{r} = \frac{a}{\beta} - (1-\delta) \tag{30}$$

Using equation (23) we get

$$\lambda a Z = \beta\tau \frac{U_c Y}{1-\beta\omega} \tag{31}$$

Whereas, using equation (31) in (18), (19), (20) and (21) we get the following expressions.

$$\bar{w} = \frac{Y}{N}\left[\phi + \frac{\mu(1-\omega)\,\beta\tau}{1-\beta\omega}\right] \tag{32}$$

$$\bar{r} = \frac{Y}{K}\left[(1-\phi-\tau) + \frac{(1-\mu)\,(1-\omega)\beta\tau}{1-\beta\omega}\right] \tag{33}$$

$$\frac{(1-s_N)}{s_N} = \frac{\mu\beta\tau(1-\omega)}{\phi(1-\beta\omega)} \tag{34}$$

$$\frac{(1-s_K)}{s_K} = \frac{\beta\tau(1-\mu)(1-\omega)}{(1-\phi-\tau)(1-\beta\omega)} \tag{35}$$

It is interesting to note that the above steady state solutions of the wage, rental on physical capital, income shares of labor and capital are independent of intangible capital.

The factor share of labor and capital in the steady state can be easily estimated from equation (34) and (35). Using equations (34) and (35) in (27), we get the steady state intangible investment to output ratio as

$$\frac{I_z}{Y} = \frac{\beta\tau(1-\omega)}{1-\beta\omega}$$

The ratio of investment in intangible capital to output depends on two important parameters, i.e., τ and ω, where τ is the key parameter. There is a one-to-one correspondence between intangible investment-output ratio and τ.

From equation (32), labor share of income can be estimated as follows:

$$\frac{wN}{Y} = \left[\phi + \frac{\mu(1-\omega)\,\beta\tau}{1-\beta\omega}\right] \tag{36}$$

Using equation (33), we can solve for capital-output ratio as follows:

$$\frac{K}{Y} = \frac{1}{r}\left[(1-\phi-\tau) + \frac{(1-\mu)(1-\omega)\beta\tau}{1-\beta\omega}\right] \tag{37}$$

The size of parameters μ and τ has a direct bearing on capital to output ratio and labor share of income ratio. As μ increases, the contribution of labor to the creation of intangible capital increases while the contribution of physical capital falls. Since the permanent technology shock is increasing in labor, an increase in μ further increases the ability of the shock to create intangible capital. As τ increases, the share of intangible capital increases in the production of the final good and the share of physical capital decreases. This makes investment in intangible capital more attractive as compared to investment in physical capital, which results in larger investment in intangible capital to investment in physical capital ratio.

In the steady state intangible capital equation can be written as

$$Z = \left[\left((1-s_N)N\right)^{\mu}\left((1-s_K)K\right)^{1-\mu}\right]a^{1/1-\omega} \tag{38}$$

The above equation can be used to eliminate Z from the production function. Since capital-output ratio is already determined, this equation can be used to solve for output of the final good in the steady state. Once output is determined, capital can be calculated. Given the steady state

output, wage can be determined using equation (36). Similarly consumption can be estimated from the goods market equilibrium condition. Profit in the steady state can be estimated using equation (28). Since the steady state values of capital, labor, and income shares of factor of primary inputs are already determined, the stock of intangible capital and investment in intangible capital can be determined using equations (38) and (26) respectively.

3.6 Stochastic Model

This paper employs Christiano method of undetermined coefficient to solve the dynamic model (Christiano 2002). The two Euler equations from the household problem can be used to examine the model dynamics. The system of equations that describe the problem of the firm can be reduced by few substitutions. Using equation (20) in (18), (19), (21) and (23), one can reduce this system of equations to four equations in four unknowns. The four unknowns are capital, intangible capital, income shares of labor and capital. The relationship between the income shares of capital and labor is as follows:

$$s_N = \frac{\phi(1-\mu)\, s_K}{\mu(1-\phi-\tau)(1-s_K)+\phi(1-\mu)s_K} \qquad (39)$$

Equation (39) shows that the income share of labor depends on the income share of capital. Since s_N is known, the output of the final good can be written as $Y = f(K,N,s_N,s_K,Z)$. Wage can be determined as follows:

$$w = \phi\frac{Y}{s_N N} \qquad (40)$$

The above equation suggests that the output of the final good depends on labor and income share of labor; i.e., $w=f(y,N,s_N)$. The rental rate in the dynamic equilibrium can be determined by using equation (41) as follows:

$$r = \frac{Y}{K}\left[(1-\phi-\tau)+\frac{\phi\,(1-\mu)\,(1-s_N)}{\mu s_N}\right] \qquad (41)$$

Equation (41) suggests that rental rate of physical capital is a function of output, capital and factor share of labor; i.e., $r = f(Y,K,s_N)$.

Consumption in a dynamic equilibrium can be determined either using budget constraint or goods market clearing condition. Consumption is a function of output and capital only, i.e., $C = f(Y, K)$.

$$C = Y - a'K' + (1-\delta)K \qquad (42)$$

The dynamic system consists of four equations that could be solved using the feed-back part of the stochastic model.[6] Two equations are based on the household problem and other two are based on the firm problem. These equations are as follows:

$$w = \frac{CB\eta}{1-N} \qquad (43)$$

$$a' = \beta E\left\{\frac{C}{C'}\left(r'+(1-\delta\right)\right\} \qquad (44)$$

$$Z'a' = \left[\left((1-s_N)N\right)^{\mu}\left((1-s_K)K\right)^{1-\mu}\right]^{\omega} Z^{\omega} \qquad (45)$$

$$\beta\left(\frac{CY's_N}{C'Y(1-s_N)}\right)\left[\frac{\mu(1-\omega)\tau}{\phi}+\omega\frac{(1-s_N')}{s_N'}\right]=1 \qquad (46)$$

Equations (43) to (46) are the equilibrium conditions that can be used in our calibration exercise in the next section.

3.7 Model Calibration

Eleven parameters, namely $\beta,\ \eta,\ \delta,\ \rho_b,\ \phi,\ \tau,\ \mu,\ \omega,\ \rho_b,\ \rho_\theta$ and v_a appear in equations (43) to (46). Some of these parameters also appear in standard RBC models that do not include intangible capital. As a part of the calibration exercise, we set the discount rate at β = 0.99. The depreciation of physical capital is taken as δ = 0.022. The elasticity of labor is assumed to be ϕ = .535 whereas the elasticity of capital is assumed to be 0.29. Similar parameter values are used in standard business cycle literature.

[6] The feed-back part characterizes the impact of the endogenous state variables on the current period endogenous variables. For further details, see Christiano (2002).

However, with the inclusion of intangible capital as the third input, we have to use slightly different parameter values. For example, McGrattan and Prescott (2010) assumed income share of capital as 0.26, which is slightly lower. The household intertemporal condition is used to pin down the value of utility parameter, i.e., η = 1.85. The drift in the permanent technology shock is assumed to be $v_a = 0.0034$. The persistent parameters of three shocks are: persistence in permanenttechnology shock, ρ_a = 0.5, persistence is transitory shock, ρ_θ = 0.95 and persistence in preference shock, ρ_a = 0.9767

Since this model includes intangible capital, three parameters are of special importance[7] -i.e., τ, μ and ω McGrattan and Prescott (2010) use a value of τ which is around 7.6 percent, which is somewhat lower than the other studies.[8] They assumed that input elasticities for producing both final goods and intangible capital goods are identical. They used information on elasticities along with the information on NIPA compensation to pin down all capital shares. The model presented in the present paper is slightly different from the one suggested by McGrattan and Prescott. Our elasticities are different for final goods production and production of stock of intangible capital in next period. Hashmi (2013) uses a value of 0.13, which is more close to Corrado et al. (2006). Jinnai (2009) chooses a value of 0.27 in an asset pricing intangible capital model. Hou and Johri (2009) estimated τ = 0.173 by assigning tight priors, because the value of τ is sensitive to steady state ratios. We assume this value to be 17 percent, which is the average of the values that are used in the existing literature.

The second important parameter, μ is generally assumed to have a higher value, which means more labor is allocated to produce intangible capital than physical capital. McGrattan and Prescott (2010) estimated μ = 0.646. Again, McGrattan and Prescott (2010) assumed that the elasticity of labor is same across both production technologies. However, in this paper we use different values. One would expect that the contribution of labor would be higher in producing intangible capital. In the existing literature, the value used ranges from 0.5 to 0.85. Hou and Johri (2009) estimated a value close to 0.84. The value of μ is sensitive to the steady state ratios. In this model, we assumed this parameter to be 0.85, which implies that income share of capital in intangible capital production is much lower than labor. This also implies that more capital is used in the production of the final good, which is likely to be the case. The last important parameter is ω . The existing literature offers very little guidance in this case. Hou and Johri (2009) estimated the value of ω=0.592. Their estimated standard deviation on this parameter is 0.075. By analyzing the variation in the mean value, using standard deviation, we use a value which is close to the one that is used by Hou and Johri.

The steady state ratios are not significantly different from the standard model. However, there are some ratios which are of particular importance to our model. Intangible investment to output ratio, and investment in intangible capital to investment in physical capital ratio are two such examples. In the existing literature, a number of different values are used. For example, see McGrattan and Prescott (2010), Corrado et al, 2006 and Hou and Johri (2009). We assume intangible investment to output ratio of 0.16. The investment in intangible capital to investment in physical capital ratio is assumed to be 0.68, which is higher than 0.42 assumed by McGrattan and Prescott (2010), but lower than the value assumed by Corrado et al. (2006) and Hou and Johri (2009).

4. RESULTS

This section examines the impulse response functions to analyze how the economy responds to three types of shocks in an RBC model with

[7] For the sensitivity analysis of three important parameters, see Appendix C.
[8] McGrattan and Prescott used 1990 levels of U.S. data to obtain estimates

intangible capital.

4.1 Dynamic Responses to a Transitory Shock

The standard RBC predicts that as a result of positive transitory technology shock, hours worked, capital stock, output, consumption, investment and labor productivity all go up. This type of shock is the key driving force of business cycles, which results in strong positive correlation between hours worked and labor productivity in the data. As a result of a temporary technology shock, all variables rise above their long-term trends and hence formulate into a positive deviation. In the standard RBC model, firms are producing in perfectly competitive market, so profits are essentially zero.

The distinguishing feature of this model from standard RBC is that it includes intangible capital and positive profits. The impulse response from a positive transitory technology shock in an RBC model with intangible capital is shown in Figures 1, 2 and 3. The solid line is the steady state path while the dotted line is the respective impulse response. A transitory technology shock leads to an increase in hours worked and decrease in leisure. Furthermore, it increases consumption, investment, capital stock and output as shown in Figure 1. The transitory shock also increases investment in intangible capital and so does the factor shares allocated for the creation of stock of intangible capital. The transitory shock also results in positive profits as shown in Figure 2. The effect of the shock dies off since it is transitory shock. The impulse responses die off slowly since the persistence level is high, i.e., $\rho = 0.95$.

The impulse response for the normalized series is shown in Appendix B. The solid line is the balanced growth path while the dotted line represents the movement of the respective variables. It is clearly shown that shock increases initially, but all series revert to the balanced growth path.

4.2 Dynamic Effects of Permanent Technology Shock

The dynamic effects of a permanent technology shock with persistence are shown in Figure 4. The initial effect of productivity shock results in decrease in hours, as depicted in Figure 6, which implies people are willing to substitute current labor hours for leisure. More leisure results in higher consumption. Since people tend to smooth consumption over time, investment would be more volatile than consumption. Due to permanent technology shock income effect seems to be dominating the substitution effect. Higher current consumption leads to lower investment and capital stock, as shown in Figure 4. After an initial decrease, capital stock grows above the balance growth path as shown in Figure 5. Similarly investment in physical capital decreases after a shock before reverting to a long-run growth path as depicted in Figure 4.

The inclusion of intangible capital produces interesting results. Since firms have incentive to invest in intangible capital more labor and capital are used in the creation of intangible capital. This raises the investment in intangible capital as shown in Figure 4. However, higher investment in intangible capital comes at the cost of lower current profits since there is a trade-off between profits and intangible capital. Furthermore, higher investment in intangible capital leads to higher profits in the future.

The factor share of labor (s_N) and the factor share of capital(s_K) in production are decreasing as shown in Figure 5 and Figure 6, respectively. Since firms are allocating more labor, $(1-s_N)$, and capital, $(1-s_K)$, to produce intangible capital, the factor shares are increasing as shown in Figure 8. The total contribution of labor and capital in output production can be calculated by multiplying their respective factor shares

[9] Appendix A has a discussion on how the economy responds to a trend stationary transitory shock.

with labor and capital, i.e., ($s_N \times N$) for labor, and ($s_K \times K$) for capital. Similarly, the contribution of both inputs in the creation of intangible capital can be estimated as follows: ($(1-s_N)N$)=labor share and ($(1-s_K)K$) = capital share.

The RBC model without intangible capital fails to produce effective mechanism to propagate shocks overtime because output growth is positively auto-correlated in the short-run and weakly auto-correlated in the long-run (Cogley and Nason (1995). The RBC model with intangible capital propagates shocks since firms respond to an increase in productivity by investing more in intangible capital consequently raising future productivity.

A better way to check the consistency of the RBC model with intangible capital is to look at the effect of permanent technology shocks on consumption. The permanent shock does increase consumption, which is consistent with the data as shown in Figure 4.

4.3 Dynamic Effects of Preferences Shock

Since preference shock enters the household decision in a way that they value leisure, therefore, as a result of a shock people consume more leisure and work less hours. The impulse responses from a preference shock are shown in Figure 7. Here, substitution effect is dominating since to dampen the effect of more leisure they cut back on consumption. Lower consumption results in lower output. However, in order to absorb the effect of lower output, more labor and capital are allocated to current production relative to intangible capital. The increase in relative contribution of factor shares of labor and capital in production decreases the investment in intangible capital which further decreases the stock of intangible capital in the next period. However, greater contribution in production results in increase in profits as shown in Figure.7. Remember there is a trade-off between profits and investment in intangible capital.

The labor and capital share in production and intangible capital along with the effect of preference shock on labor and capital is shown in Figure 8 and Figure 9, respectively.

5. CONCLUSION

This paper extends a standard RBC model to include intangible capital as an additional input in the production process. A key prediction of the model is the procyclical nature of investment in intangible capital. Intangible capital is found to play a significant role in producing endogenous movements in productivity. Within the context of this paper, investment in intangible capital increases as a result of both transitory and permanent shocks. However, as a result of transitory shock, the increase in investment in physical capital is greater than the investment in intangible capital. Following a transitory shock, as compared to investment in intangible capital, firms allocate more labor and capital to the production of the final good. A trend stationary productivity shock also produces the same result. On the other hand, permanent productivity shock leads to an increase in investment in intangible capital. In addition, firm profit increases after a transitory technology shock. But a permanent technology shock reduces firm profit. This follows from the fact that firms are investing in intangible capital which reduces current period profit. However, this investment leads to higher profits in the long-run. We find that a permanent productivity shock reduces employment but the impact of a transitory shock on employment is positive. The higher level of persistence in the permanent shock leads to a stronger negative employment effect. The RBC model with intangible capital is also successful in propagating shocks over a longer-horizon since firms acquire more intangible capital in response to an increase in productivity, which further raises productivity in the future. In addition, firms have an incentive to invest in intangible capital in the current period to raise future earnings.

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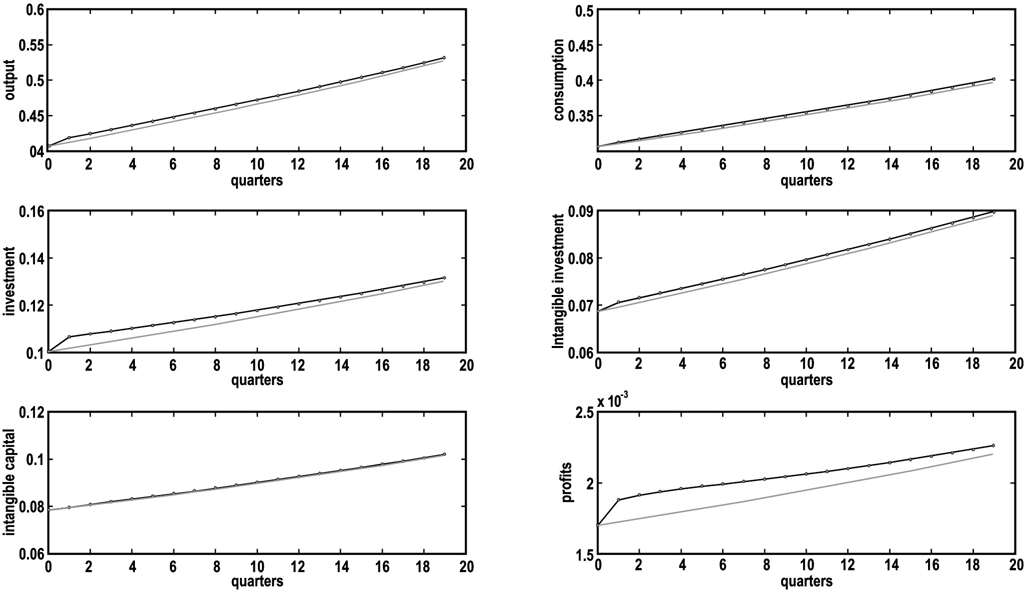

Figure 1: Impulse Responses to Transitory Technology Shocks

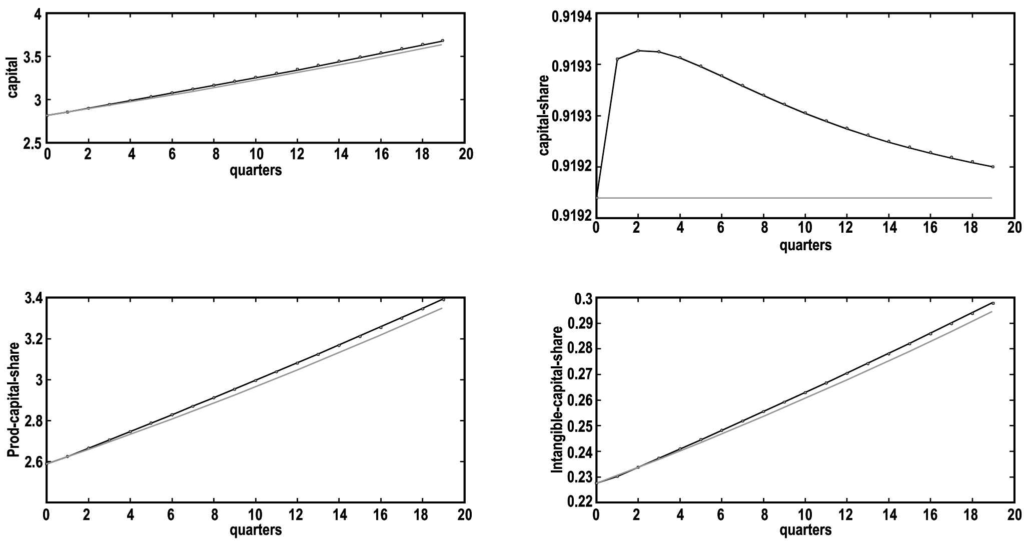

Figure 2: Impulse Responses to Transitory Technology Shocks

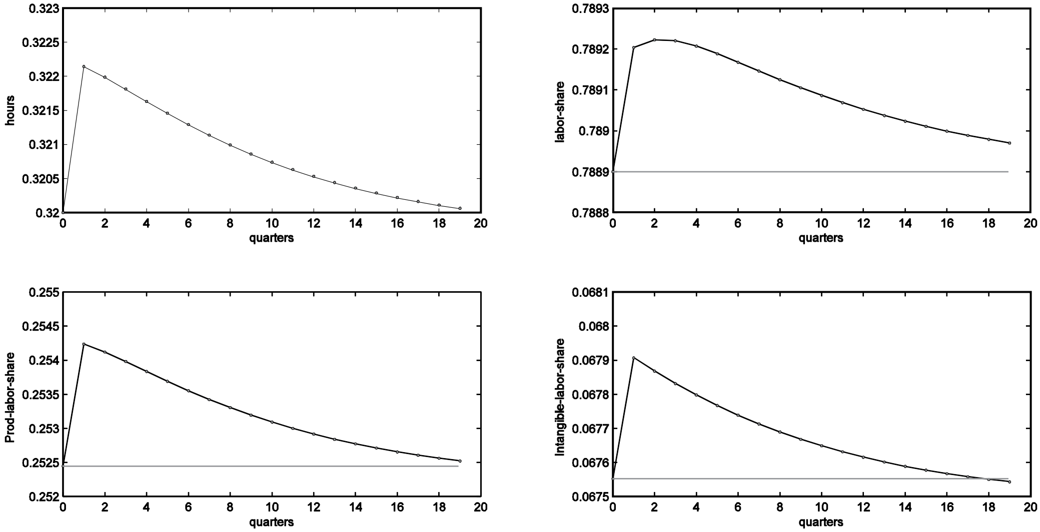

Figure 3: Impulse Responses to Transitory Technology Shocks

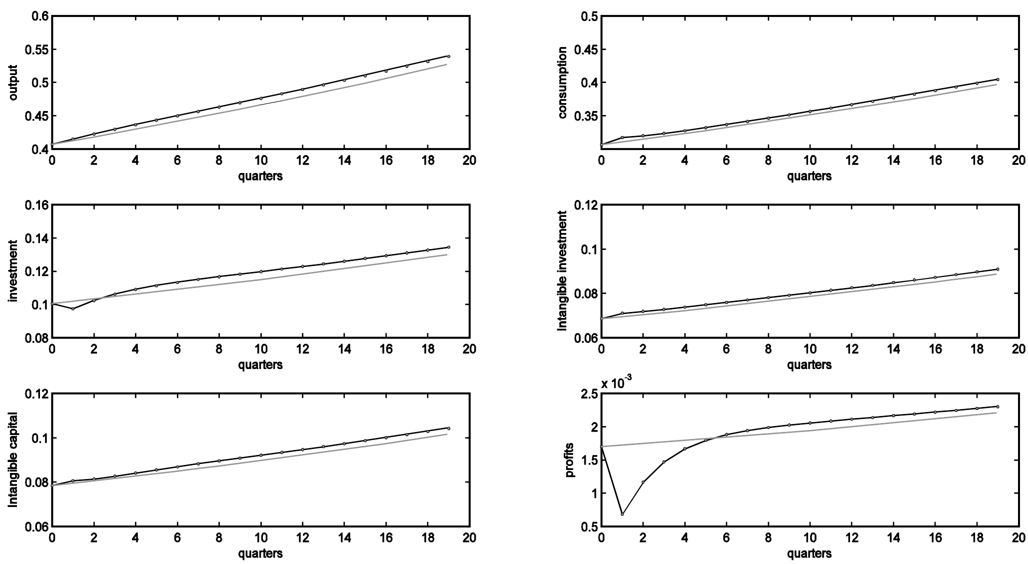

Figure 4: Impulse Responses to Permanent Technology Shocks

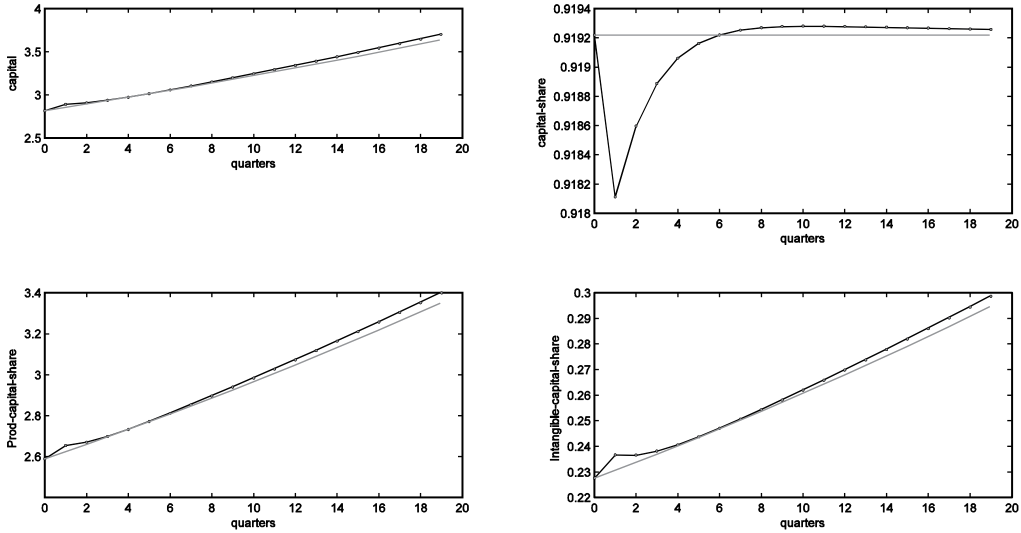

Figure 5: Impulse Responses to Permanent Technology Shocks

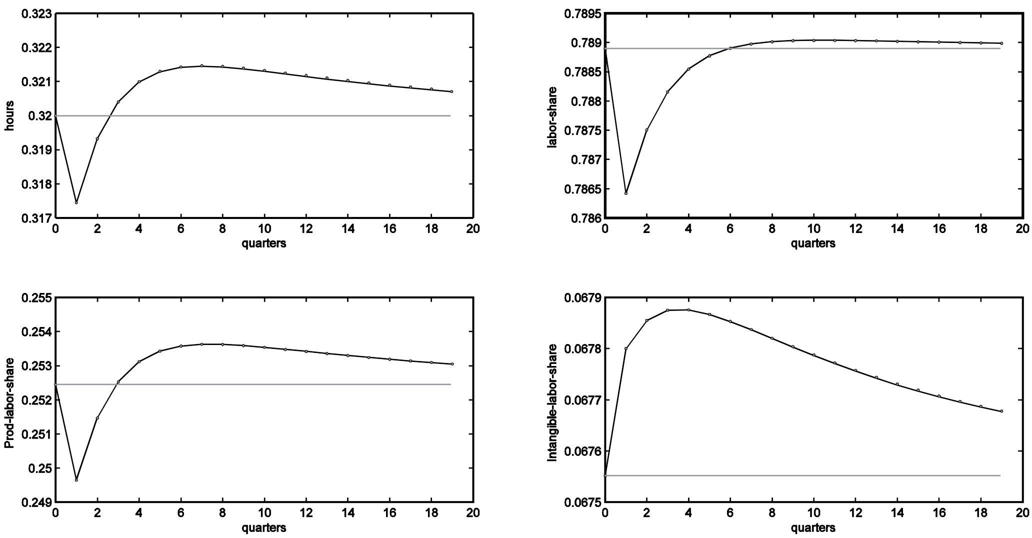

Figure 6: Impulse Responses to Permanent Technology Shocks

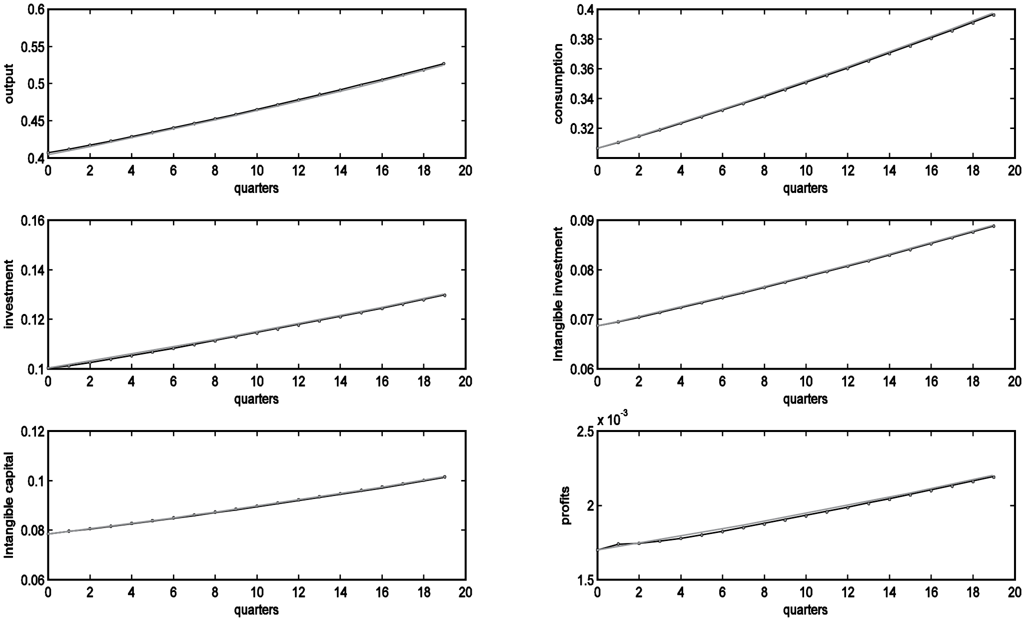

Figure 7: Impulse Responses to Preference Shocks

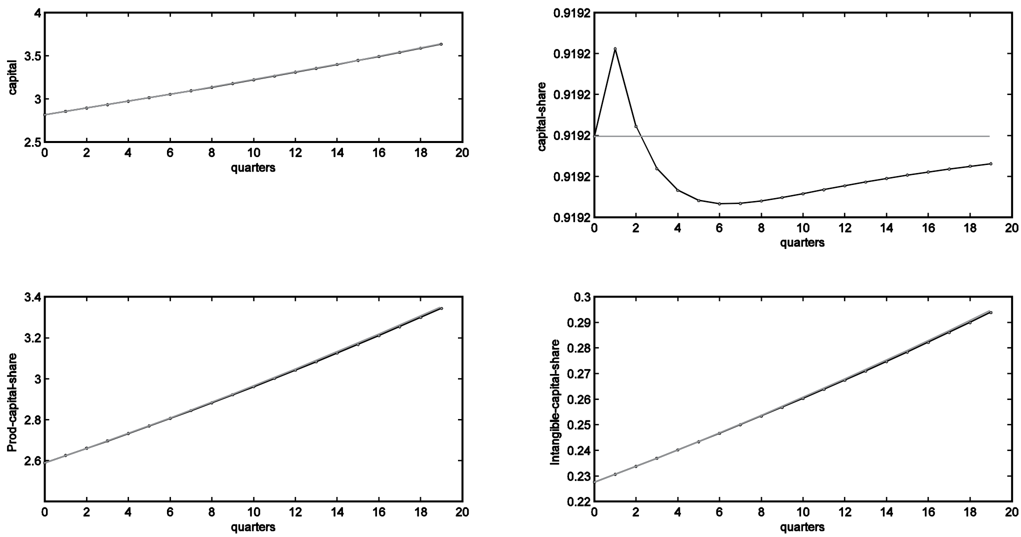

Figure 8: Impulse Responses to Preference Shocks

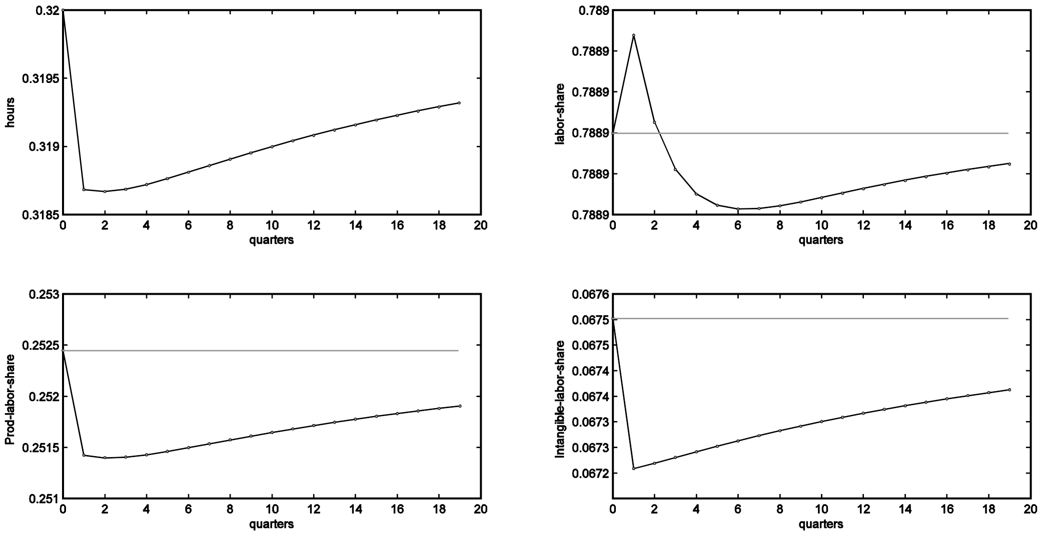

Figure 9: Impulse Responses to Preference Shocks

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Abstract

Recent empirical studies have shown that intangible capital plays an important role in explaining productivity gains that have occurred during the last two decades. By introducing intangible capital in an otherwise standard theoretical real business cycle model, this paper aims to provide a theoretical foundation of the empirical findings. Our results indicate that investment in intangibles is pro-cyclical. Both transitory as well as permanent productivity shocks increase intangible capital. A permanent technology shock increases the income share of labor and capital allocated to the creation of intangible capital which decreases profit in the current period. We find that investment in intangible capital also plays an important role in producing endogenous movements in productivity.

Key Words: Intangible Capital, Real Business Cycle, Productivity Shocks

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