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From AdS/CFT correspondence to hydrodynamics. II. Sound waves

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Abstract: As a non-trivial check of the non-supersymmetric gauge/gravity duality, we use a near-extremal black brane background to compute the retarded Green's functions of the stress-energy tensor in $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory at finite temperature. For the long-distance, low-frequency modes of the diagonal components of the stress-energy tensor, hydrodynamics predicts the existence of a pole in the correlators corresponding to propagation of sound waves in the $\mathcal{N} = 4$ SYM plasma. The retarded Green's functions obtained from gravity do indeed exhibit this pole, with the correct values for the sound speed and the rate of attenuation.

Keywords: AdS/CFT correspondence, thermal field theory.

Contents

1. Introduction

Any finite-temperature medium conducts sound. This simple fact provides a nontrivial method to check the conjecture of gauge theory/gravity correspondence [1] at finite temperatures. Indeed, if one can compute the correlators of the components of the stress-energy tensor $T^{\mu\nu}$ using an AdS/CFT prescription [2, 3], one should recognize in them a pole corresponding to sound-wave propagation, expected from hydrodynamics [4].

The techniques necessary for performing this check have been developed in refs. [5, 6]. In ref. [5] we formulated a prescription for computing the retarded Green's functions from gravity. In ref. [6] we use this prescription to compute the correlators of the shear (non-propagating) modes, and show that the results agree with hydrodynamic expectation. In this paper, we conclude the program by finding the propagating sound waves from gravity.

The paper is organized as follows. In section 2 we discuss the Ward identities and show how leading behaviors of the Green's functions in the infrared are completely determined. Section 3 is devoted to the calculation of the Green's functions from gravity. We show that these correlators have the same form as expected from hydrodynamics. Section 4 contains concluding remarks. The Appendix deals with the gauge invariance of the gravitational action.

2. Field theory (hydrodynamic) picture

Let us first recall (see e.g., section 2 of ref. [6]) that hydrodynamics predicts that some elements of the stress-energy tensor have correlators with a sound-wave pole at

$$\omega(q) = v_s q - \frac{i}{2}\frac{1}{\epsilon + P}\left(\zeta + \frac{4}{3}\eta\right) q^2, \qquad v_s^2 = \frac{\partial P}{\partial \epsilon} \tag{2.1}$$

where ϵ, P η and ζ are the energy density, pressure, shear and bulk viscosities, respectively. In conformal theories $\epsilon = 3P$, $v_s = 1/\sqrt{3}$, and $\zeta = 0$. We shall now show that the infrared form of the correlators relevant for sound waves can be completely found from the Ward identities. Our treatment closely follows ref. [7], with the addition of conformal Ward identities.

To derive the Ward identities it is convenient to temporarily consider the theory in curved space-time and set the space-time to flat at the end. For definiteness, we first consider Euclidean correlators, and perform analytical continuation afterwards. By considering our field theory in a general metric, we can define a generating function $W[g_{\mu\nu}]$ so that the correlators of $T^{\mu\nu}$ can be found by differentiating W with respect to $g_{\mu\nu}$. In particular,

$$\langle T^{\mu\nu}(x)\rangle = \left.\frac{\delta W[g]}{\delta g_{\mu\nu}(x)}\right|_{g_{\mu\nu}=\eta_{\mu\nu}}, \qquad \langle T^{\mu\nu}(x) T^{\lambda\rho}(y)\rangle = \left.\frac{\delta^2 W[g]}{\delta g_{\mu\nu}(x)\,\delta g_{\lambda\rho}(y)}\right|_{g_{\mu\nu}=\eta_{\mu\nu}}. \tag{2.2}$$

Assuming W to be invariant under general coordinate transformations,

$$W[g_{\mu\nu}] = W[g_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu] + O(\xi^2)\,, \tag{2.3}$$

by taking variation over ξ_μ, we find

$$\nabla_\mu \langle T^{\mu\nu}\rangle|_{g_{\mu\nu}} \equiv \nabla_\mu \left\langle \frac{\delta W[g]}{\delta g_{\mu\nu}} \right\rangle = 0\,. \tag{2.4}$$

Putting the metric to be flat, $g_{\mu\nu} = \eta_{\mu\nu}$, eq. (2.4) becomes $\partial_\mu \langle T^{\mu\nu}\rangle = 0$, which is conservation of energy and is trivially satisfied in thermal equilibrium where $T^{\mu\nu}$ is constant. Differentiating eq. (2.4) once more with respect to $g_{\lambda\rho}$, taking into account that ∇_μ depends (via the Christoffel symbols) on the metric, and then setting $g_{\mu\nu} = \eta_{\mu\nu}$, one finds

$$q_\mu \left(G_E^{\mu\nu\lambda\rho}(q) + \eta^{\nu\lambda}\langle T^{\mu\rho}\rangle + \eta^{\nu\rho}\langle T^{\mu\lambda}\rangle - \eta^{\mu\nu}\langle T^{\lambda\rho}\rangle \right) = 0\,, \tag{2.5}$$

where $G_E^{\mu\nu\lambda\rho}(q)$ is the Euclidean Green's function in momentum space,

$$G_E^{\mu\nu\lambda\rho}(q) = \int d^4x\, e^{-iq\cdot x} \langle T^{\mu\nu}(x) T^{\lambda\rho}(0)\rangle\,. \tag{2.6}$$

Performing analytic continuation to Minkowski space, assuming that $G_E(q)$ becomes a Minkowski Green's function $-G(q)$ (i.e., up to a sign change), one finally obtains the following Ward identity:

$$q_\mu \left(G^{\mu\nu\lambda\rho}(q) - \eta^{\nu\lambda}\langle T^{\mu\rho}\rangle - \eta^{\nu\rho}\langle T^{\mu\lambda}\rangle + \eta^{\mu\nu}\langle T^{\lambda\rho}\rangle\right) = 0\,. \tag{2.7}$$

We note here the appearance of the contact terms in eq. (2.7). In the supersymmetric vacuum all these contact terms vanish, hence eq. (2.7) becomes $q_\mu G^{\mu\nu\lambda\rho} = 0$. However, at finite temperatures the contact terms are present.

The conformal invariance of the $\mathcal{N} = 4$ SYM theory leads to an additional Ward identity. Instead of eq. (2.4), one writes

$$\langle T^\mu{}_\mu\rangle \equiv g_{\mu\nu}\left\langle \frac{\delta W[g]}{\delta g_{\mu\nu}}\right\rangle = \mathcal{O}(R^2) \tag{2.8}$$

where $\mathcal{O}(R^2)$ is the conformal anomaly that is quadratic in the elements of the Riemann tensor. Now differentiating eq. (2.8) with respect to $g_{\mu\nu}$, and then putting $g_{\mu\nu} = \eta_{\mu\nu}$, one obtains, after going to Minkowski space,

$$\eta_{\mu\nu} G^{\mu\nu\lambda\rho}(q) = 2\langle T^{\lambda\rho}\rangle\,. \tag{2.9}$$

Again, the contact term in eq. (2.9) vanishes in the supersymmetric vacuum, but is nonzero at finite temperature. Equations (2.7) and (2.9) are the Ward identities we sought to establish.

We now show that the Ward identities can be used to completely determine the forms of the correlators in the hydrodynamic limit. Assuming, without loss of generality, that $\boldsymbol{q}$ is aligned along the z axis, $\boldsymbol{q} = (0, 0, q)$, one can classify the elements of the stress-energy tensor with respect to the O(2) rotation around the z axis. In this paper we are interested only in the components of $T^{\mu\nu}$ that are invariant under this rotation: T^{tt}, T^{tz}, T^{zz}, and $T^{aa} = T^{xx} + T^{yy}$. Therefore, there are 10 independent correlators: G^{AB}, where A and B can be tt, tz, zz, or aa. The diffeomorphism Ward identities are

$$\begin{aligned}
-\omega(G^{tttt} + \epsilon) + qG^{tttz} &= 0\,, & -\omega G^{tttz} + q(G^{ttzz} + \epsilon) &= 0\,,\\
-\omega G^{tttz} + q(G^{tztz} + P) &= 0\,, & -\omega(G^{ttzz} - P) + qG^{tzzz} &= 0\,,\\
-\omega(G^{tztz} - \epsilon) + qG^{tzzz} &= 0\,, & -\omega G^{tzzz} + q(G^{zzzz} - P) &= 0\,,\\
-\omega(G^{ttaa} - 2P) + qG^{tzaa} &= 0\,, & -\omega G^{tzaa} + q(G^{zzaa} + 2P) &= 0\,.
\end{aligned} \tag{2.10}$$

The arguments of G's in eqs. (2.10), as well as in (2.11) below, are ω, q. The conformal Ward identities are

$$\begin{aligned}
-G^{tttt} + G^{ttzz} + G^{ttaa} &= 2\epsilon\,, & -G^{tttz} + G^{tzzz} + G^{tzaa} &= 0\,,\\
-G^{ttzz} + G^{zzzz} + G^{zzaa} &= 2P\,, & -G^{ttaa} + G^{zzaa} + G^{aaaa} &= 4P\,.
\end{aligned} \tag{2.11}$$

The presence of contact terms in eqs. (2.10) implies that the Green's function G obtained from the generating functional $W[g]$ does not concide with the retarded Green's function, which is defined as

$$G_R^{\mu\nu\lambda\rho}(q) = -i \int d^4x \, e^{-iq\cdot x} \, \theta(x^0)[T^{\mu\nu}(x), \, T^{\lambda\rho}(0)] \, . \tag{2.12}$$

To see that, let us first show that $G_R^{t\nu\lambda\rho}(\omega, \mathbf{0}) = 0$. Indeed, according to the definition (2.12),

$$G_R^{t\nu\lambda\rho}(\omega, \mathbf{0}) = -i \int dt \, e^{i\omega t} \langle [P^\nu, \, T^{\lambda\rho}(0)] \rangle \tag{2.13}$$

where $P^\nu = \int d^3\boldsymbol{x} \, T^{t\nu}(t, \boldsymbol{x})$ is the conserved momentum. Now since $-i[P^\nu, \, T^{\lambda\rho}] = \partial^\nu T^{\lambda\rho}$, the right hand side of eq. (2.13) vanishes if translational invariance is respected (as it is for a plasma in thermal equilibrium). Now, by putting $q = 0$, $\omega \neq 0$ into eqs. (2.10), one immediately sees that G cannot coincide with the retarded Green function G_R. Rather, the two should differ by contact terms. If one is interested only in the regime of small ω and q, the leading infrared behavior of some of these contact terms can be found,

$$G^{tttt} = G_R^{tttt} - \epsilon \, , \qquad G^{ttij} = G_R^{ttij} + P\delta^{ij} \, , \qquad G^{titj} = G_R^{titj} + \epsilon\delta^{ij} \, . \tag{2.14}$$

The restriction coming from the Ward identities is so severe that the Green's function can be found explicitly, if one assumes, based on hydrodynamic considerations, that the only singularities of the Green's functions are simple poles at $\omega = \pm q/\sqrt{3}$. (For eqs.(2.15) and (2.16) below we neglect the imaginary part in the pole (2.1), which is small compared to the real part when q is small.) Then the Green's functions are

$$G^{\mu\nu\lambda\rho}(\omega, q) = \frac{P}{3\omega^2 - q^2} P^{\mu\nu\lambda\rho}(\omega, q) \, , \tag{2.15}$$

where $P^{\mu\nu\lambda\rho}(\omega, q)$ are polynomials of ω and q that can be found by successively applying the Ward identities,

$$\begin{aligned}
P^{tttt} &= 3(5q^2 - 3\omega^2) \, , & P^{tttz} &= 12\omega q \, , & P^{ttzz} &= 3(q^2 + \omega^2) \, , \\
P^{tztz} &= q^2 + 9\omega^2 \, , & P^{tzzz} &= 4\omega q \, , & P^{zzzz} &= -q^2 + 7\omega^2 \, , \\
P^{ttaa} &= 6(q^2 + \omega^2) \, , & P^{tzaa} &= 8\omega q \, , & P^{zzaa} &= 2(q^2 + \omega^2) \, , \\
P^{aaaa} &= 16\omega^2 \, .
\end{aligned} \tag{2.16}$$

These expressions are valid up to corrections of higher orders in ω and q (negligible in the infrared). We now show that eqs. (2.15) and (2.16) are reproduced from gravity.

3. Gravity picture

According to the gauge theory/gravity correspondence, the near-horizon limit of the non-extremal gravitational background of N black three-branes is dual to the $\mathcal{N}=4$ $SU(N)$ SYM at finite temperature in the limit $N\to\infty$, $g^2_{YM}N\to\infty$. The relevant $5d$ part of the background metric is given by

$$ds^2_{10} = \frac{(\pi T R)^2}{u}\left(-f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{R^2}{4u^2 f(u)}du^2\,, \tag{3.1}$$

where $f(u) = 1-u^2$. We use the same notations and conventions as in [6]. As in [6], we shall consider a small perturbation of the background (3.1), $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$, where we assume $h_{\mu\nu}$ to be dependent only on t and z. There are three types of perturbations (classified by the spin under the O(2) rotations in the xy plane) listed explicitly in ref. [6]. The tensor and the vector perturbations were treated in ref. [6], where it was shown that the calculations for the vector perturbations concides with the hydrodynamic expectation for the diffusive shear modes. Here, we are interested in the scalar perturbations where the only non-zero elements of $h_{\mu\nu}$ are h_{tt}, $h_{xx}=h_{yy}$, h_{zz} and h_{tz}. These perturbations correspond to sound waves in field theory.

3.1 Linearized equations of motion

It will be convenient to use the Fourier decomposition

$$h_{\mu\nu}(t,z) = \int \frac{d\omega\, dq}{(2\pi)^2} e^{-i\omega t + iqz} h_{\mu\nu}(\omega, q, u)\,, \tag{3.2}$$

and to introduce the dimensionless energy and momentum,

$$\mathfrak{w} = \frac{\omega}{2\pi T}\,, \qquad \mathfrak{q} = \frac{q}{2\pi T}\,. \tag{3.3}$$

We work in the gauge $h_{u\mu}=0$ for all μ (including $\mu=u$). Then, to the first order in perturbation, the Einstein equations read

$$H''_{tt} - \frac{3u}{f}H'_{tt} - H''_{ii} + \frac{u}{f}H'_{ii} = 0\,, \tag{3.4a}$$

$$\mathfrak{w}\Big(H'_{ii} + \frac{u}{f}H_{ii}\Big) + \mathfrak{q}\Big(H'_{tz} + \frac{2u}{f}H_{tz}\Big) = 0\,, \tag{3.4b}$$

$$\mathfrak{q}\,(fH'_{tt} - uH_{tt}) + \mathfrak{w}H'_{tz} - \mathfrak{q}fH'_{aa} = 0\,, \tag{3.4c}$$

$$H''_{tt} - \frac{1+u^2}{2uf}(3H'_{tt} + H'_{ii}) - \frac{1}{uf^2}(\mathfrak{q}^2 f H_{tt} + \mathfrak{w}^2 H_{ii} + 2\mathfrak{w}\mathfrak{q}H_{tz}) = 0\,, \tag{3.4d}$$

$$H''_{tz} - \frac{H'_{tz}}{u} + \frac{\mathfrak{w}\mathfrak{q}}{uf} H_{aa} = 0\,, \tag{3.4e}$$

$$H''_{aa} - \frac{2}{uf} H'_{aa} + \frac{H'_{tt} - H'_{zz}}{u} + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f}{uf^2} H_{aa} = 0\,, \tag{3.4f}$$

$$H''_{zz} - \frac{3+u^2}{2uf} H'_{zz} + \frac{H'_{tt}-H'_{aa}}{2u} + \frac{\mathfrak{w}^2 H_{zz} + 2\mathfrak{w}\mathfrak{q} H_{tz} + \mathfrak{q}^2 f(H_{tt}-H_{aa})}{uf^2} = 0, \tag{3.4g}$$

where we have defined $H_{tt} = uh_{tt}/f(\pi TR)^2$, $H_{tz} = uh_{tz}/(\pi TR)^2$, $H_{ij} = uh_{ij}/(\pi TR)^2$, $H_{aa} = H_{xx} + H_{yy}$, $H_{ii} = H_{aa} + H_{zz}$.

Not all equations in the system (3.4) are independent. This can be seen already from the fact that the number of equations (7) is larger than the number of unknown functions (4). Indeed, all equations in (3.4) can be derived from the following system of four equations

$$H''_{tt} - \frac{3u}{f} H'_{tt} - H''_{ii} + \frac{u}{f} H'_{ii} = 0\,, \tag{3.5a}$$

$$\mathfrak{w}\Big(H'_{ii} + \frac{u}{f} H_{ii}\Big) + \mathfrak{q}\Big(H'_{tz} + \frac{2u}{f} H_{tz}\Big) = 0\,, \tag{3.5b}$$

$$\mathfrak{q}\,(fH'_{tt} - uH_{tt}) + \mathfrak{w}H'_{tz} - \mathfrak{q} f H'_{aa} = 0\,, \tag{3.5c}$$

$$H'_{ii} - \frac{3\,fH'_{tt}}{3-u^2} - \frac{2}{f(3-u^2)}\Big[\mathfrak{w}^2 f H_{ii} + 2\mathfrak{w}\mathfrak{q} H_{tz} + \mathfrak{q}^2 f(H_{tt} - H_{aa})\Big] = 0\,. \tag{3.5d}$$

We have checked that the left hand sides of eqs. (3.4) are linear combinations of the left hand sides of eqs. (3.4) and their derivatives. The number of integration constants for eqs. (3.5) is 5, which corresponds to four Dirichlet boundary conditions at $u = 0$ and one condition of incoming waves at $u = 1$.

Equations (3.4) (and hence (3.5)) are invariant under residual gauge transformations, i.e., those gauge transformations which do not break the gauge choice $h_{u\mu} = 0$. These residual gauge transformations are written explicitly in the Appendix. By performing the residual gauge transformations on the trivial solution $H_{\mu\nu} = 0$, one can obtain the pure-gauge solutions to eqs. (3.5). These pure-gauge solutions are linear combinations of H^I, H^{II} and H^{III} whose explicit forms are (only nonzero components are written down)

$$H^I_{tz} = \mathfrak{w}\,, \tag{3.6a}$$

$$H^I_{zz} = -2\,\mathfrak{q}\,, \tag{3.6b}$$

$$H^{II}_{tt} = -2\,\mathfrak{w}\,, \tag{3.7a}$$

$$H^{II}_{tz} = \mathfrak{q}\, f\,, \tag{3.7b}$$

$$H^{III}_{tt} = \frac{1+u^2+2\mathfrak{w}^2 u}{\sqrt{f}}\,, \tag{3.8a}$$

$$H^{III}_{tz} = -\mathfrak{q}\,\mathfrak{w}\,\arcsin u - \mathfrak{q}\,\mathfrak{w}\,u\,\sqrt{f}\,, \tag{3.8b}$$

$$H^{III}_{aa} = -2\,\sqrt{f}\,, \tag{3.8c}$$

$$H^{III}_{zz} = 2\,\mathfrak{q}^2\,\arcsin u - \sqrt{f}\,. \tag{3.8d}$$

As we shall see below, the two remaining independent solutions correspond to the incoming and outgoing waves.

Our first step towards solving eqs. (3.5) is to determine the behavior of the solution near the horizon $u=1$. For this end, it is useful to temporarily abandon eqs. (3.5) and start again from eqs. (3.4), which we rewrite as a system of six first-order differential equations,[1]

$$H'_{tt} = \frac{1}{f}P_{tt}\,, \tag{3.9a}$$

$$H'_{aa} = \frac{1}{f}P_{tt} - \frac{u}{f}H_{tt} + \frac{\mathfrak{w}}{\mathfrak{q}f}P_{tz}\,, \tag{3.9b}$$

$$H'_{ii} = -\frac{u}{f}H_{ii} - \frac{\mathfrak{q}}{\mathfrak{w}}P_{tz} - \frac{2\mathfrak{q}u}{\mathfrak{w}f}H_{tz}\,, \tag{3.9c}$$

$$H'_{tz} = P_{tz}\,, \tag{3.9d}$$

$$P'_{tz} = \frac{1}{u}P_{tz} - \frac{\mathfrak{w}\mathfrak{q}}{uf}H_{aa}\,, \tag{3.9e}$$

$$P'_{tt} = -\frac{u^2-3}{2uf}P_{tt} + \frac{\mathfrak{q}^2}{u}H_{tt} + \frac{\mathfrak{q}(1+u^2)}{2\mathfrak{w}u}P_{tz} + \frac{u+u^3+2\mathfrak{w}^2}{uf}\left[\frac{H_{ii}}{2} + \frac{\mathfrak{w}}{\mathfrak{q}}H_{tz}\right]. \tag{3.9f}$$

In matrix notation, eqs. (3.9) read

$$X' = A(u)\,X\,. \tag{3.10}$$

where $X^T = (H_{tt}, H_{aa}, H_{ii}, H_{tz}, P_{tz}, P_{tt})$, and $A[u]$ is a 6×6 matrix which is singular at the horizon $u=1$.

To find the indices characterizing the behavior of the solution there, one substitutes the ansatz $X = (1-u)^r F[u]$ into (3.10) and matches the coefficients of the leading

[1] The sixth spurious equation is needed to avoid an irregular singularity. The spurious solution can be eliminated at the end by a direct check.

singular terms on both sides. This amounts to finding the eigenvalues of the matrix $\bar{A} = -\lim_{u\to 1}(1-u)A[u]$. The six eigenvalues are $r_1 = r_2 = 0$, $r_3 = i\omega/2$, $r_4 = -i\omega/2$, $r_5 = -1/2$, $r_6 = 1/2$. Five of the corresponding eigenvectors are given by $F_1 = (0, 0, -2\mathfrak{q}, \mathfrak{w}, 0, 0)$, $F_2 = (\mathfrak{w}, 0, 0, 0, \mathfrak{q}, 0)$, $F_3 = (1, 0, 0, 0, 0, 1)$, $F_4 = (0, i, 0, 0, \mathfrak{q}, 0)$, $F_5 = (0, -i, 0, 0, \mathfrak{q}, 0)$. Then a linear combination of $X_k = (1-u)^{r_k} F_k$, $k = 1, ...5$, represents a local solution near the horizon. The sixth eigenvector, $F_6 = (1+\mathfrak{w}^2, 2, 2, 0, 2\mathfrak{w}\mathfrak{q}, -1-\mathfrak{w}^2)$, is a spurious one (one can check that $\sqrt{1-u}F_6$ is not a local solution of eqs. (3.9)), and should be discarded. The eigenvectors with $r = 0, 0, -1/2$ correspond to the three independent pure gauge solutions given explicitly in eqs. (3.6), (3.7) and (3.8), respectively. The solutions with indices $\mp i\omega/2$ represent incoming/outgoing waves.

Having found the indices and local solutions near the horizon, we can now return to eqs. (3.5) and solve them perturbatively in the hydrodynamic regime $\mathfrak{w} \ll 1$ and $\mathfrak{q} \ll 1$. Our knowledge of the local behavior of the solutions near the horizon allows us to isolate the solution corresponding to the incoming wave. To the second order in $\mathfrak{w}$ and $\mathfrak{q}$ it is given by $H^{inc} = (H^{inc}_{tt}, H^{inc}_{ii}, H^{inc}_{tz}, H^{inc}_{aa})$, where

$$H^{inc}_{tt} = \frac{\mathfrak{q}^2}{3}(1-u) \,, \tag{3.11a}$$

$$H^{inc}_{ii} = \mathfrak{q}^2(1-u) \,, \tag{3.11b}$$

$$H^{inc}_{tz} = -\frac{i\mathfrak{q}}{2}\left(1-u^2\right) + \frac{\mathfrak{w}\mathfrak{q}}{2}u(1-u) - \frac{\mathfrak{w}\mathfrak{q}}{4}\left(1-u^2\right)\ln\frac{2(1-u)}{1+u} \,, \tag{3.11c}$$

$$\begin{aligned} H^{inc}_{aa} = {} & 1 - \frac{i\mathfrak{w}}{2}\ln\frac{1-u^2}{2} - \frac{\mathfrak{w}^2}{8}\ln^2(1-u) - \frac{\mathfrak{w}^2}{4}\ln(1-u)\ln\frac{1+u}{2} \\ & + \frac{2\mathfrak{q}^2}{3}(1-u) + \frac{3\mathfrak{w}^2+\mathfrak{q}^2}{3}\ln\frac{1+u}{2} + \frac{\mathfrak{w}^2}{8}\ln^2\frac{1+u}{2} - \frac{\mathfrak{w}^2}{2}\mathrm{Li}_2\frac{1-u}{2} \,. \end{aligned} \tag{3.11d}$$

Thus, to the second order in $\mathfrak{w}$ and $\mathfrak{q}$, the most general solution to eqs. (3.5) which satisfies the incoming-wave boundary condition at the horizon is given by

$$H(u) = a\,H^{inc}(u) + b\,H^{I}(u) + c\,H^{II}(u) + d\,H^{III}(u) \,, \tag{3.12}$$

where a, b, c and d are constants, and $H^{I,II,III}$ are the pure-gauge solutions given in eqs. (3.6), (3.7) and (3.8). The constants a, b, c and d are determined from the Dirichlet conditions at the boundary $u = \epsilon \to 0$ and can be expressed in terms of the boundary values H^0_{tt}, H^0_{aa}, H^0_{ii}, H^0_{tz}.

3.2 The action

The action is given by the sum of three terms,

$$S = S_M + S^{(1)}_{\partial M} + S^{(2)}_{\partial M} \,, \tag{3.13}$$

where $S^{(1)}_{\partial M}$ is the Gibbons-Hawking boundary term, and $S^{(2)}_{\partial M}$ is proportional to the volume of the boundary. Explicitly,

$$S = \frac{\pi^3 R^5}{2\kappa_{10}^2}\left[\int_0^1 du\, d^4x\,\sqrt{-g}\,(\mathcal{R}-2\Lambda) + 2\int d^4x\,\sqrt{-h}K + a\int d^4x\,\sqrt{-h}\right]. \qquad (3.14)$$

Here $\kappa_{10} = \sqrt{8\pi G}$ is the ten-dimensional gravitational constant, related to the parameter R of the non-extremal geometry and the number N of coincident branes by $\kappa_{10} = 2\pi^2\sqrt{\pi}R^4/N$ [8]. Following ref. [9], we choose $a = -6/R$ to cancel the volume divergence in eq. (3.14). On shell, the action reduces to the surface terms, $S = S_{horizon} + S_\epsilon$, where

$$\begin{aligned} S_\epsilon = \frac{\pi^2 N^2 T^4}{8}\int d^4x \Bigg[& -1 + \frac{1}{2}\left(3H_{tt} + H_{ii}\right) \\ & + \frac{1}{8}\left(3H_{tt}^2 - 12H_{tz}^2 + 2H_{tt}H_{ii} + 2H_{zz}H_{aa} - H_{zz}^2\right) \\ & - \frac{1}{2\epsilon}\left(H_{tz}^2 + \frac{1}{4}H_{aa}^2 - H_{tt}H_{ii} + H_{zz}H_{aa}\right)'\Bigg]. \end{aligned} \qquad (3.15)$$

We can now substitute our solution (3.12) into eq. (3.15) and compute the correlators. At leading order, we retain only terms linear in $\mathfrak{w}$ and $\mathfrak{q}$ in eq. (3.12). We have[2]

$$\begin{aligned} S_\epsilon = \frac{\pi^2 N^2 T^4}{8}\Bigg[& -V_4 + \frac{1}{2}\left(3H^0_{tt}(0) + H^0_{ii}(0)\right) \\ & + \frac{1}{2(\mathfrak{q}^2 - 3\mathfrak{w}^2)}\left(\left(2\mathfrak{q}H^0_{tz} + \mathfrak{w}H^0_{ii}\right)^2 + H^0_{tt}\left(3\mathfrak{q}^2 H^0_{tt} + 12\mathfrak{q}\mathfrak{w}H^0_{tz} + (3\mathfrak{w}^2 + \mathfrak{q}^2)H^0_{ii}\right)\right) \\ & + \frac{1}{8}\left(3(H^0_{tt})^2 - 12(H^0_{tz})^2 + 2H^0_{tt}H^0_{ii} + 2H^0_{zz}H^0_{aa} - (H^0_{zz})^2\right)\Bigg]. \end{aligned} \qquad (3.16)$$

The constant term, $-\pi^2 N^2 T^4 V_4/8$, is the free energy density (i.e., the pressure with a minus sign) [10], times the four-volume.

3.3 The correlators

As noticed in ref. [5], in general it is not possible to calculate the Minkowski two-point Green's function by differentiating the gravitational action with respect to the

[2] Terms quadratic in H in (3.16) should be understood as products $H(\omega, \boldsymbol{q})H(-\omega, -\boldsymbol{q})$, and integration over ω and $\boldsymbol{q}$ is implied.

boundary values of fields, since the result of this differentiation is necessarily real while the retarded Green's function is in general complex. In other words, in Minkowski AdS/CFT, the boundary action in general fails to be a generating functional for the Green's functions, and a prescription formulated in ref. [5] should be used to obtain the correct results. We remark, however, that to leading order this problem does not arise, because the correlators are real (cf. eqs. (2.15) and (2.16)). Thus S_ϵ in eq. (3.16) can be used as the generating functional to this order.[3] The coefficient of proportionality is easily determined by using the naive equality

$$\langle e^{i\int \phi_0 \mathcal{O}} \rangle = e^{iS_\epsilon} \,. \tag{3.17}$$

In our case, the boundary value of the metric perturbation h^i_j acts as a source for the stress-energy tensor [9], with the coupling given by

$$\frac{1}{2}\int dt\, d^3x h^\nu_\mu T^\mu_\nu = \frac{1}{2}\int dt\, d^3x \left(H^0_{tt} T^{tt} + \frac{1}{2} H^0_{aa} T^{aa} + H^0_{zz} T^{zz} + 2 H^0_{tz} T^{tz} \right) . \tag{3.18}$$

Taking the derivatives of S_ϵ with the normalization ensured by (3.18), one finds the following one-point functions

$$\langle T^{tt} \rangle = 2 \frac{\delta S_\epsilon}{\delta H^0_{tt}} = \frac{3\pi^2}{8} N^2 T^4 \,, \tag{3.19a}$$

$$\langle T^{aa} \rangle = 4 \frac{\delta S_\epsilon}{\delta H^0_{aa}} = \frac{\pi^2}{4} N^2 T^4 \,, \tag{3.19b}$$

$$\langle T^{zz} \rangle = 2 \frac{\delta S_\epsilon}{\delta H^0_{zz}} = \frac{\pi^2}{8} N^2 T^4 \,. \tag{3.19c}$$

These results are in complete agreement with the known thermodynamics of our theory: indeed, $\langle T^{tt} \rangle = \epsilon = 3P$, $\langle T^{aa} \rangle = \langle T^{xx} + T^{yy} \rangle = 2P$ and $\langle T^{zz} \rangle = P$ with the pressure obtained in ref. [8], $P = \pi^2 N^2 T^4/8$.

The two-point functions easily follow from S_ϵ by taking into account eq. (3.17) and eq. (3.18). The results are in complete agreement with hydrodynamics, eqs. (2.15) and (2.16). For example,

$$G^{tttt}(\omega, q) = -4 \frac{\delta^2 S_\epsilon}{\delta (H^0_{tt})^2} = \frac{3N^2\pi^2 T^4 q^2}{2(3\omega^2 - q^2)} - \frac{3\pi^2}{8} N^2 T^4 = 3P \frac{5q^2 - 3\omega^2}{3\omega^2 - q^2} \,. \tag{3.20}$$

Other correlators are computed similarly. The singularity structures, and more unexpectedly, all contact terms are correctly reproduced by gravity. These formulas are

[3] As discussed in ref. [5], the surface term $S_{horizon}$ must be dropped.

valid up to the corrections of order $O(\mathfrak{w}^2, \mathfrak{q}^2, \mathfrak{w}\mathfrak{q})$ (the corrections are discussed below in section 3.4). We observe that the correlation functions exhibit a pole corresponding to the propagation of the sound wave in the hot $\mathcal{N} = 4$ SYM plasma, with the correct value for the speed of sound, $v_s = 1/\sqrt{3}$. It is truly amazing that the *quantitative* prediction of field theory follows from the 5d non-supersymmetric gravity background. This result boosts our confidence in the validity of the gauge/gravity correspondence in general.

Since the Green's functions coincide with the forms expected from hydrodynamics, they automatically satisfy the Ward identities (2.10)–(2.11). This is the consequence of the invariance of the boundary gravitational action (3.16), which serves as the generating functional,[4] under the gauge transformations (A.14), (A.15) and (A.16)

3.4 Sound attenuation

To find the imaginary part of the dispersion relation (2.1), one has to perform calculation beyond leading order. That is done by using the full solution (3.12) and keeping the terms quadratic in $\mathfrak{w}$ and $\mathfrak{q}$. We have to follow the prescription of ref. [5], since to this order in $\mathfrak{w}$, $\mathfrak{q}$ the boundary action S_ϵ is no longer a generating functional for the Green's functions.[5]

The resulting analytic expressions for the correlators are rather cumbersome, and will not be presented here. The real and imaginary parts of a typical correlator are plotted in figs. 1,2. We will discuss only the location of the pole to this next-to-leading order. It is now given by the equation

$$\mathfrak{w}^2 - \frac{\mathfrak{q}^2}{3} + \frac{2i}{3}\,\mathfrak{w}\,\mathfrak{q}^2 - \frac{i\mathfrak{w}\log 2}{6}\left(\mathfrak{q}^2 - 3\,\mathfrak{w}^2\right) = 0\,. \tag{3.21}$$

Near the pole we have $\mathfrak{w} \approx \mathfrak{q}/\sqrt{3}$. Thus, in this region the last term in (3.21) is of order $\mathfrak{w}^2\mathfrak{q}^2$, and can be neglected. We obtain

$$\mathfrak{w} = \frac{\mathfrak{q}}{\sqrt{3}} - \frac{i\mathfrak{q}^2}{3} + O(\mathfrak{q}^3)\,, \tag{3.22}$$

or, in terms of original variables,

$$\omega = \frac{q}{\sqrt{3}} - \frac{iq^2}{6\pi T} + O(q^3)\,. \tag{3.23}$$

This is in complete agreement with hydrodynamics, which predicts (see eq. (2.1) and recall that $\zeta = 0$ in conformal theories) that the imaginary part of the sound-wave

[4] With the reservations discussed above.

[5] We have not checked the Ward identities in this approximation.

dispersion relation is

$$-\frac{2i}{3}\frac{\eta}{\epsilon+P}q^2 = -\frac{iq^2}{6\pi T} \tag{3.24}$$

where we have used the value of $\eta/(\epsilon+P) = (4\pi T)^{-1}$ computed in refs. [11, 6].

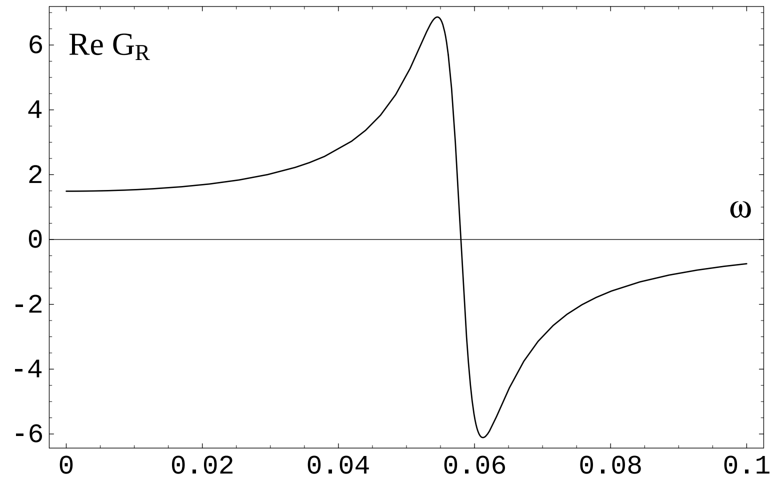

Figure 1: The real part of the correlator $G^R_{tt,\,tt}(\mathfrak{w},\mathfrak{q})$ as a function of (real) $\mathfrak{w}$ at $\mathfrak{q} = 1/10$.

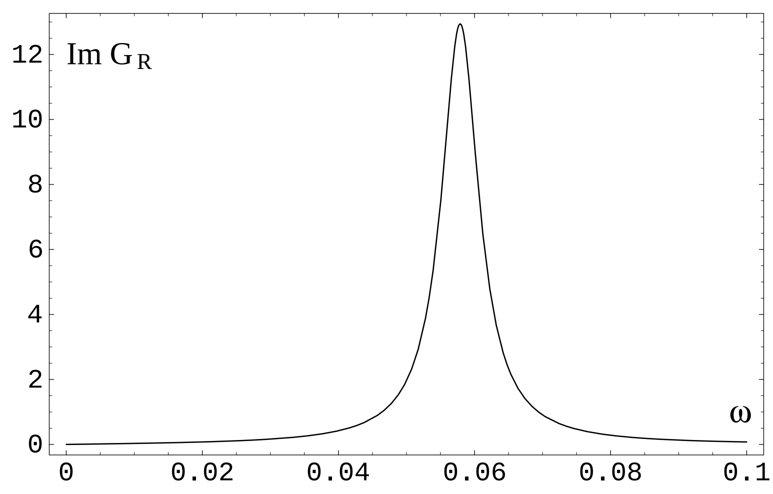

Figure 2: The imaginary part of the correlator $G^R_{tt,\,tt}(\mathfrak{w},\mathfrak{q})$ as a function of (real) $\mathfrak{w}$ at $\mathfrak{q} = 1/10$. The peak corresponds to the (attenuated) pole at $\mathrm{Re}\,\mathfrak{w} = \mathfrak{q}/\sqrt{3} \approx 0.0577$.

4. Conclusion

In this paper we have demonstrated how sound waves emerge from gravity, with a sound speed in agreement with field-theoretical expectations. Together with the result of our previous work [6] we have found that the AdS/CFT correspondence is capable of reproducing the hydrodynamic regime of the correlators in $\mathcal{N} = 4$ SYM theory at finite temperature, which is another testimony to the validity of the correspondence.

The calculations of ref. [6] has been extended in ref. [12] to M2 and M5 branes of M-theory, where hydrodynamic behavior is also observed, although the worldvolume theories in these cases are not well understood.[6] It should be possible to find the sound waves in these cases too.

It would be interesting to investigate whether the hydrodynamic behaviors observed so far are generic for all black p-branes solutions, i.e., are inherent properties of general relativity (as, for example, the entropy), or are specific properties of metrics which have dual field-theoretical description.

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A. Gauge transformations

The requirement that the infinitesimal diffeomorphism

$$x^\mu \to x^\mu + \xi^\mu \,, \tag{A.1}$$

$$g_{\mu\nu} \to g_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu \tag{A.2}$$

preserves the gauge conditions $h_{u\mu} = 0$ leads to the equation

$$\partial_\mu \xi_u + \partial_u \xi_\mu - 2\Gamma^\rho_{\mu u} \xi_\rho = 0 \,, \tag{A.3}$$

which constrains possible choices of ξ_μ for residual gauge transformations. Since the covariant derivative in eq. (A.2) is taken with respect to the full metric, the residual

[6] Methods for computing the absorption cross section by non-extremal D- and M-branes were discussed in detail in [13].

gauge transformations are dependent on the fluctuations $h_{\mu\nu}$. It will be useful to introduce a book-keeping parameter κ associated with the fluctuations $h_{\mu\nu}$ and expand both ξ_μ and the Christoffel symbols in series over κ,

$$\xi_\mu = \xi_\mu^{(0)} + \kappa\, \xi_\mu^{(1)} + \cdots\,, \tag{A.4}$$

$$\Gamma = \Gamma^0 + \kappa\, \Gamma^{(1)} + \cdots\,. \tag{A.5}$$

Then, to the linear order in fluctuations, a general residual gauge transformation is linear combinations of the following three types of transformations:

1) The transformations generated by

$$\xi_z = \frac{C_z(t,z)}{u} + \xi_z^{(1)}(u,t,z)\,, \tag{A.6a}$$

$$\xi_t = \xi_t^{(1)}(u,t,z)\,, \tag{A.6b}$$

where $\xi_z^{(1)}$ and $\xi_t^{(1)}$ satisfy, respectively,

$$\partial_u \xi_z^{(1)} + \frac{1}{u}\xi_z^{(1)} = \frac{C_1 H'_{zz}}{u}\,, \tag{A.7a}$$

$$\partial_u \xi_t^{(1)} + \frac{1+u^2}{uf}\xi_t^{(1)} = \frac{fC_1}{u}\left(\frac{H_{tz}}{f}\right)'\,, \tag{A.7b}$$

They are given by $H_{\mu\nu} \to H_{\mu\nu} + H^{gauge}_{I\,\mu\nu}$, where the only nonzero terms in $H^{gauge}_{I\,\mu\nu}$ are[7]

$$H^{gauge}_{I\,tt} = \frac{2\mathfrak{w}u}{f}\xi_t^{(1)} - \mathfrak{q}\, C_1 H_{tt} - \frac{2\,\mathfrak{w}\, C_1}{f} H_{tz}\,, \tag{A.8a}$$

$$H^{gauge}_{I\,tz} = \mathfrak{w} C_1 + u\,\mathfrak{w}\,\xi_z^{(1)} - u\mathfrak{q}\,\xi_t^{(1)} - \mathfrak{w}\, C_1 H_{zz}\,, \tag{A.8b}$$

$$H^{gauge}_{I\,aa} = -\mathfrak{q}\, C_1 H_{aa}\,, \tag{A.8c}$$

$$H^{gauge}_{I\,zz} = -2\mathfrak{q} C_1 - 2\,u\,\mathfrak{q}\,\xi_z^{(1)} + \mathfrak{q}\, C_1 H_{zz}\,. \tag{A.8d}$$

Here and later we use $C_1 = i(2\pi T)C_z$, $C_2 = i(2\pi T)C_t$ and $C_3 = i(2\pi T)C_u$.

2) The transformations generated by

$$\xi_t = -\frac{fC_t(t,z)}{u} + \xi_t^{(1)}(u,t,z)\,, \tag{A.9a}$$

$$\xi_z = \xi_z^{(1)}(u,t,z)\,, \tag{A.9b}$$

[7] In all formulas in this Appendix products involving H_{ij} should be understood as convolutions. For example, $(\omega\, C_1\, H_{zz})(\omega, q) = \int \omega'\, C_1(\omega', q')\, H_{zz}(\omega-\omega', q-q')\, d\omega' dq'$.

where $\xi_t^{(1)}$ and $\xi_z^{(1)}$ obey the following inhomogeneous equations

$$\partial_u \xi_t^{(1)} + \frac{1+u^2}{uf}\xi_t^{(1)} = \frac{fC_2}{u} H'_{tt}\,, \tag{A.10a}$$

$$\partial_u \xi_z^{(1)} + \frac{1}{u}\xi_z^{(1)} = \frac{C_2 H'_{tz}}{u}\,. \tag{A.10b}$$

To the linear order in κ, the transformations are given by $H_{ij} \to H_{ij} + H^{gauge}_{II\,ij}$, where

$$H^{gauge}_{II\,tt} = -2\,\mathfrak{w}\,C_2 + \frac{2\mathfrak{w}u}{f}\xi_t^{(1)} - \mathfrak{w}\,C_2\,H_{tt}\,, \tag{A.11a}$$

$$H^{gauge}_{II\,tz} = \mathfrak{q}\,f\,C_2 - u\,\mathfrak{q}\,\xi_t^{(1)} + u\,\mathfrak{w}\,\xi_z^{(1)} + \mathfrak{q}\,f\,C_2\,H_{tt}\,, \tag{A.11b}$$

$$H^{gauge}_{II\,aa} = \mathfrak{w}\,C_2\,H_{aa}\,, \tag{A.11c}$$

$$H^{gauge}_{II\,zz} = -2\,u\,\mathfrak{q}\,\xi_z^{(1)} + 2\,\mathfrak{q}\,C_2\,H_{tz} + \mathfrak{w}\,C_2\,H_{zz}\,. \tag{A.11d}$$

3) A third group of transformations is generated by

$$\xi_u = \frac{C_u(t,z)}{u\sqrt{f}}\,, \tag{A.12a}$$

$$\xi_z = -\partial_z C_u(t,z)\frac{\arcsin u}{u}\,, \tag{A.12b}$$

$$\xi_t = -\partial_t C_u(t,z)\sqrt{f}\,, \tag{A.12c}$$

and is given by $H_{ij} \to H_{ij} + H^{gauge}_{III\,ij}$, where

$$\begin{aligned} H^{gauge}_{III\,tt} =& \frac{C_3\,(1+u^2+2\mathfrak{w}^2 u)}{\sqrt{f}} + \frac{C_3\,u^2}{\sqrt{f}}\left(\frac{f H_{tt}}{u}\right)' \\ &+ \frac{C_3\,u\,\mathfrak{w}^2\,H_{tt}}{\sqrt{f}} + \frac{C_3\,\arcsin u}{f}\left(\mathfrak{q}^2 f H_{tt} + 2\,\mathfrak{q}\,\mathfrak{w}\,H_{tz}\right)\,, \end{aligned} \tag{A.13a}$$

$$\begin{aligned} H^{gauge}_{III\,tz} =& -C_3\,\mathfrak{q}\,\mathfrak{w}\,(1-H_{zz})\arcsin u + C_3\,u^2\,\sqrt{f}\,\left(\frac{H_{tz}}{u}\right)' \\ &- C_3\,\mathfrak{q}\,\mathfrak{w}\,u\,\sqrt{f}\,(1+H_{tt})\,, \end{aligned} \tag{A.13b}$$

$$\begin{aligned} H^{gauge}_{III\,aa} =& -2\,C_3\,\sqrt{f} - \frac{C_3\,u\,\mathfrak{w}^2\,H_{aa}}{\sqrt{f}} + u^2\,\sqrt{f}\,C_3\left(\frac{H_{aa}}{u}\right)' \\ &+ \mathfrak{q}^2\,C_3\,H_{aa}\arcsin u\,, \end{aligned} \tag{A.13c}$$

$$\begin{aligned} H^{gauge}_{III\,zz} =& C_3\,\mathfrak{q}^2\,\arcsin u\,(2-H_{zz}) - C_3\sqrt{f} + u^2\,\sqrt{f}C_3\left(\frac{H_{zz}}{u}\right)' \\ &- \frac{C_3 u}{\sqrt{f}}\left(2\,\mathfrak{q}\,\mathfrak{w}\,H_{tz} + \mathfrak{w}^2\,H_{zz}\right)\,. \end{aligned} \tag{A.13d}$$

At the boundary, the gauge transformations to the first order in $\mathfrak{w}$, $\mathfrak{q}$, and to the linear order in fluctuations, reduce to the following three independent sets:
Set I:

$$H_{tt}^0 \to H_{tt}^0 - \mathfrak{q}\, C_1\, H_{tt}^0 - 2\,\mathfrak{w}\, C_1\, H_{tz}^0\,, \tag{A.14a}$$

$$H_{tz}^0 \to H_{tz}^0 + \mathfrak{w}\, C_1\, - \mathfrak{w}\, C_1\, H_{zz}^0\,, \tag{A.14b}$$

$$H_{aa}^0 \to H_{aa}^0 - \mathfrak{q}\, C_1\, H_{aa}^0\,, \tag{A.14c}$$

$$H_{zz}^0 \to H_{zz}^0 - 2\,\mathfrak{q}\, C_1 + \mathfrak{q}\, C_1\, H_{zz}^0 \tag{A.14d}$$

Set II:

$$H_{tt}^0 \to H_{tt}^0 - 2\mathfrak{w}\, C_2 - \mathfrak{w}\, C_2\, H_{tt}^0\,, \tag{A.15a}$$

$$H_{tz}^0 \to H_{tz}^0 + \mathfrak{q}\, C_2 + \mathfrak{q}\, C_2\, H_{tt}^0\,, \tag{A.15b}$$

$$H_{aa}^0 \to H_{aa}^0 + \mathfrak{w}\, C_2\, H_{aa}^0\,, \tag{A.15c}$$

$$H_{zz}^0 \to H_{zz}^0 + 2\mathfrak{q}\, C_2\, H_{tz}^0 + \mathfrak{w}\, C_2\, H_{zz}^0\,, \tag{A.15d}$$

Set III:

$$H_{tt}^0 \to H_{tt}^0 + C_3 - C_3 H_{tt}^0\,, \tag{A.16a}$$

$$H_{tz}^0 \to H_{tz}^0 - C_3 H_{tz}^0\,, \tag{A.16b}$$

$$H_{aa}^0 \to H_{aa}^0 - 2C_3 - C_3 H_{aa}^0\,, \tag{A.16c}$$

$$H_{zz}^0 \to H_{zz}^0 - C_3 - C_3 H_{zz}^0\,. \tag{A.16d}$$

From the point of view of four dimensions, eqs. (A.14) and (A.15) are diffeomorphisms, while eqs. (A.16) correspond to dilatation when C_3 is constant. One can check that to the linear order in fluctuations, the generating functional (3.16) is invariant under the gauge transformations of Sets I, II and III.

References

[1] J. Maldacena, *The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys.* **2** (1998) 231 [`hep-th/9711200`].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory, Phys. Lett.* **B 428** (1998) 105 [`hep-th/9802109`].

[3] E. Witten, *Anti de Sitter space and holography, Adv. Theor. Math. Phys.* **2** (1998) 253 [`hep-th/9802150`].

[4] L.D. Landau and E.M. Lifshitz, *Fluid mechanics*, Pergamon Press, New York 1987, 2nd ed.

[5] D.T. Son and A.O. Starinets, *Minkowski space correlators in AdS/CFT correspondence: recipe and applications, J. High Energy Phys.* **09** (2002) 42 [hep-th/0205051].

[6] G. Policastro, D.T. Son and A.O. Starinets, *From AdS/CFT correspondence to hydrodynamics, J. High Energy Phys.* **09** (2002) 43 [hep-th/0205052].

[7] L.G. Yaffe, *Hydrodynamic fluctuations in relativistic theories* (1992), unpublished.

[8] S.S. Gubser, I.R. Klebanov and A.W. Peet, *Entropy and temperature of black 3-branes, Phys. Rev.* **D 54** (1996) 3915 [hep-th/9602135].

[9] H. Liu and A.A. Tseytlin, $D = 4$ *super Yang-Mills,* $D = 5$ *gauged supergravity and* $D = 4$ *conformal supergravity, Nucl. Phys.* **B 533** (1998) 88 [hep-th/9804083].

[10] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *Coupling constant dependence in the thermodynamics of* $\mathcal{N} = 4$ *supersymmetric Yang-Mills theory, Nucl. Phys.* **B 534** (1998) 202 [hep-th/9805156].

[11] G. Policastro, D.T. Son and A.O. Starinets, *Shear viscosity of strongly coupled* $\mathcal{N} = 4$ *supersymmetric Yang-Mills plasma, Phys. Rev. Lett.* **87** (2001) 081601 [hep-th/0104066].

[12] C.P. Herzog, *The hydrodynamics of M-theory,* hep-th/0210126.

[13] G. Policastro and A. Starinets, *On the absorption by near-extremal black branes, Nucl. Phys.* **B 610** (2001) 117 [hep-th/0104065].